# Spintronics in Nanostructures (16877-01)

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| | days | time | place |
|---|---|---|---|
| Lectures | **every Wednesday** | **10:15-12:00** | **Room 4.1** (Institute of Physics) |
| Exercises | **every Tuesday** | **10:00-11:00** | **Room 4.1** (Institute of Physics) |

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## I. SYMMETRY IN SOLID STATE PHYSICS

### Lecture 1

#### 1. Introduction to Symmetry in Physics

Symmetry is most basic and important concept in physics. Every process in physics is governed by selection rules that are the consequence of symmetry requirements. For example, **momentum conservation** is a consequence of translational symmetry of space ($\mathbf{r} \to \mathbf{r} + \mathbf{a}$, $\mathbf{a} \in \mathbb{R}^3$), **energy conservation** is due to translation symmetry of time ($t \to t + \tau$, $\tau \in \mathbb{R}$), and **angular momentum conservation** is the consequence of spacial rotation invariance. These are so called **continuous symmetries**. There are **discrete symmetries**: time reversal symmetry ($t \to -t$), spacial inversion symmetry ($\mathbf{r} \to -\mathbf{r}$), charge reversal symmetry ($q \to -q$), etc.

In quantum mechanics eigenstate properties of a Hamiltonian (or an other operator which describe a quantum system) and the degeneracy of eigenvalues are governed by symmetry considerations. Beauty of theory group is that we can translate the heavy and complex language of symmetry operations into a language of a very simple linear algebra.

#### 2. Basics of Group Theory

**Definition 1.** ***A group*** *$G$ is a (finite or infinite) set of elements $g_1, g_2, g_3, \ldots$ having the following properties:*
*1) The product of any two elements of the group is itself an element of the group: $\forall g_1, g_2 \in G\, \exists g_3 \in G:\ g_1 g_2 = g_3$.*
*2) The associative law of multiplication holds: $g_1(g_2 g_3) = (g_1 g_2) g_3$.*
*3) Among the elements of a group there is one and only one element, called the identity of unit element $e$, which has the property $ge = eg = g\ \forall g \in G$.*
*4) Each group element $g$ has an inverse element $g^{-1}$ such that $gg^{-1} = e$: $\forall g \in G, \exists g^{-1} \in G:\ gg^{-1} = e$.*

In general, the elements of a group do not commute: $g_1 g_2 \neq g_2 g_1$.

**Definition 2.** *Group $G_A$ in which $g_1 g_2 = g_2 g_1\ \forall g_1, g_2 \in G_A$ called as* ***an Abelian group****.*

**Definition 3.** ***Order of a group*** *$G$, denoted as $|G|$, is the number of elements in the set $G$.*

**Definition 4.** ***A subgroup*** *is a collection of elements within a group that by themself form a group.*

**Theorem 1.** *If in a finite group an element $x$ is multiplicated by itself enough times ($n$), the identity $x^n = e$ is eventually recovered.*

**Definition 5.** ***The order of an element*** *$x$ is the smallest value of $n$ such that $x^n = e$.*

**Definition 6.** *Each element of a finite group may be represented as a power or product of powers of a certain finite number of elements called* ***generators*** *of the group.*

**Definition 7.** *An element $b \in G$* ***conjugate to*** *$a \in G$ is by definition $b = xax^{-1}\ \exists x \in G$.*

**Definition 8.** ***A class*** *is the total set of conjugated elements.*

**Theorem 2.** *All elements of the same class have the same order.*

**Definition 9.** *A group consisting of elements $e, a, a^2, a^3, \ldots, a^{n-1}$ ($a^n = e$) is said to be* ***cyclic group****.*

## 3. Point Groups

Any transformation which brings a body into coincidence with itself can be decomposed into elementary transformations:
1) rotation about an axis,
2) reflection in a plane,
3) translation by some vector.

The symmetry group of a body of finite dimensions cannot contain a translation, since for any finite body there must be a fixed point, namely, the center mass.

**Definition 10.** *A symmetry group in which there is a fixed point common to all transformations of the group is called* ***a point group****.*

Here is the list of all "elementary" elements of point groups:
$e$ is the identity,
$c_n$ is the rotation through $2\pi/n$ angle,
$\sigma$ is the reflection in a plane,
$\sigma_h$ is the reflection in a *horizontal* plane (perpendicular to the axis of highest rotational symmetry),
$\sigma_v$ is the reflection in a *vertical* plane (parallel to that axis),
$\sigma_d$ is the diagonal reflection in a plane through the origin and the axis with the 'highest' symmetry, but also bisecting the
$i$ is the inversion $(x \to -x, y \to -y, z \to -z)$,
$s_n$ is the improper rotation through $2\pi/n$ angle ($s_n = \sigma_h c_n$ is the rotation followed by reflection in horizontal plane),
$ic_n$ is the compound rotation-inversion.

There are only **fourteen types** of finite point groups: $C_n, S_{2n}, C_{nh}, C_{nv}, D_n, D_{nh}, D_{nd}, T, T_d, T_h, O, O_h, Y, Y_h$.