Departement of Physics, University of Basel

Dr. Rakesh P. Tiwari (rakesh.tiwari@unibas.ch)

Nanophysics — Fall 2016

Exercise 1

(1) **Entangled Bell states**

In the lecture we defined the Bell states as

$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$,

$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$,

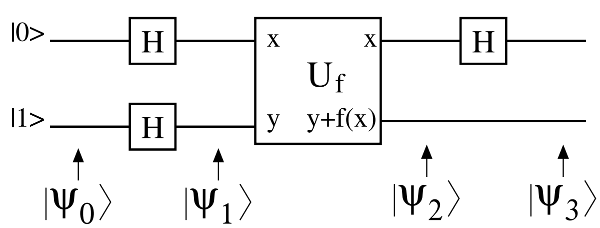
$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$,

$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.

Imagine that one of the qubits of a Bell state is sent to Alice and the other one from the same Bell state is sent to Bob. If both Alice and Bob apply a Hadamard gate H to their qubit, show that two of the Bell states are interchanged, and two of the states are left unchanged by this transformation. Remember: $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(2) **Deutsch's Algorithm**

Consider a binary classical function $f(x) : \{0,1\} \rightarrow \{0,1\}$. A quantum circuit that implements Deutsch's Algorithm is shown below



where $U_f : |x, y\rangle \rightarrow |x, y + f(x)\rangle$. The input state $|\psi_0\rangle = |01\rangle$.

(a) Write down the state $|\psi_1\rangle$ obtained when the input state is sent through two Hadamard gates (H).

(b) Show that

$$|\psi_2\rangle = \begin{cases} \pm\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) & \text{if } f(0) = f(1) \\ \pm\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) & \text{if } f(0) \neq f(1) \end{cases}, \qquad (1)$$

(c) Show that $|\psi_3\rangle = \pm|f(0) \oplus f(1)\rangle \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$, as mentioned in the lecture.

(3) **Two qubit gate**

The SWAP-gate is defined through the relation

$$U_{\text{SWAP}}|\psi_1\psi_2\rangle = |\psi_2\psi_1\rangle,$$

where two qubits swap their states. This two-qubit gate is not by itself sufficient for universal quantum computation, but if the gate is pulsed for half-a-period, the resulting 'square-root-of-SWAP' (or $\sqrt{\text{SWAP}} \equiv U_{\text{SWAP}}^{1/2}$) becomes useful since one can obtain an XOR-gate (up to some single-qubit rotations).

(a) Show that the matrix form of the $U_{\text{SWAP}}^{1/2}$ takes the following form in the computational basis ($|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$):

$$U_{\text{SWAP}}^{1/2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(1+i) & \frac{1}{2}(1-i) & 0 \\ 0 & \frac{1}{2}(1-i) & \frac{1}{2}(1+i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Using the state $|\psi\rangle = |10\rangle$ as an input of $\sqrt{\text{SWAP}}$ gate, what is the output state? Are the input and/or output states entangled?

(c) Repeat (b) using states $|\psi\rangle = |00\rangle$, $|\psi\rangle = |01\rangle$ and $|\psi\rangle = |11\rangle$ as input states.