Majorana fermions in a ferromagnetic wire on the surface of a bulk spin-orbit coupled *s*-wave superconductor

Hoi-Yin Hui, P. M. R. Brydon, Jay D. Sau, S. Tewari,
and S. Das Sarma
arXiv:1407.7519v1

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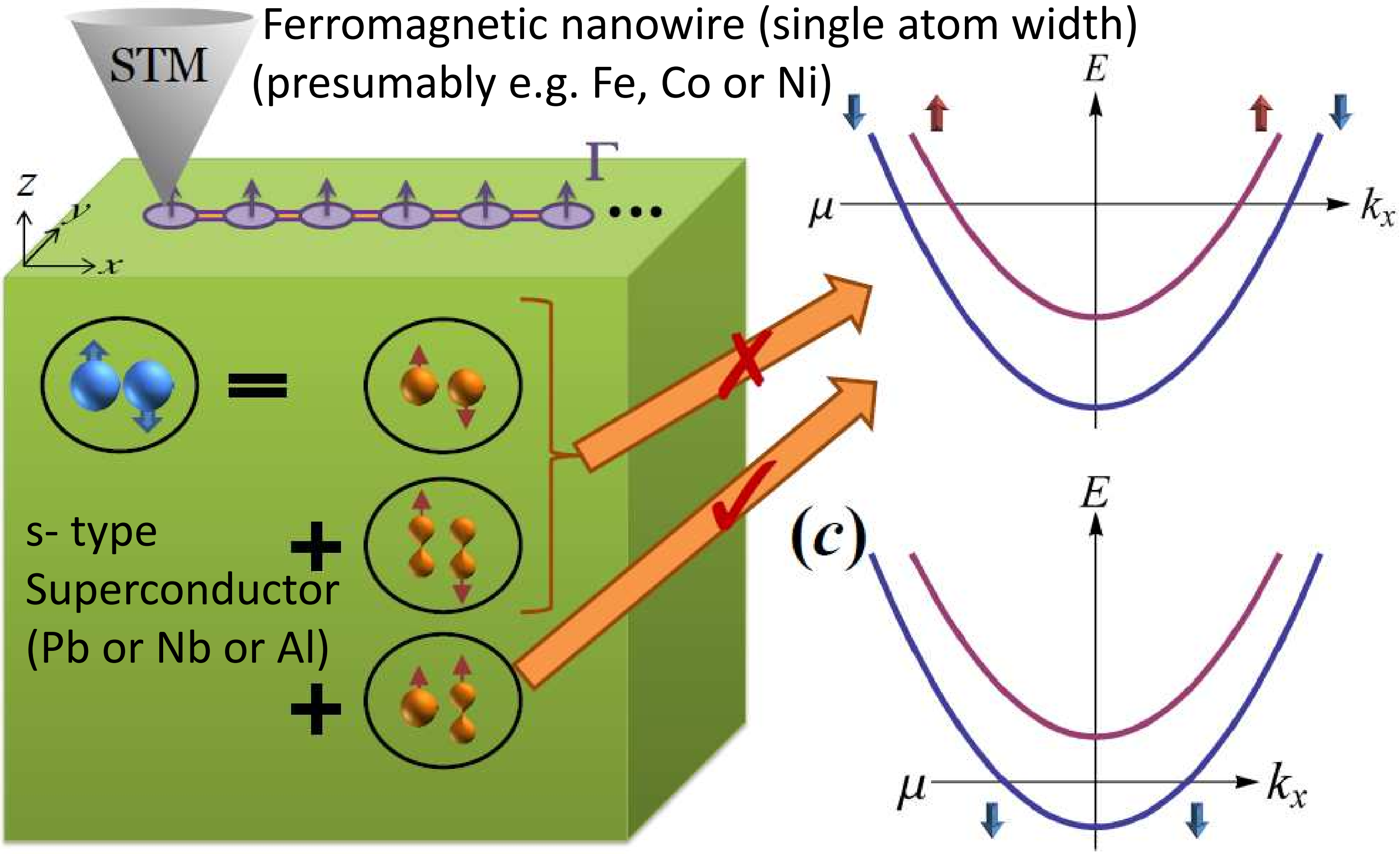
CMT Journal Club 09.09.2014

System - ingredients



2 MFs at each end

Half-metal regime

1 MF at each end

Assumptions & conditions

- Bandwidth of the electronic states in the wire is expected to be much greater (orders of magnitude) than the gap in the host superconductor $W >> \Delta$ (which is in contrast to arrays of magnetic atoms on surface of a superconductor)

- Strong SO in the bulk of the host s-wave superconductor

- Orbitals of different parity (s & p) both make a significant contribution to the states near the Fermi surface

$$\Downarrow$$

Proximity induced triplet gap in the wire required for TS

Consequences

- Spin is not a good quantum number
- Presence of *T* –symmetry and *I* – symmetry => pseudospin $\varsigma = \pm 1$

$$T\left|k,\varsigma\right\rangle = \varsigma\left|-k,-\varsigma\right\rangle \quad I\left|k,\varsigma\right\rangle = \varsigma\left|-k,\varsigma\right\rangle$$

- General form of pseudospin state

$$|\mathbf{k},\varsigma\rangle = \sum_{\sigma=\uparrow,\downarrow} \left\{ B^{s}_{\varsigma,\sigma}(\mathbf{k})|s,\mathbf{k},\sigma\rangle + B^{p}_{\varsigma,\sigma}(\mathbf{k})|p,\mathbf{k},\sigma\rangle \right\}$$

- conventional spin singlet gap => pseudospin singlet pairing state contains both intra-orbital (s&p) spin-singlet & inter-orbital spin-triplet terms

Pseudospin basis - symmetries

$$|\mathbf{k},\varsigma\rangle = \sum_{\sigma=\uparrow,\downarrow} \left\{ B^s_{\varsigma,\sigma}(\mathbf{k})|s,\mathbf{k},\sigma\rangle + B^p_{\varsigma,\sigma}(\mathbf{k})|p,\mathbf{k},\sigma\rangle \right\}$$

$$\mathcal{T}|s,\mathbf{k},\sigma\rangle = \sigma|s,-\mathbf{k},-\sigma\rangle\,, \qquad \mathcal{I}|s,\mathbf{k},\sigma\rangle = |s,-\mathbf{k},\sigma\rangle\,,$$
$$\mathcal{T}|p,\mathbf{k},\sigma\rangle = \sigma|p,-\mathbf{k},-\sigma\rangle\,, \qquad \mathcal{I}|p,\mathbf{k},\sigma\rangle = -|p,-\mathbf{k},\sigma\rangle\,,$$

$$\text{inversion:} \quad \Rightarrow \quad B^s_{\varsigma,\sigma}(\mathbf{k}) = B^s_{\varsigma,\sigma}(-\mathbf{k})\,, \quad B^p_{\varsigma,\sigma}(\mathbf{k}) = -B^p_{\varsigma,\sigma}(-\mathbf{k})\,,$$
$$\text{time-reversal:} \quad \Rightarrow \quad B^s_{\varsigma,\sigma}(\mathbf{k}) = \varsigma\sigma[B^s_{-\varsigma,\overline{\sigma}}(-\mathbf{k})]^*\,, \quad B^p_{\varsigma,\sigma}(\mathbf{k}) = \varsigma\sigma[B^p_{-\varsigma,\overline{\sigma}}(-\mathbf{k})]^*$$

$$\hat{B}^s(\mathbf{k}) = \alpha^s_{\mathbf{k}} + i\boldsymbol{\beta}^s_{\mathbf{k}}\cdot\hat{\boldsymbol{\sigma}}\,,$$
$$\hat{B}^p(\mathbf{k}) = \alpha^p_{\mathbf{k}}\hat{\boldsymbol{\sigma}}\cdot\mathbf{k} + \boldsymbol{\beta}^p_{\mathbf{k}}\cdot(\hat{\boldsymbol{\sigma}}\times\mathbf{k}) + i\boldsymbol{\gamma}^p_{\mathbf{k}}\cdot\mathbf{k}$$

$$\breve{B}_{\mathbf{k}} = \left(\hat{B}^s(\mathbf{k}) \;\; \hat{B}^p(\mathbf{k}) \right)$$

Superconductor-nanowire heterostructure

- Tunneling between superconductor and the nanowire

$$H_{\text{tun}} = \sum_{\mathbf{r}\in\text{wire}} \sum_{\sigma} \{ f^{\dagger}_{\mathbf{r},\sigma} [t_s s_{\mathbf{r},\sigma} + t_p p_{\mathbf{r},\sigma}] + \text{H.c.} \}$$

- Proximity effect in the nanowire - Self energy correction

$$\Sigma(x, x'; \omega) = \mathbf{T} G_{\text{orb}}(x, x'; \omega) \mathbf{T}^{\dagger} \quad \mathbf{T} = \begin{pmatrix} t_s \hat{1} & t_p \hat{1} & 0 & 0 \\ 0 & 0 & -t_s \hat{1} & -t_p \hat{1} \end{pmatrix}$$

- Green's function of the superconductor in orbital basis

$$G_{\text{orb}}(\mathbf{k}, \omega) = \begin{pmatrix} \breve{B}^T_{\mathbf{k}} & 0 \\ 0 & \breve{B}^T_{\mathbf{k}} \end{pmatrix} G_{\text{pseudo}}(\mathbf{k}, \omega) \begin{pmatrix} \breve{B}^*_{\mathbf{k}} & 0 \\ 0 & \breve{B}^*_{\mathbf{k}} \end{pmatrix} \quad G_{\text{pseudo}}(\mathbf{k}, \omega) = \frac{\omega \hat{\tau}_0 + \xi_{\mathbf{k}} \hat{\tau}_z + \Delta_0 \hat{\tau}_x}{\omega^2 - \xi^2_{\mathbf{k}} - \Delta^2_0}$$

Effective Hamiltonian of the wire

$$H_{\text{wire}}^{\text{eff}}(x,x') = H_{\text{wire}}^{(0)}(x,x') + \Sigma(x,x';\omega=0)$$

Zeeman splitting due to FM

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$$H_{\text{wire}}^{\text{eff}}(k_x) = (-2t\cos k_x - \mu)\,\hat{\tau}_z + \boldsymbol{\Gamma}\cdot\hat{\boldsymbol{\sigma}}$$
$$+ \left(\Delta + \tilde{\Delta}\cos k_x\right)\hat{\tau}_x$$
$$+ \tilde{\Delta}^{(t)}\sin k_x\hat{\sigma}_y\hat{\tau}_x$$

Proximity induced singlet gap

Proximity induced triplet gap

Topological properties of the wire Hamiltonian

- Particle hole symmetry $\left\{H_{\text{wire}}^{\text{eff}}, \hat{\Xi}\right\} = 0 \quad \hat{\Xi} = \sigma_y \tau_y K$
- Chiral symmetry $\left\{H_{\text{wire}}^{\text{eff}}, \hat{\mathcal{C}}\right\} = 0 \quad \hat{\mathcal{C}} = \sigma_y \tau_y$

BDI – symmetry class TC

- Toplogical index Q=n number of zero-energy Majorana Fermion end modes

$$\tilde{H}_{\text{wire}}^{\text{eff}} = \hat{U} H_{\text{wire}}^{\text{eff}} \hat{U}^\dagger = \begin{pmatrix} 0 & A_{k_x} \\ A_{-k_x}^T & 0 \end{pmatrix}$$

$$\hat{U} = e^{-i\frac{\pi}{4}\tau_x \sigma_y}$$

$$Q = \frac{1}{\pi} \int_0^\pi dk_x \frac{d \arg\left(\det A_{k_x}\right)}{dk_x}$$

S. Tewari and J. D. Sau, Phys. Rev. Lett. **109**, 150408 (2012).
S. Tewari, T. D. Stanescu, J. D. Sau and S. Das Sarma, Phys. Rev. B **86**, 024504 (2012).

Topological phase diagram of the wire

$$\mathbf{\Gamma} = \Gamma_z \mathbf{e}_z$$

Chiral symmetry

$\mathbf{\Gamma}$ in (x-z) plane

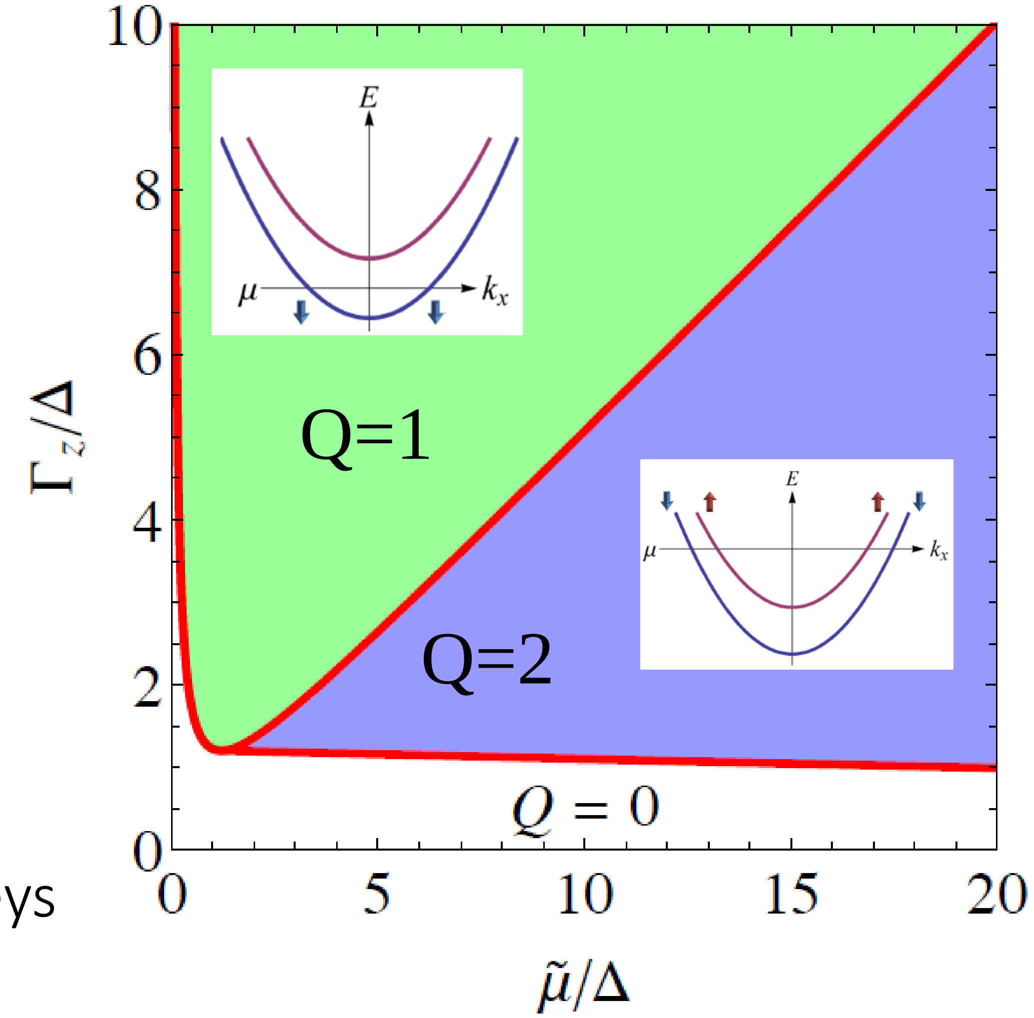
$$\tilde{\mu} = (\mu + 2t + \Gamma_z)$$

- Topological phases

$$\Delta < \Gamma_z < \tilde{\mu}/2 \qquad Q = 2$$

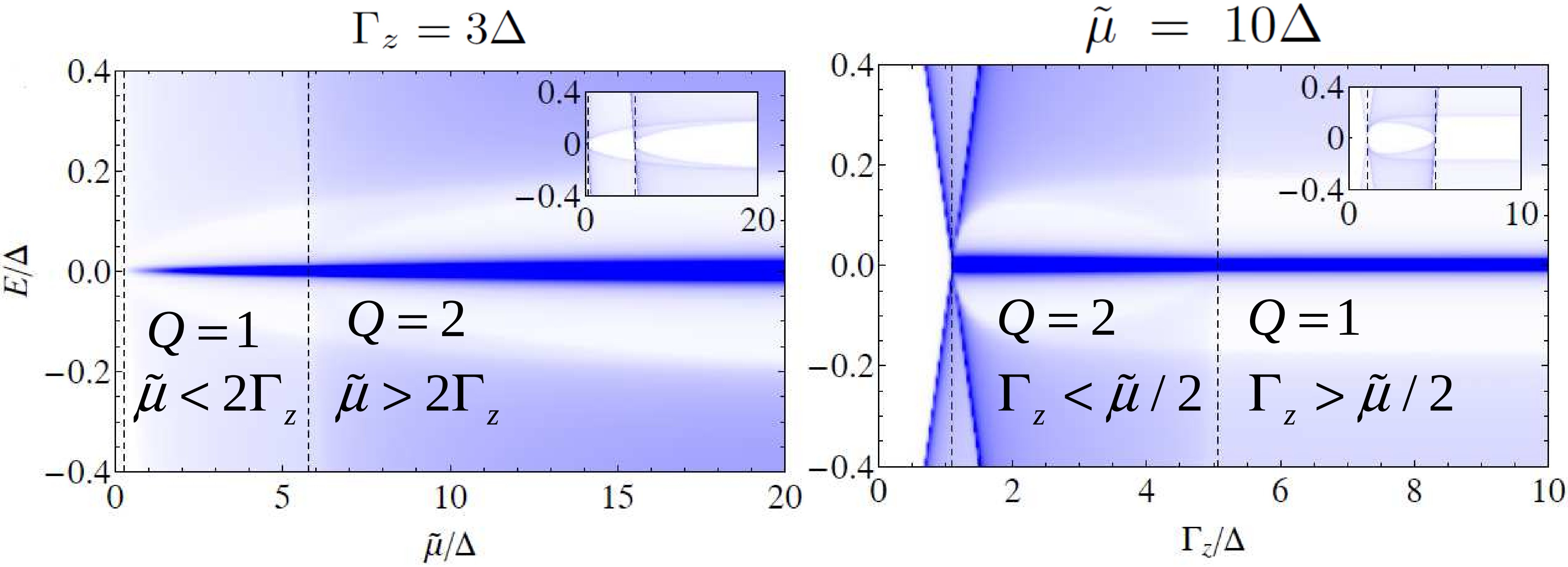
$$\Delta, \tilde{\mu}/2 < \Gamma_z \qquad Q = 1$$

$Q = 1$ Q-odd Majorana multiplet obeys non-abelian braiding statistics



LDOS at the ends of the FM wire

$$\nu(x,\omega) = \frac{-1}{2\pi}\mathrm{Im}\mathrm{Tr}\left[\omega + i\delta - H_{\mathrm{wire}}^{\mathrm{eff}}(x,x)\right]^{-1}(1+\hat{\tau}_z)$$

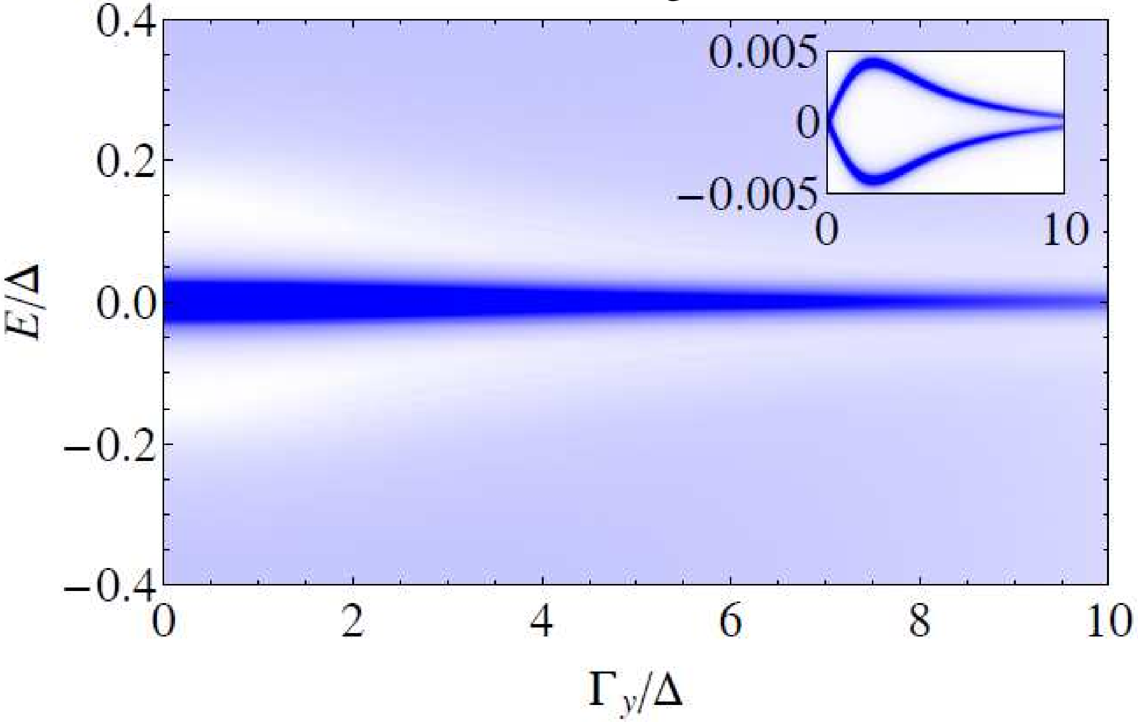


LDOS at one end of the FM wire

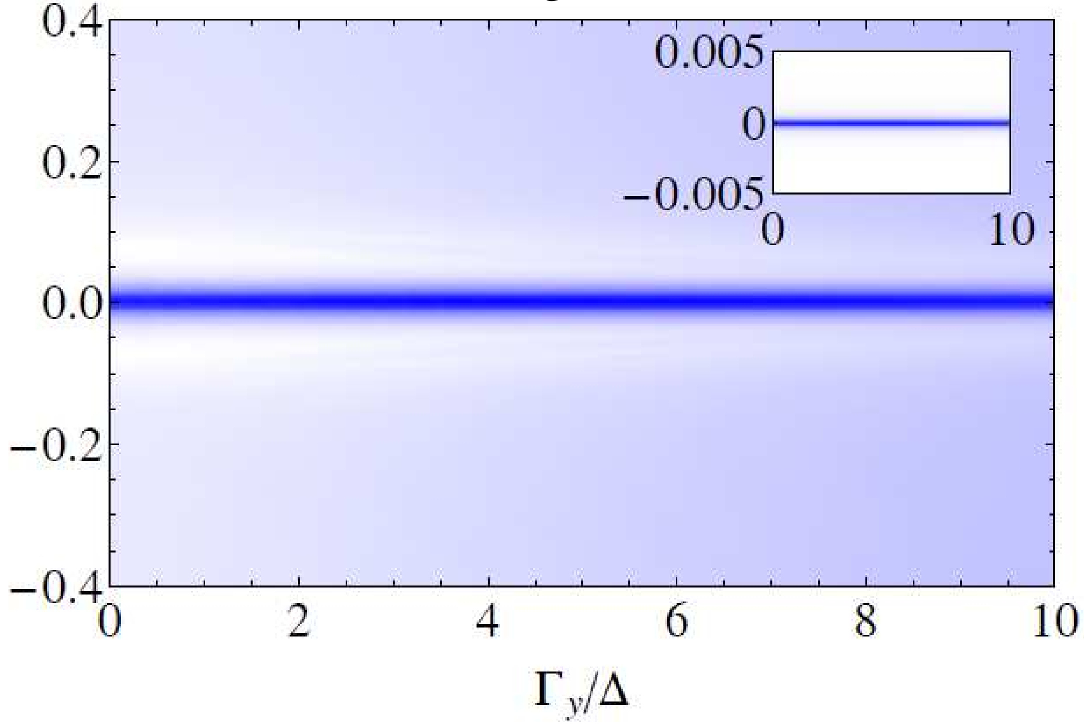
$\mathbf{\Gamma} = \Gamma_y \mathbf{e}_y + 3\Delta \mathbf{e}_z$ Chiral symmetry is broken

BDI
$\Downarrow$
D-class TS

Phase $\mathbf{Q}$=2



Phase $\mathbf{Q}$=1



Conclusions

- Ferromagnetic nanowire

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- s-wave superconductor with strong SO

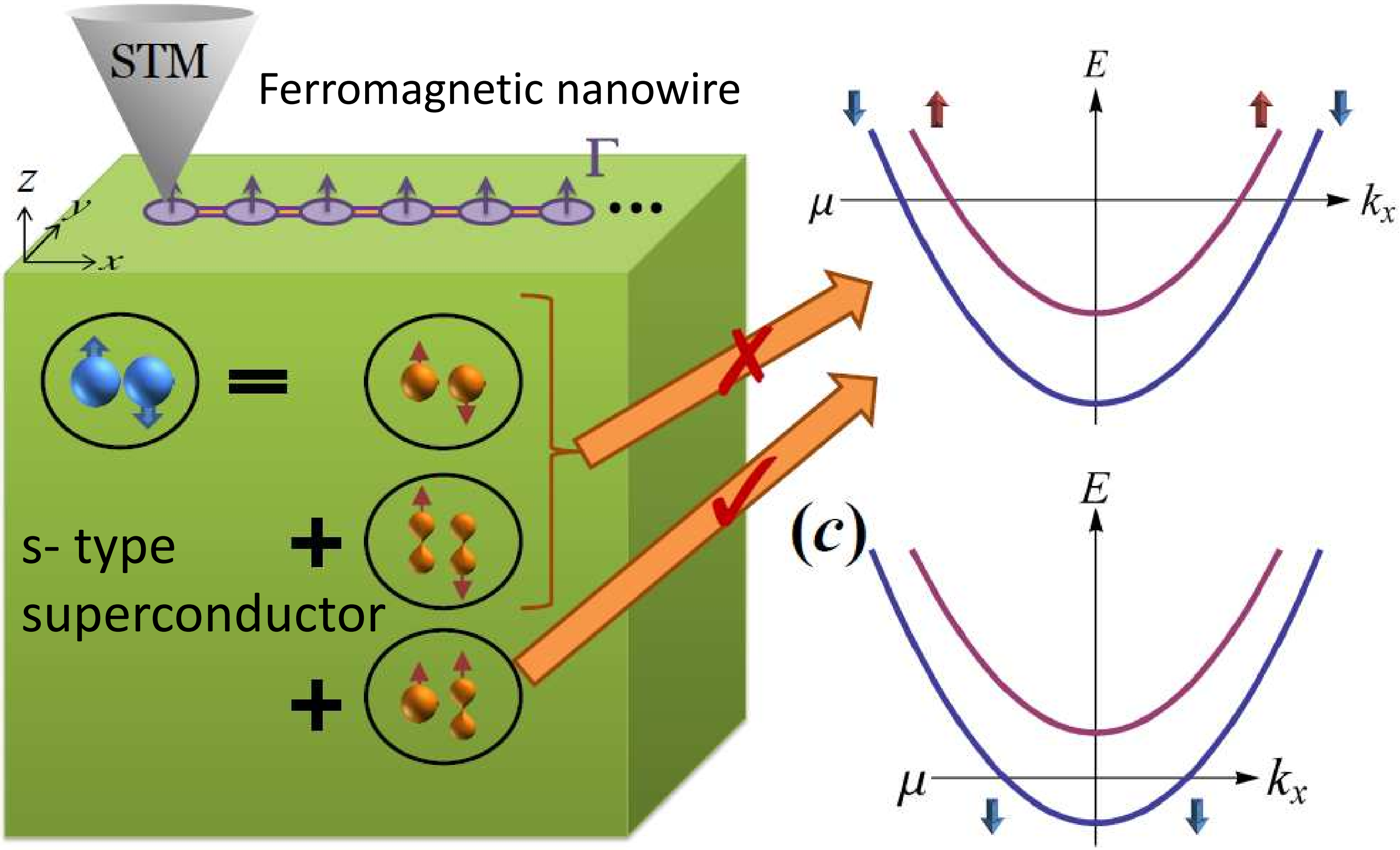
  =

- One (odd) or two(even) MFs localized and the same end (protected by topological chiral symmetry) and accessible for wide range of parameters

- Zero energy peak in the LDOS

- Experimental signatures of ZBPs observed in Princeton Group (A. Yazdani)

Thank you for your attention



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Green functions & Proximity Effect

$$G_{\mathrm{orb}}(\mathbf{k},\omega) = \begin{pmatrix} \check{B}_{\mathbf{k}}^T & 0 \\ 0 & \check{B}_{\mathbf{k}}^T \end{pmatrix} G_{\mathrm{pseudo}}(\mathbf{k},\omega) \begin{pmatrix} \check{B}_{\mathbf{k}}^* & 0 \\ 0 & \check{B}_{\mathbf{k}}^* \end{pmatrix}$$

$$\Sigma(x,x';\omega) = \mathbf{T} G_{\mathrm{orb}}(x,x';\omega) \mathbf{T}^\dagger$$

$$\mathbf{T} = \begin{pmatrix} t_s \hat{1} & t_p \hat{1} & 0 & 0 \\ 0 & 0 & -t_s \hat{1} & -t_p \hat{1} \end{pmatrix}$$

Proximity Effect

- Tunneling SC - FW

$$\Sigma(x,x';\omega) = \int \frac{d^3k}{(2\pi)^3} \mathbf{T} G_{\text{orb}}(\mathbf{k},\omega)\mathbf{T}^\dagger e^{ik_x(x-x')} = \int \frac{d^3k}{(2\pi)^3} \Sigma(\mathbf{k},\omega) e^{ik_x(x-x')}$$

$$\Sigma(\mathbf{k},\omega) = \frac{1}{\omega^2 - \xi_{\mathbf{k}}^2 - \Delta_0^2} \begin{pmatrix} (\omega+\xi_{\mathbf{k}})\hat{\Xi}(\mathbf{k}) & -\Delta_0\hat{\Xi}(\mathbf{k}) \\ -\Delta_0\hat{\Xi}(\mathbf{k}) & (\omega-\xi_{\mathbf{k}})\hat{\Xi}(\mathbf{k}) \end{pmatrix}$$

$$\begin{aligned} \hat{\Xi}(\mathbf{k}) = {} & t_s^2[\hat{B}^s(\mathbf{k})]^T[\hat{B}^s(\mathbf{k})]^* + t_p^2[\hat{B}^p(\mathbf{k})]^T[\hat{B}^p(\mathbf{k})]^* \\ & + t_s t_p[\hat{B}^s(\mathbf{k})]^T[\hat{B}^p(\mathbf{k})]^* + t_s t_p[\hat{B}^p(\mathbf{k})]^T[\hat{B}^s(\mathbf{k})]^* \end{aligned}$$

Effective Hamiltonian

- Correction to the bare wire Hamiltonian

$$H_{\text{wire}}^{\text{eff}}(x,x') = H_{\text{wire}}^{(0)}(x,x') + \Sigma(x,x';\omega=0)$$

- Correction to the bare wire Hamiltonian

$$H_{\text{wire}}^{(0)}(x,x') = t(x,x')\hat{\tau}_z + \mathbf{\Gamma}\cdot\hat{\boldsymbol{\sigma}}\delta_{x,x'} \quad t(x,x') = -\mu\delta_{x,x'} - t(\delta_{x,x'+1} + \delta_{x,x'-1})$$

- Self energy

$$\Sigma(x,x';\omega=0) = \delta t(x,x')\hat{\tau}_z + \mathbf{g}(x,x')\cdot\hat{\boldsymbol{\sigma}}\hat{\tau}_z + \Delta^s(x,x')\hat{\tau}_x + \mathbf{d}(x,x')\cdot\hat{\boldsymbol{\sigma}}\hat{\tau}_x$$

Topological properties of the wire Hamiltonian

$$\delta t(x,x') = -\int \frac{d^3k}{(2\pi)^3} e^{ik_x(x-x')} \frac{\xi_{\mathbf{k}}}{\xi_{\mathbf{k}}^2 + \Delta_0^2} \left\{ t_s^2 [\hat{B}^s(\mathbf{k})]^T [\hat{B}^s(\mathbf{k})]^* + t_p^2 [\hat{B}^p(\mathbf{k})]^T [\hat{B}^p(\mathbf{k})]^* \right\}$$

$$\mathbf{g}(x,x') \cdot \hat{\boldsymbol{\sigma}} = -\int \frac{d^3k}{(2\pi)^3} e^{ik_x(x-x')} \frac{\xi_{\mathbf{k}}}{\xi_{\mathbf{k}}^2 + \Delta_0^2} \left\{ t_s t_p [\hat{B}^s(\mathbf{k})]^T [\hat{B}^p(\mathbf{k})]^* + t_s t_p [\hat{B}^p(\mathbf{k})]^T [\hat{B}^s(\mathbf{k})]^* \right\}$$

$$\Delta^s(x,x') = \int \frac{d^3k}{(2\pi)^3} e^{ik_x(x-x')} \frac{\Delta_0}{\xi_{\mathbf{k}}^2 + \Delta_0^2} \left\{ t_s^2 [\hat{B}^s(\mathbf{k})]^T [\hat{B}^s(\mathbf{k})]^* + t_p^2 [\hat{B}^p(\mathbf{k})]^T [\hat{B}^p(\mathbf{k})]^* \right\}$$

$$\mathbf{d}(x,x') \cdot \hat{\boldsymbol{\sigma}} = \int \frac{d^3k}{(2\pi)^3} e^{ik_x(x-x')} \frac{\Delta_0}{\xi_{\mathbf{k}}^2 + \Delta_0^2} \left\{ t_s t_p [\hat{B}^s(\mathbf{k})]^T [\hat{B}^p(\mathbf{k})]^* + t_s t_p [\hat{B}^p(\mathbf{k})]^T [\hat{B}^s(\mathbf{k})]^* \right\}$$

Further assumptions

- Simplifaction

$$t_s^2[\hat{B}^s(\mathbf{k})]^T[\hat{B}^s(\mathbf{k})]^* + t_p^2[\hat{B}^p(\mathbf{k})]^T[\hat{B}^p(\mathbf{k})]^* = t_s^2(\alpha^s)^2 + t_p^2(\beta^p)^2(k_x^2 + k_y^2)\,,$$

$$t_s t_p[\hat{B}^s(\mathbf{k})]^T[\hat{B}^p(\mathbf{k})]^* + t_s t_p[\hat{B}^p(\mathbf{k})]^T[\hat{B}^s(\mathbf{k})]^* = 2t_s t_p \alpha^s \beta^p(\hat{\sigma}_x k_y + \hat{\sigma}_y k_x)\,.$$

$$\Delta^s(x,x') = \Delta\delta_{x,x'} + \frac{1}{2}\tilde{\Delta}\left(\delta_{x,x'+1} + \delta_{x,x'-1}\right)$$

$$\mathbf{d}(x,x')\cdot\hat{\boldsymbol{\sigma}} = \frac{1}{2}\tilde{\Delta}^{(t)}\left\{\delta_{x,x'-1} - \delta_{x,x'+1}\right\}\hat{c}$$

$$\Delta = \int \frac{d^3k}{(2\pi)^3}\frac{\Delta_0}{\xi_{\mathbf{k}}^2 + \Delta_0^2}\left\{t_s^2(\alpha^s)^2 + t_p^2(\beta^p)^2(k_x^2 + k_y^2)\right\}\,,$$

$$\tilde{\Delta} = \int \frac{d^3k}{(2\pi)^3}e^{ik_x a}\frac{\Delta_0}{\xi_{\mathbf{k}}^2 + \Delta_0^2}\left\{t_s^2(\alpha^s)^2 + t_p^2(\beta^p)^2(k_x^2 + k_y^2)\right\}$$

$$\tilde{\Delta}^{(t)} = \int \frac{d^3k}{(2\pi)^3}e^{ik_x a}\frac{\Delta_0}{\xi_{\mathbf{k}}^2 + \Delta_0^2}\left\{2t_s t_p \alpha^s \beta^p k_x\right\}\,.$$

Effective Hamiltonian for the nanowire

$$H^{\mathrm{eff}}_{\mathrm{wire}}(k_x) = (-2t\cos k_x - \mu)\,\hat{\tau}_z + \mathbf{\Gamma}\cdot\hat{\boldsymbol{\sigma}} + \left(\Delta + \tilde{\Delta}\cos k_x\right)\hat{\tau}_x + \tilde{\Delta}^{(t)}\sin k_x\hat{\sigma}_y\hat{\tau}_x$$