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Mechanically probing coherent tunnelling in a double quantum dot

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We study theoretically the interaction between the charge dynamics of a few-electron double quantum dot and a capacitively-coupled AFM cantilever, a setup realized in several recent experiments. We demonstrate that the dot-induced frequency shift and damping of the cantilever can be used as a sensitive probe of coherent inter-dot tunnelling, and that these effects can be used to quantitatively extract both the magnitude of the coherent interdot tunneling and (in some cases) the value of the double-dot T_1 time. We also show how the adiabatic modulation of the double-dot eigenstates by the cantilever motion leads to new effects compared to the single-dot case.

Introduction

- Applications of QDs both for QI and as a laboratory for studies of fundamental physics.
- Self-assembled, epitaxial QDs have advantages over lithographically defined dots: size, confinement, scalability.
- Direct electrical characterization (transport exp.) difficult due to their small size.
- Alternative approach using an AFM, which couples capacitively to the QD charge. The QD dynamics alters the frequency and the damping rate of the cantilever.
- It provides informations on the QD.

Proposal:

- Consider a low-frequency cantilever coupled to a double QD.
- Investigate the dependence to the stength of the coherent tunneling between the QDs.
- Results are derived using a linear-response, quantum master-equation calculation. (In extent to the often used semi-classical Fokker-Planck treatment)

New Effects

- In the vicinity of the charge transfer line (*almost degenerate DQD configurations with the same total charge)*, new mechanism for the DQD-induced cantilever damping, that is enhanced by the relatively long charge relaxation.
  $\rightarrow$ Characterization of T_1 time of the DQD.

- Another new mechanism for the DQD-induced cantilever frequency shift near the charge transfer line. Due to coherent tunneling, the DQD energy eigenstates are superpositions of charge states. The cantilever motion could adiabatically modulate these wavefunctions.
  $\rightarrow$ Allows to probe the interdot tunneling.

Model

- A self-assembled DQD capacitively coupled to an oscillating metallic cantilever is considered. The dots are also tunnel-coupled to a 2DEG.
- Coulomb blockade Hamiltonian with coherent interdot tunneling.

$$\hat{H}_{DQD} = \hat{H}_c + t_c \left(\hat{d}_L^\dagger \hat{d}_R + \hat{d}_R^\dagger \hat{d}_L \right) + \hat{H}_{res},$$

$$\hat{H}_c = \sum_{\alpha=L,R} E_{C\alpha} (\hat{n}_\alpha - \mathcal{N}_\alpha)^2 + E_{Cm} (\hat{n}_L - \mathcal{N}_L)(\hat{n}_R - \mathcal{N}_R)$$

  $\hat{H}_{res}$ describes a free 2DEG and dot-2DEG tunneling.
- Assumptions:
  - spinless electrons
  - few-electron regime (at most one electron per dot), only a single orbital in each dot is retained.
  - Coulomb blockade regime $E_{C\alpha} \gg k_B T$
  - 2DEG in equilibrium at T
  - dot-2DEG tunnel matrix element is the same for both dots. ($\sim \Gamma$)

- The dimensionless gate voltages $\mathcal{N}_\alpha$ depend on the cantilever position $\vec{r}_{tip}$,

$$\mathcal{N}_\alpha = -\frac{V_B C_{tip,\alpha}(|\vec{r}_{tip} - \vec{r}_\alpha|)}{e}, \qquad (C_{tip,\alpha} : \text{cantilever-dot capacitance})$$

- Letting the cantilever height z_m oscillate. The coordinate z_m is described as an harmonic oscillator with frequency ω_m and mass m. Considering small oscillations, the dependence of $\mathcal{N}_\alpha$ is linearized in z_m. The DQD-cantilever interaction is

$$\hat{H}_{int} = -\hat{z}_m \sum_{\alpha=L,R} A_\alpha \hat{n}_\alpha \equiv -\hat{z}_m \hat{F},$$

$$A_\alpha = 2E_{C\alpha}\frac{\partial \mathcal{N}_\alpha}{\partial z_m} + E_{Cm}\frac{\partial \mathcal{N}_{\bar{\alpha}}}{\partial z_m} = -\frac{\partial E_{+\alpha}}{\partial z_m}.$$

Electron addition energy: $E_{+\alpha} = E_{C\alpha}(1 - 2\mathcal{N}_\alpha) - E_{Cm}\mathcal{N}_{\bar{\alpha}}$.

- Mostly focus on the regime where the total DQD charge is fixed at 1 and where $\delta = \frac{E_{+L} - E_{+R}}{2}$ is small (electrostatic energy detuning between state $|01\rangle$ and $|10\rangle$.) States with $\hat{n}_{tot} > 2$ can be neglected.
- In self-assembled QDs, $t_c \gg \Gamma, \omega_m$ is satisfied and it is useful to work in the basis of adiabatic eigenstates of $\hat{H}_{DQD}$, determined by the instantaneous value of z_m.

$$|1[z_m]\rangle = |11\rangle, \qquad |2[z_m]\rangle = -\sin(\theta/2)|10\rangle + \cos(\theta/2)|01\rangle,$$
$$|3[z_m]\rangle = \cos(\theta/2)|10\rangle + \sin(\theta/2)|01\rangle, \qquad |4[z_m]\rangle = |00\rangle,$$

where $\tan\theta[z_m] = t_c/\delta[z_m] = t_c/\left(\delta - \frac{1}{2}z_m(A_L - A_R)\right)$. Adiabatic eigenenergies,

$$E_{2,3}[z_m] = \left(\frac{E_{+L} + E_{+R} - z_m(A_L + A_R)}{2}\right) \mp \sqrt{(\delta[z_m])^2 + t_c^2}$$
$$\equiv \varepsilon[z_m] \mp \Delta[z_m].$$

- For $z_m = 0$, the states $|2\rangle$ and $|3\rangle$ will primarily be occupied by the DQD. Approximately the physics of a two-level system.

Calculation

- Because of the DQD-cantilever coupling $\hat{H}_{int} = -\hat{z}_m \hat{F}$, the average force $\langle \hat{F} \rangle$ will respond with a delay to the motion of the oscillator resulting in both a spring-constant shift k_{dot} and extra damping γ_{dot}.
- In the weak coupling limit, it is well described within linear response. k_{dot} and γ_{dot} are calculated by replacing $\hat{z}_m \to z_m(t) = z_0 \cos(\omega_m t)$.
- A Lindblad master equation is derived to describe these effects in the regime $\omega_m \ll T$.

$$\frac{\partial \hat{\rho}_{rot}}{\partial t} = \frac{1}{i\hbar}[\hat{H}_{eff}, \hat{\rho}_{rot}] + \sum_{j,k=1}^{4} \Gamma_{jk} \mathcal{D}[\hat{S}_{jk}]\hat{\rho}_{rot},$$

$$\hat{H}_{eff} = \varepsilon \hat{n}_{tot} + E_m|4\rangle\langle 4| + \Delta \hat{\sigma}_z - \frac{\hbar}{2}\frac{\partial z_m}{\partial t}\frac{\partial \theta}{\partial z_m}\hat{\sigma}_y,$$

$$\mathcal{D}[\hat{S}_{jk}]\hat{\rho}_{rot} = \hat{S}_{jk}\hat{\rho}_{rot}\hat{S}_{jk}^{\dagger} - \frac{1}{2}(\hat{S}_{jk}^{\dagger}\hat{S}_{jk}\hat{\rho}_{rot} + \hat{\rho}_{rot}\hat{S}_{jk}^{\dagger}\hat{S}_{jk}), \qquad \hat{S}_{jk} = |j\rangle\langle k.|$$

where

$$\hat{\rho}_{rot}(t) = \hat{U}[z_m(z)]^{\dagger}\hat{\rho}(t)\hat{U}[z_m(z)] \quad \text{and} \quad \hat{U}[x]|j[0]\rangle = |j[x]\rangle,$$

$$\hat{\sigma}_z = |3\rangle\langle 3| - |2\rangle\langle 2|, \qquad \hat{\sigma}_y = i(|2\rangle\langle 3| - |3\rangle\langle 2|).$$

To obtain k_{dot} and γ_{dot}, the ME is used to find the first-order-correction to $\hat{\rho}_{rot}$ in z and calculate the corresponding change in $\langle \hat{F}(t) \rangle$ to infer k_{dot} and γ_{dot}.

Basic Mechanisms

- Low-frequency limit, the linear response results have the form:

$$m\gamma_{dot} = \tau \frac{\partial \langle \hat{F} \rangle}{\partial z_m}, \qquad k_{dot} = -\frac{\partial \langle \hat{F} \rangle}{\partial z_m}.$$

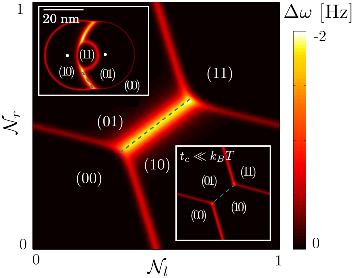
- Single dot case $\frac{\partial \langle \hat{F} \rangle}{\partial z_m} \propto \frac{\partial \langle \hat{n}_{tot} \rangle}{\partial \mathcal{N}}$, thus γ_{dot} and k_{dot} are only significant when the QD total charge can fluctuate via 2DEG-QD tunneling (charge addition lines).
- In the DQD case, γ_{dot} and k_{dot} are determined by the dynamics of the DQD charge distribution.
- Near the charge transfer line, the DQD-induced force operator is

$$\hat{F} - \frac{A_L + A_R}{2} \simeq A_\delta(\hat{n}_L - \hat{n}_R) = A_\delta(\cos\theta\hat{\sigma}_z - \sin\theta\hat{\sigma}_x), \quad A_\delta = \frac{A_L - A_R}{2}.$$

- Three different mechanisms : $\partial_{z_m}\langle\hat{\sigma}_x\rangle$, $\partial_{z_m}\langle\hat{\sigma}_z\rangle$ and $\partial_{z_m}\theta$. The first is strongly suppressed as $\omega_m \ll t_c$.

Adiabatic Frequency Shift

- $\hat{F}$ has a dependence on θ, which has an intrinsic z_m-dependence, causing a modulation of $\hat{F}$.
- It corresponds to the adiabatic modulation of the DQD eigenstates by the cantilever oscillation, via the cantilever's modulation of the electrostatic detuning δ.
- The corresponding oscillation in $\langle \hat{n}_L - \hat{n}_R \rangle$ causes a force oscillation, in phase with $z_m(t)$.



$\Delta\omega = k_{dot}/(2m\omega_m)$
$\omega_m = 160$ kHz, $k_0 = 7$ N/m,
$\Gamma = 10$ kHz, $T = 4.2$ K,
$t_c = 1$ meV, $E_{CL} = 20$ meV,
$E_{CR} = 25$ meV, $E_{Cm} = 12$ meV.

$$k_{dot} = -A_\delta \langle \hat{\sigma}_z \rangle \frac{\partial \cos\theta}{\partial z_m} = -\frac{A_\delta^2 \sin^2\theta \tanh(\Delta/k_B T)}{\Delta}.$$

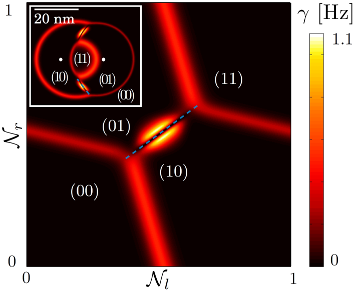
Effective TLS Damping

- Second mechanism near the charge transfer line: the cantilever's modulation of $\langle \hat{\sigma}_z \rangle$, that is the population asymmetry of the two low-energy DQD eigenstates.
- The DQD splitting Δ oscillates due to the cantilever oscillations. Thus the occupancy of the states $|2\rangle$ and $|3\rangle$ also oscillates.
- The corresponding oscillations in $\langle \hat{\sigma}_z \rangle$, hence in $\langle \hat{F} \rangle$, are phase-shifted with respect to $z_m(t)$ due to the finite DQD T_1 time.
- This mechanism contributes both to k_{dot} and γ_{dot} and is suppressed at low temperatures $T \ll \Delta$.
- DQD-induced damping

$$m\gamma_{dot} = \left(\frac{T_1}{1 + \omega_m^2 T_1^2} \right) \frac{A_\delta^2 \cos^2 \theta}{k_B T \cosh^2(\Delta / k_B T)}$$

Dot-induced Damping

- Dot-induced damping not only occurs near the charge addition lines, where DQD-2DEG tunneling is strong. ($\gamma_{dot} \propto 1/\Gamma$)
- For low frequency cantilever, near charge transfer line, $\gamma_{dot} \propto T_1$.
- If $T_1\Gamma > 1$, the "TLS damping" mechanism can be greater in magnitude than the conventional damping peaks found near charge addition lines.
- This effect vanishes at $\delta = 0$ due to the presence of coherent tunneling.



$\omega_m = 75$ kHz, $k_0 = 3$ N/m, $\Gamma = 10$ MHz, $T = 8.4$ K, $t_c = 0.3$ meV, $E_{CL} = 20$ meV, $E_{CR} = 25$ meV, $E_{Cm} = 12$ meV.

Measuring T_1

- For low frequency cantilever and $T \gg \Delta$,
- near the charge transfer line

$$\frac{m\gamma_{dot}}{k_{dot}} \simeq -\cos^2\theta\, T_1.$$

- Allows direct measure of the DQD T_1 time.

Conclusion

- Charge dynamics in DQD can influence damping and frequency shifts of a low-frequency resonator (AFM tip).
- Compared to single QD, qualitatively new effects arise due to the cantilever's sensitivity to charge distribution and to the presence of coherent interdot tunneling.
- These effects allow to access the DQD T_1 time near the charge transfer line and to probe the strength of the coherent tunneling.