Controlling non-Abelian statistics of Majorana fermions in semiconductor nanowires

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Outline

- Non-Abelian quasiparticles
- Characteristics of non-Abelian states
- Majorana fermions in 1D wires

- Majorana fermions transport
- Majorana fermions in dimer lattice
- Exchange of the Majorana fermions

Non-Abelian quasiparticles

$$|\psi_1\psi_2\rangle = e^{i\theta}\,|\psi_2\psi_1\rangle$$

Anyons in 2D

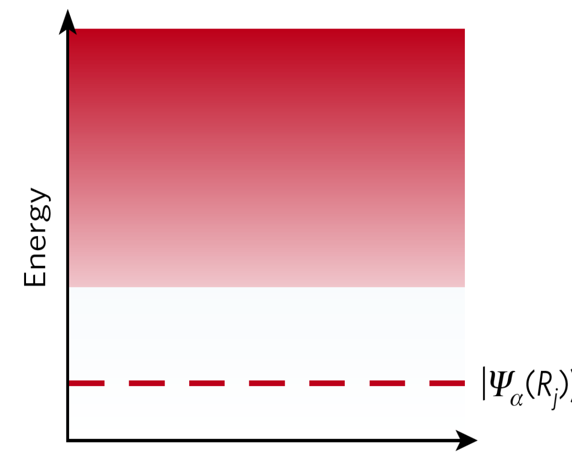
Composites of the elementary particles

- phase may take any values
- new ground state

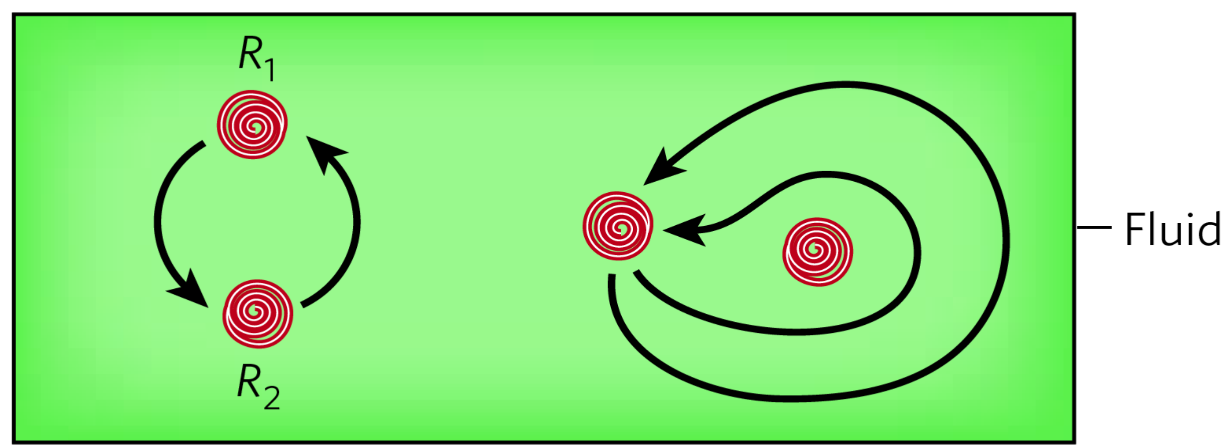
Characteristics of non-Abelian states

Example: two-dimensional fluid with several vortices

- Energy gap
- Degeneracy
- Insensitive to weak noise and perturbations
- Braiding



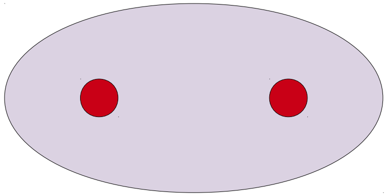
$$|\Psi_\alpha(R_j)\rangle$$
$$\downarrow$$
$$|\Psi_\beta(R_j)\rangle$$



Majorana fermions in 1D wires

The Majorana fermions operators:

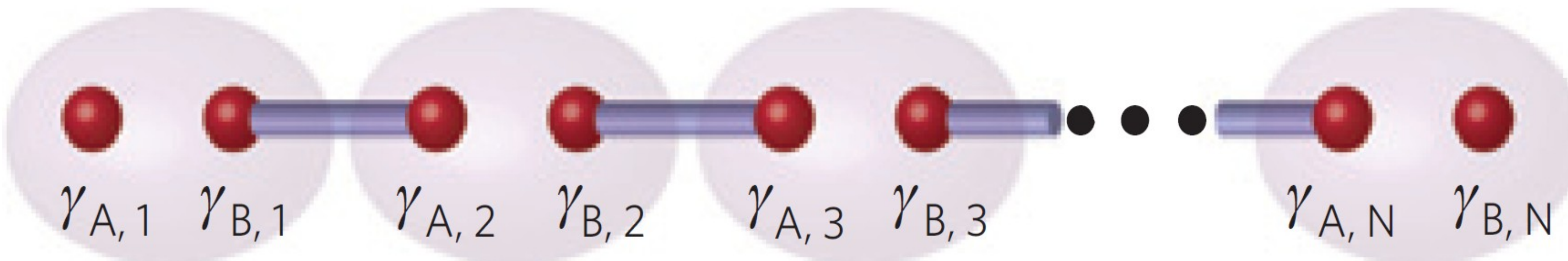
$$c_x = \frac{1}{2}(\gamma_{B,x} + i\gamma_{A,x})$$
$$c_x^\dagger = \frac{1}{2}(\gamma_{B,x} - i\gamma_{A,x})$$



$$\gamma_{\alpha,x} = \gamma_{\alpha,x}^\dagger$$
$$\{\gamma_{\alpha,x}, \gamma_{\alpha',x'}\} = 2\delta_{\alpha,\alpha'}\delta_{x,x'}$$

Majorana fermions in 1D wires

$$H = 2t \sum_{x=1}^{N-1} \left(d_x^\dagger d_x - \frac{1}{2} \right) \qquad d_x = \frac{1}{2}(\gamma_{A,x+1} + i\gamma_{B,x})$$



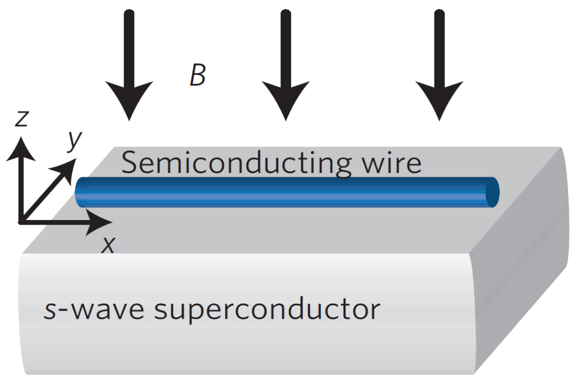
$$d_{\text{end}} = \frac{1}{2}(\gamma_{A,1} + i\gamma_{B,N})$$

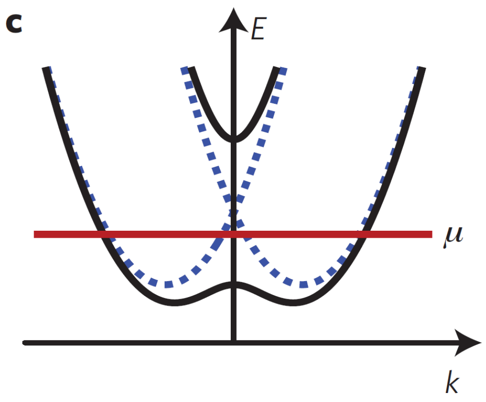
$$|1\rangle = d_{\text{end}}{}^\dagger |0\rangle \qquad d_{\text{end}}|0\rangle = 0$$

A. Y. Kitaev, Phys. Usp. 44, 131 (2001)

Majorana fermions in 1D wires

Semiconducting nanowire with proximity induced s-wave superconductivity





The Zeeman splitting V_Z

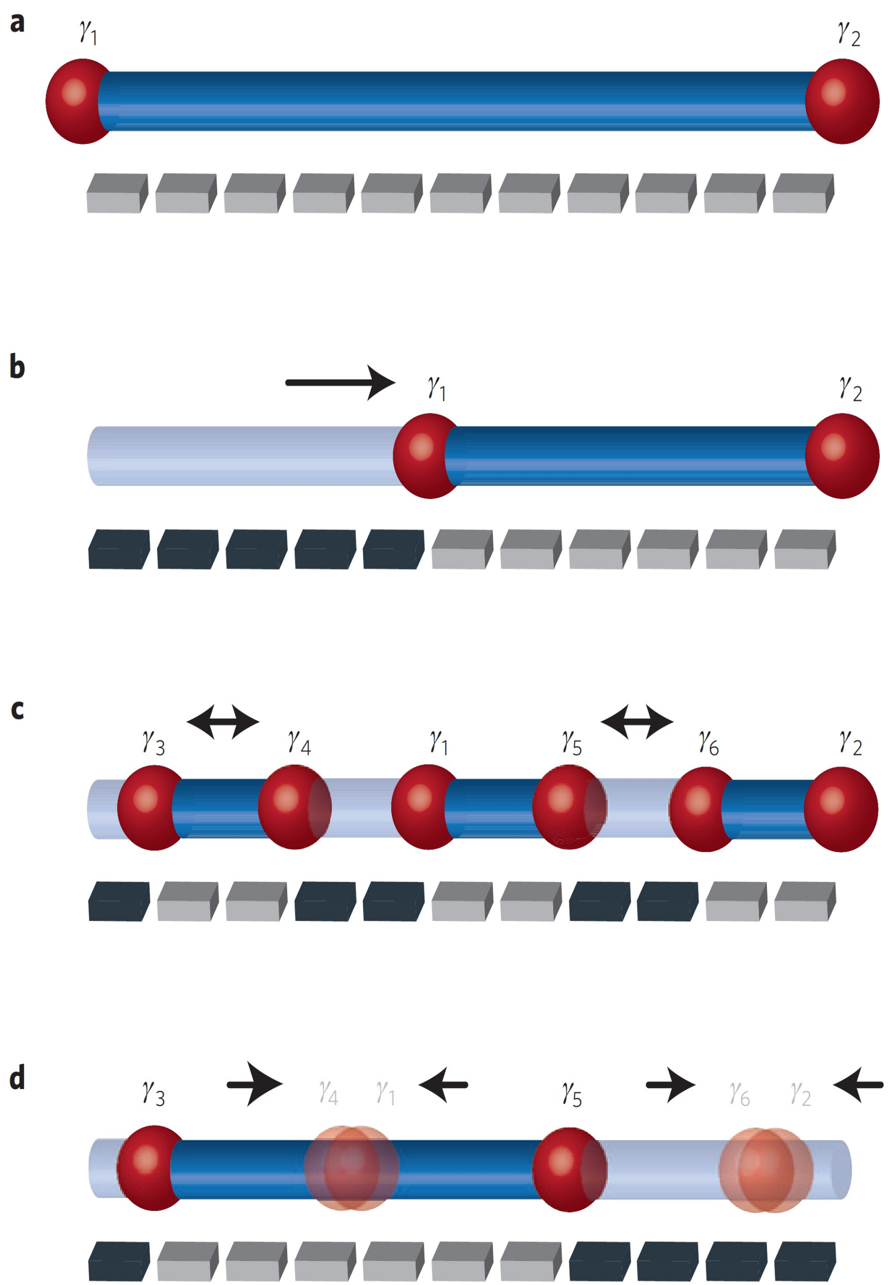
The superconducting pair potential Δ

The chemical potential μ

The topological phase $$V_Z^2 > \Delta^2 + \mu^2$$

$$\mathcal{H} = \int \mathrm{d}x \Big[\psi_x^{\dagger} \Big(-\frac{\hbar^2 \partial_x{}^2}{2m} - \mu - i\hbar u \hat{\mathbf{e}} \cdot \boldsymbol{\sigma} \partial_x - \frac{g \mu_{\mathrm{B}} B_z}{2} \sigma^z \Big) \psi_x + (|\Delta| \mathrm{e}^{i\varphi} \psi_{\downarrow x} \psi_{\uparrow x} + h.c.) \Big]$$

R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010); Y. Oreg, G. Refael, and F. von Oppen, ibid. 105, 177002 (2010).

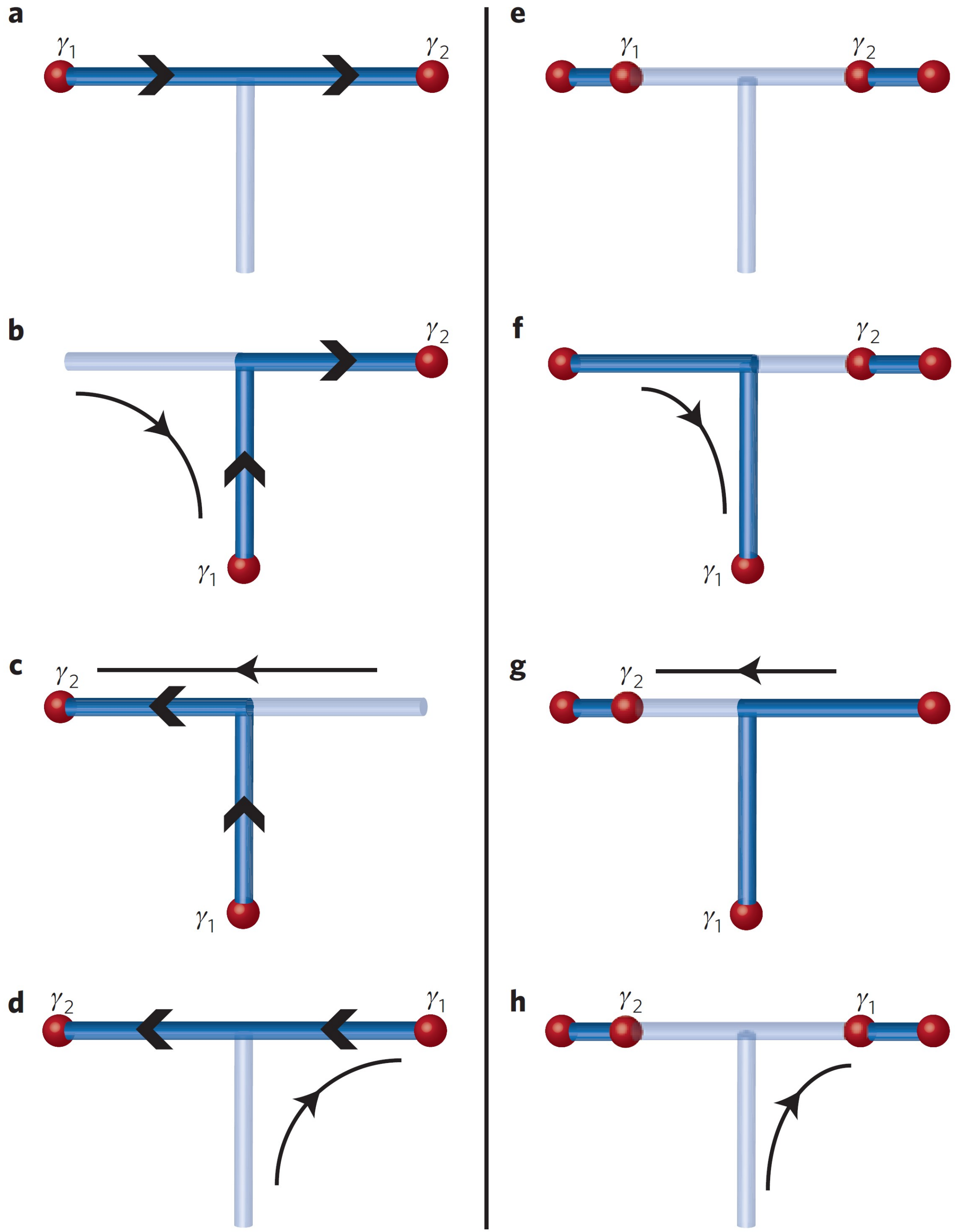


2n Majoranas

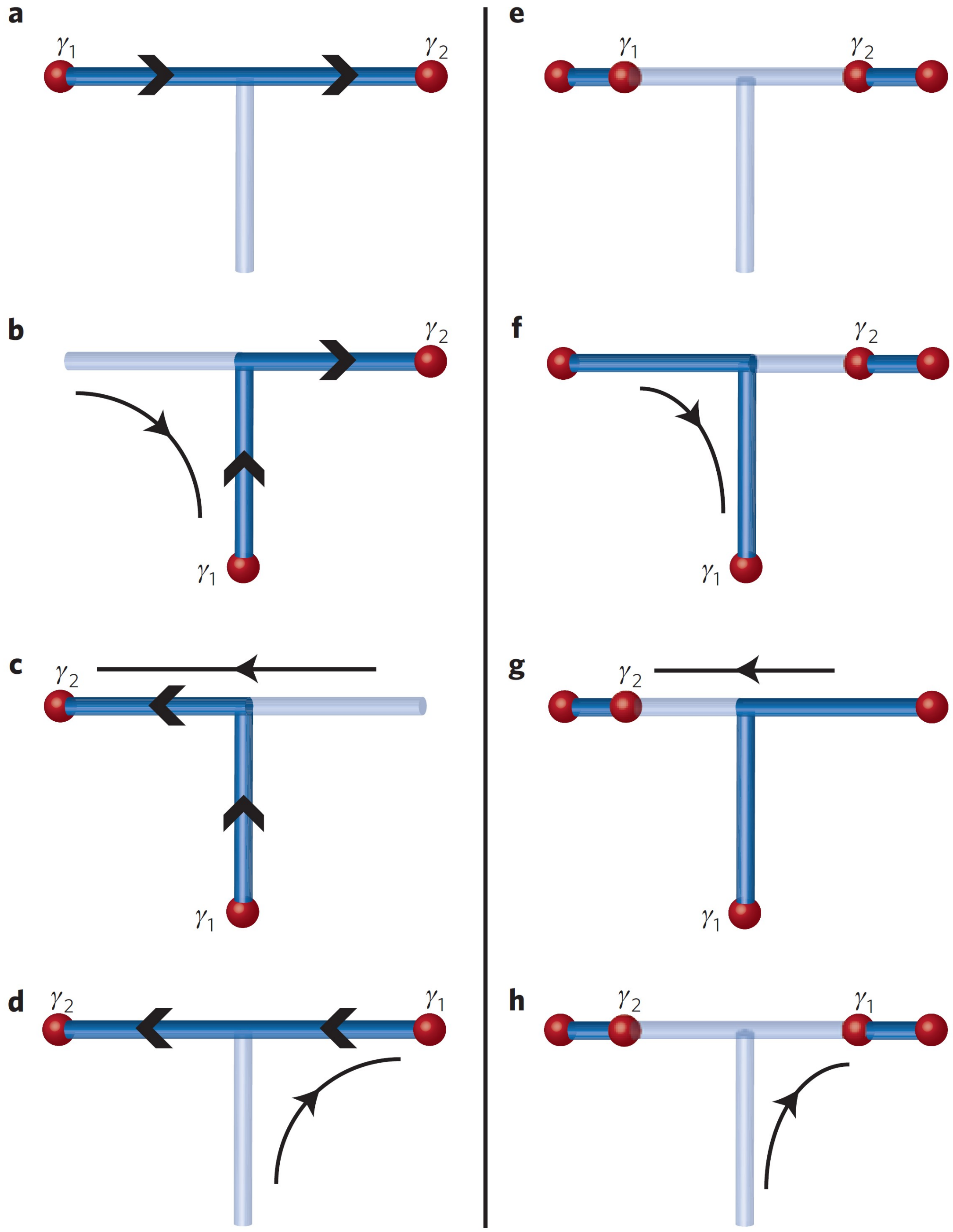
generate

n ordinary zero-energy fermions

J. Alicea, Y. Oreg, G. Refael, F. von Oppen, M. P. A. Fisher, Nat. Phys. 7, 412 (2011).

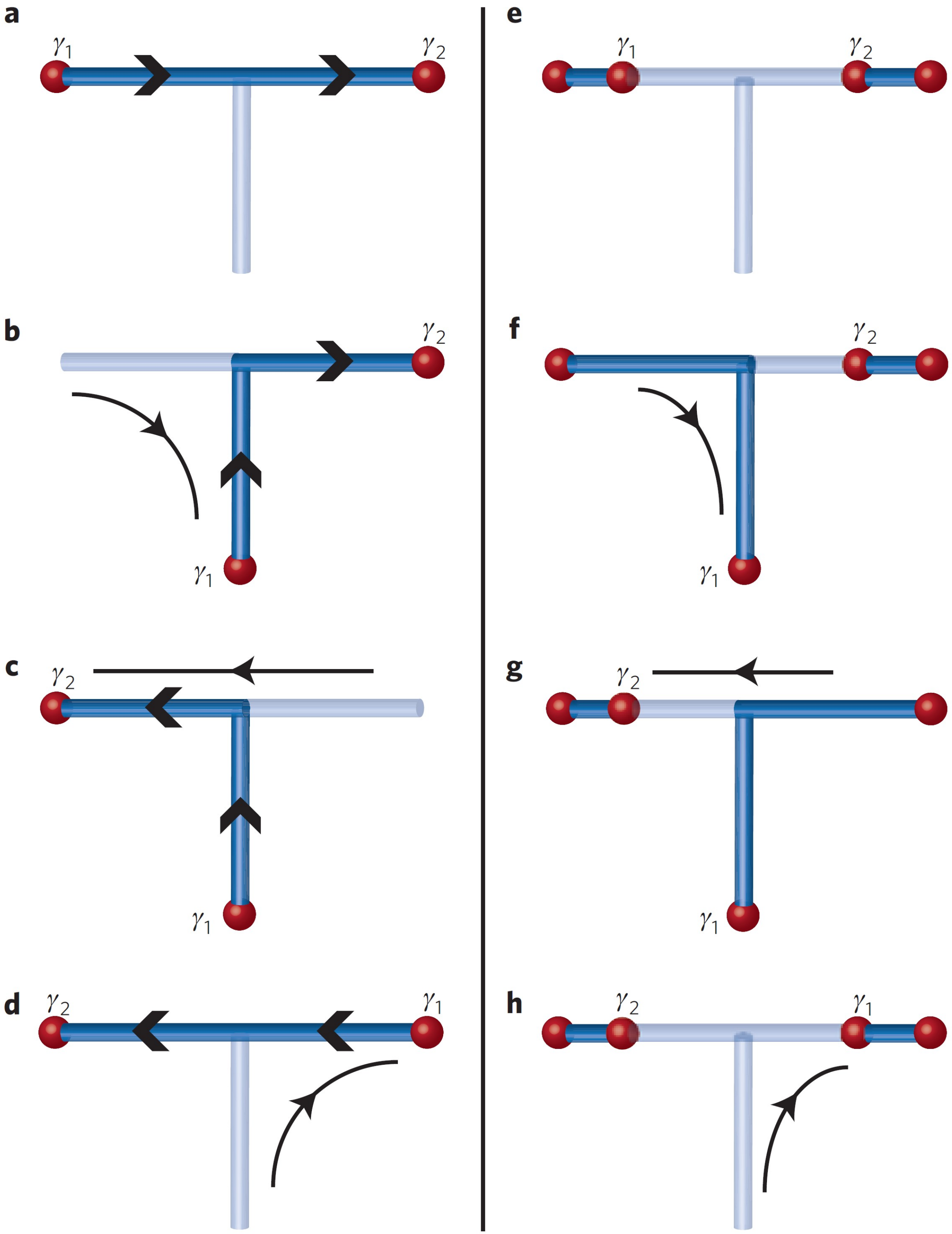


J. Alicea, Y. Oreg, G. Refael, F. von Oppen, M. P. A. Fisher, Nat. Phys. 7, 412 (2011).



A trijunction between two topological nanowires is potentially a more complex topological object than the simple topological nanowire. The continuous transport of MFs through such a trijunction is potentially dependent on details of the junction that may be difficult to control.

J. Alicea, Y. Oreg, G. Refael, F. von Oppen, M. P. A. Fisher, Nat. Phys. 7, 412 (2011).



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An alternative scheme to transport Mfs: the ends of the nanowires remain fixed, but the tunneling amplitudes between the end MFs are varied.

J. Alicea, Y. Oreg, G. Refael, F. von Oppen, M. P. A. Fisher, Nat. Phys. 7, 412 (2011).

Tunneling amplitudes

$$c^\dagger = \gamma_1 + i\gamma_2 \qquad\qquad n = c^\dagger c = \frac{1+i\gamma_1\gamma_2}{2}$$

$$H_{\text{tunneling}} = i\zeta_{12}(x)\gamma_1\gamma_2$$

$\zeta_{12}(x)$ is the tunneling matrix element for the MFs

For $x \gg \xi$ matrix element vanishes because of the localization of the MF wave functions

The tunneling of Mfs:

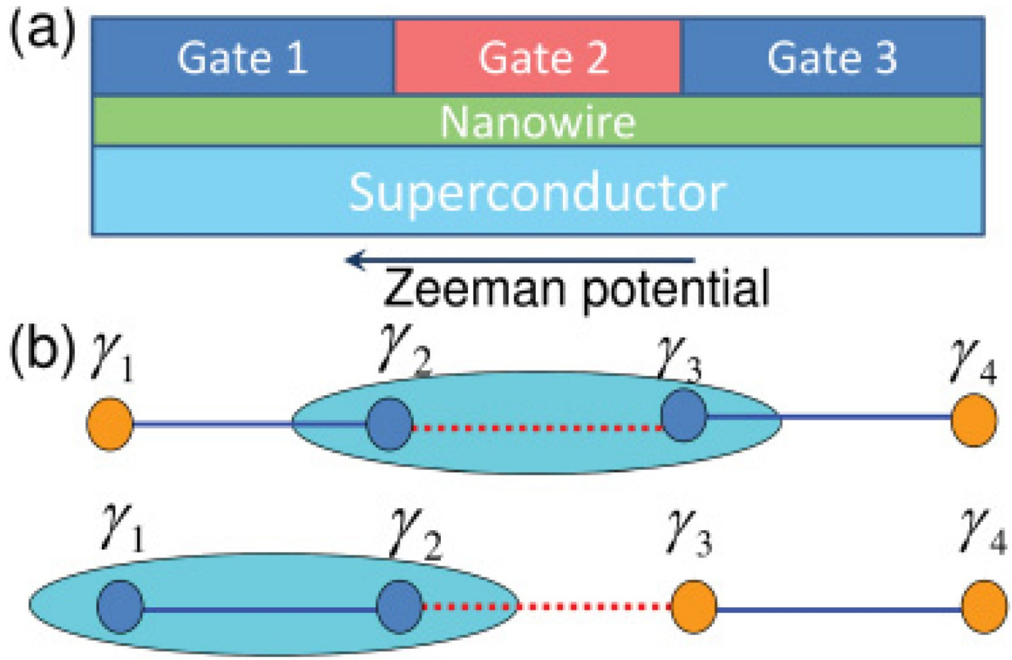
at the ends of different wires

gate-controllable tunnel barrier

at the ends of the same TS segment

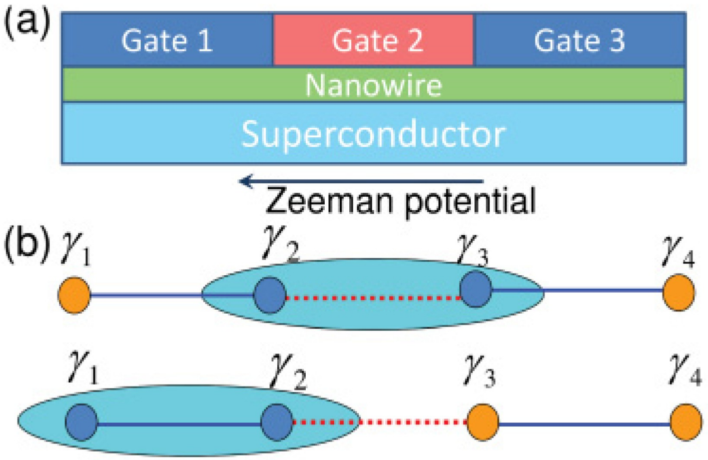
tuning the nanowire close to a TS-NTS

Majorana fermions transport



$$H = [\zeta_{12}\alpha(t)\gamma_1\gamma_2 + \zeta_{23}(1-\alpha(t))\gamma_2\gamma_3]$$

Majorana fermions transport



$$H = [\zeta_{12}\alpha(t)\gamma_1\gamma_2 + \zeta_{23}(1-\alpha(t))\gamma_2\gamma_3]$$

$$\gamma_j(t) = U^\dagger(t)\gamma_j U(t)$$

$$U(t) = T e^{-i\int_0^t H(\tau)d\tau}$$

The Heisenberg equation of motion

$$\dot{\gamma}_j(t) = i[H^{(H)}(t), \gamma_j(t)]$$

$$H^{(H)}(t) = \sum_{a,b,c=1,2,3} \epsilon_{abc} B_a(t)\gamma_b(t)\gamma_c(t)$$

$$\boldsymbol{B}(t) = [1-\alpha(t)]\zeta_{23}(1,0,0) + \alpha(t)\zeta_{1,2}(0,0,1)$$

Majorana fermions transport

$$\dot{\gamma}_a = 2\epsilon_{abc} B_b(t) \gamma_c(t)$$

spin-1/ 2 particle in a time-dependent magnetic field $\boldsymbol{B}(t)$

The initial condition:

$$\alpha(0) = 0$$

$$\boldsymbol{B}(0) = \zeta_{23}(1,0,0)$$

$$\sigma(t) = \sigma_1(0)$$

Majorana fermions transport

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The final state:

$$\alpha(t_1) = 1$$

$$\boldsymbol{B}(t_1) = \zeta_{12}(0,0,1)$$

$$\gamma_3(t_1) = \mathrm{sgn}(\zeta_{12}\zeta_{23})\gamma_1(0)$$

Majorana fermions transport

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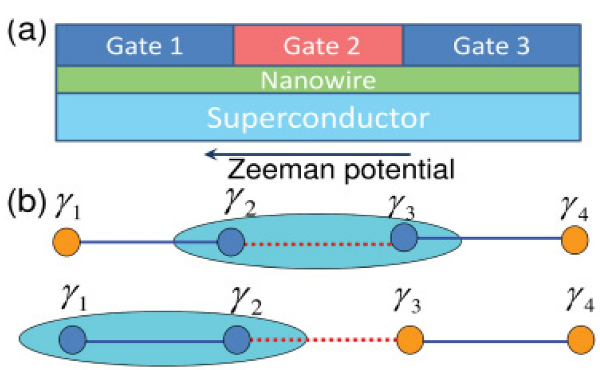
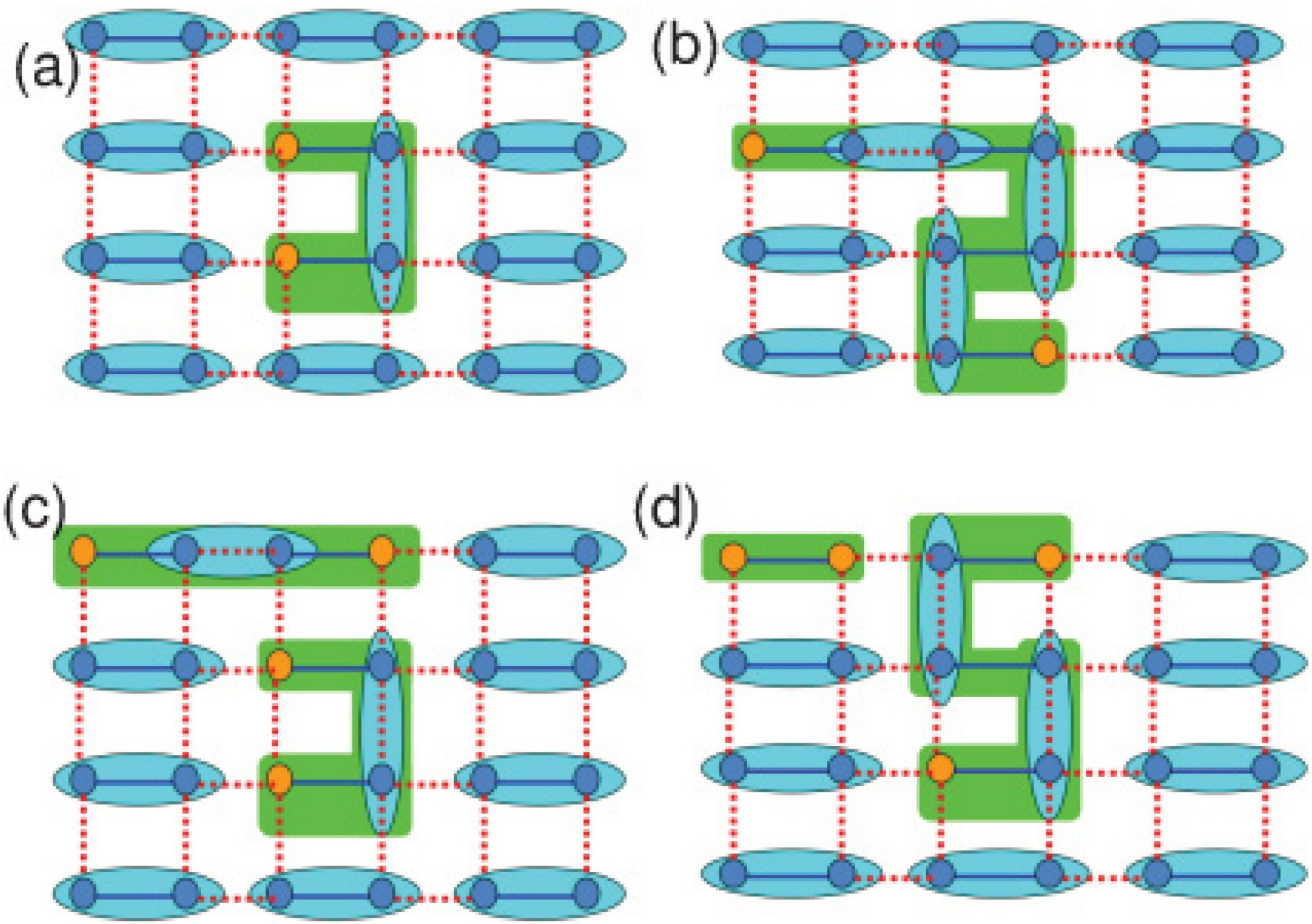


$$1 \xrightarrow{2} 3$$

Majorana fermions in dimer lattice



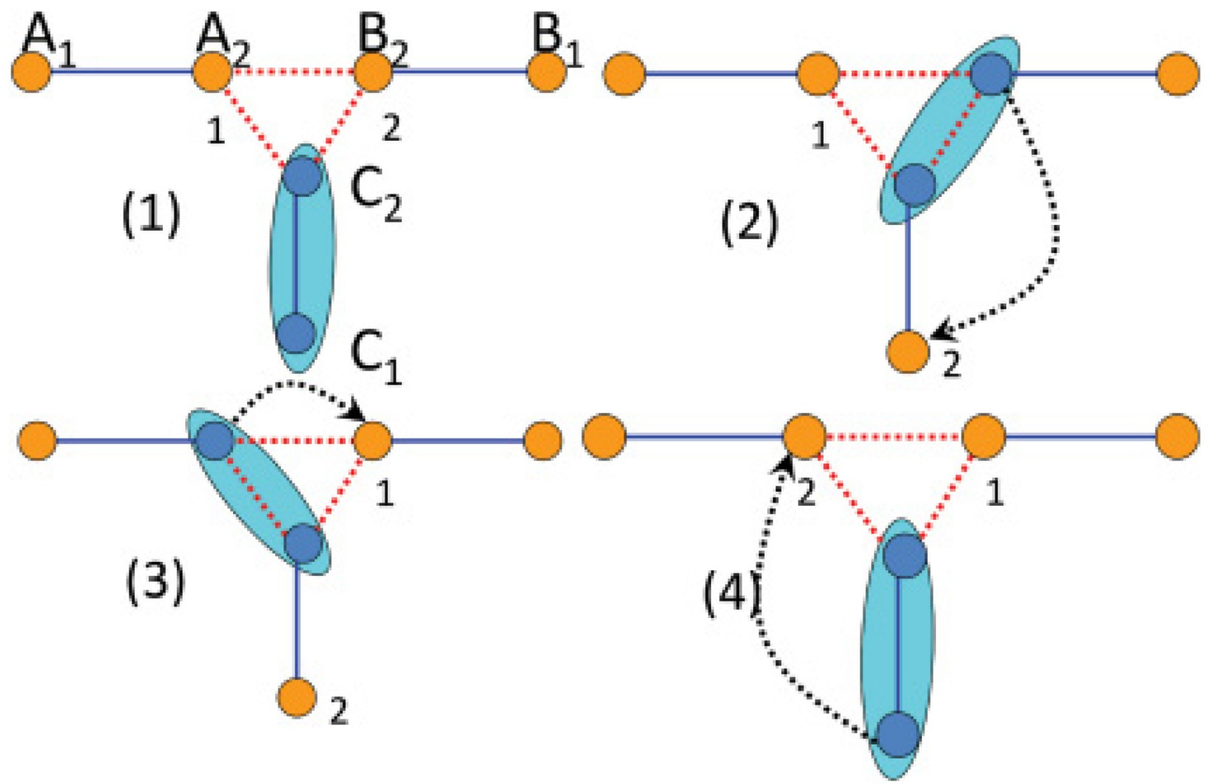
Non-Abelian statistics

$$\gamma_1(t_{\text{final}}) = \lambda \gamma_2(0), \quad \gamma_2(t_{\text{final}}) = \tilde{\lambda} \gamma_1(0)$$

$$\lambda \tilde{\lambda} = -1$$

$$U = e^{\frac{\pi}{4} \lambda \gamma_1 \gamma_2}$$

Exchange of the MFs at the ends of different segments



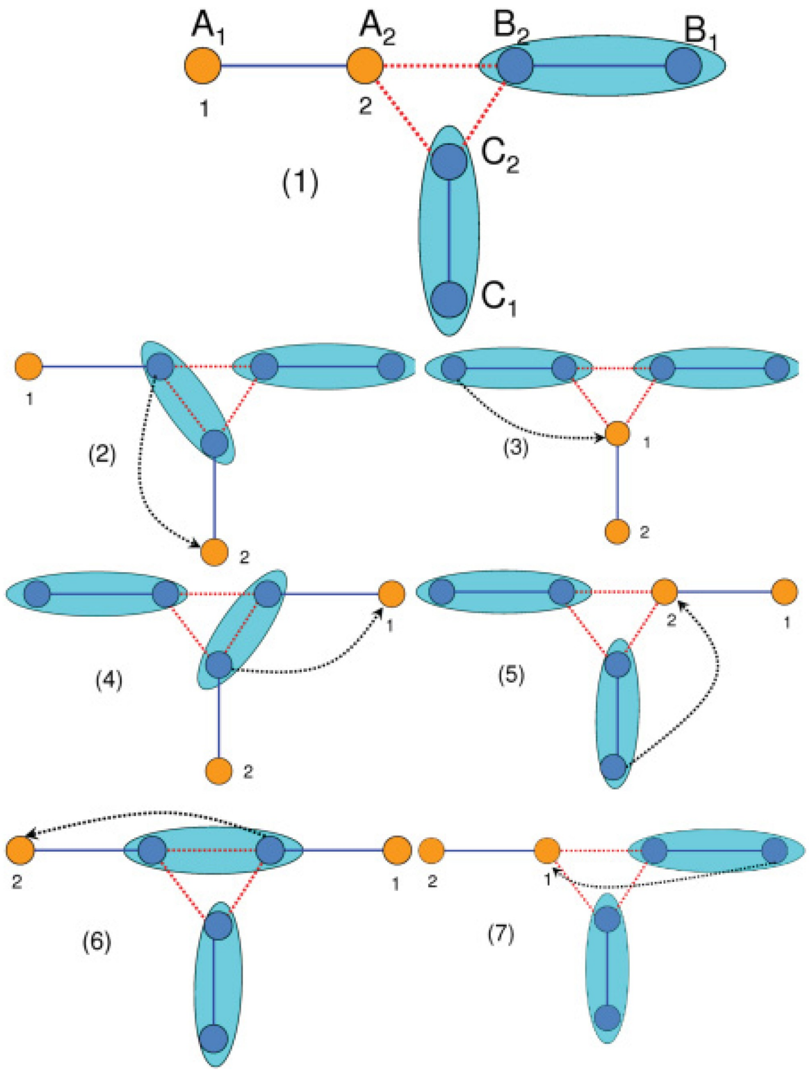
$$\text{MF 1: } A_2 \xrightarrow[(3)]{C_2} B_2$$

$$\text{MF 2: } B_2 \xrightarrow[(2)]{C_2} C_1 \xrightarrow[(4)\equiv(1)]{C_2} A_2$$

$$\lambda = -\tilde{\lambda} = \mathrm{sgn}(\zeta_{A_2 B_2})\chi$$

$$\chi = \mathrm{sgn}(\zeta_{A_2 B_2}\zeta_{B_2 C_2}\zeta_{C_2 A_2})$$

Exchange of the MFs at the ends of the same segment



$$\text{MF 1: } A_1 \xrightarrow[(3)]{A_2} C_2 \xrightarrow[(4)]{B_2} B_1 \xrightarrow[(7)\equiv(1)]{B_2} A_2$$

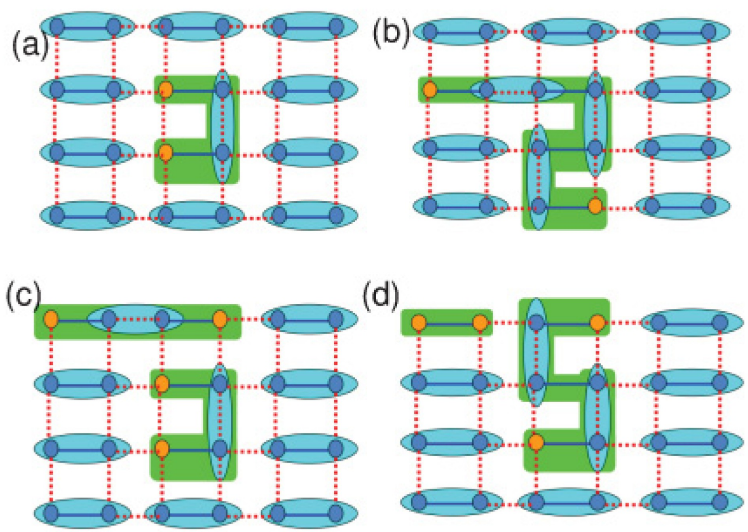
$$\text{MF 2: } A_2 \xrightarrow[(2)]{C_2} C_1 \xrightarrow[(5)]{C_2} B_2 \xrightarrow[(6)]{A_2} A_1$$

$$\lambda = -\tilde{\lambda} = \operatorname{sgn}(\zeta_{A_1A_2})\chi$$

$$U = e^{\frac{\pi}{4}\chi \operatorname{sgn}(\zeta_{12})\gamma_1\gamma_2}$$

Summary

Non-Abelian statistics for MFs at the ends of TS nanowire segments can be realized by introducing time-varying gate controllable tunnelings between MFs.



Thank you for your attention