Enhanced transport when Anderson localization is destroyed Y. Krivolapov, L. Levi, S. Fishman, M. Segev, and M. Wilkinson, arXiv:1110.3024

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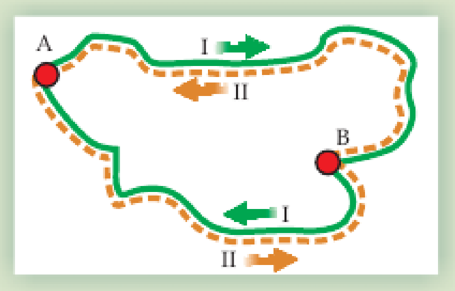
Anderson localization: the basics

"Absence of diffusion in certain random lattices",
P. W. Anderson, Phys. Rev. **109**, 1492 (1958) ($>$ $\mathbf{5000}$ citations!)

a wave-packet (or a particle) moving in a spatially-disordered,
(time-independent) potential exhibits localization!



After: Lagendijk et al., Phys. Today (2009)

A. L. most easily demonstrated in optical- and matter-wave systems
(potential proportional to the optical-field intensity: $V \propto |E|^2$)

What happens when localization gets destroyed?

Q: When Anderson localization is destroyed by time-dependent potentials, will the transport become diffusive?

reminder: $\langle x^2 \rangle \propto t$ (diffusive)
$\langle x^2 \rangle \propto t^2$ (ballistic)

there are models that predict 'anomalous behavior' (e.g., superdiffusion):
L. Golubović, S. Feng, and F.-A. Zeng, PRL **67**, 2115 (1991)

THIS PAPER: study of potentials that are random also in time!

- The resulting transport is faster than diffusion, and can even be faster than ballistic!
- "Radical differences" between 1d and 2d !

The (classical) model

Objective: Study the rate of spreading of the initially localized wave-packet (when A.L. is destroyed)

$$H = \frac{p^2}{2} + V(x,t)$$

one-dimensional (quasi-periodic ??) potential

$$V(x,t) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} A_m \exp[i(k_m x - \omega_m t)]$$

N finite, but large!

A_m – independent complex random numbers that satisfy

$$\langle A_m \rangle = \langle A_m A_n \rangle = 0\,;\;\; \langle A_m A_n^* \rangle = A^2 \delta_{mn}$$

$k_m,\ \omega_m$ distributed according to $P(k, \omega)$

classical model?

Earlier studies

studied for small $|A_m|$: B. V. Chirikov, Phys. Rep. **52**, 263 (1979)

non-overlapping Chirikov resonances:

$$\frac{d}{dt}(k_m x - \omega_m t) = k_m p - \omega_m = 0 \;\Rightarrow\; p_m^{\mathrm{res}} = \frac{\omega_m}{k_m}$$

a particle with an initial momentum close to a resonance
shows bounded pendulum-like motion near that resonance!

non-overlapping:

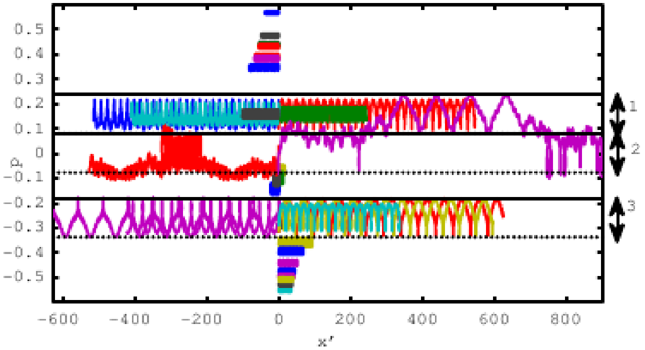
$$\Delta_m + \Delta_{m-1} \leq p_m^{\mathrm{res}} - p_{m-1}^{\mathrm{res}} \qquad \Delta_m = \sqrt{8|A_m|/\sqrt{N}}$$

Q: What happens if the resonances have overlaps?

A: random walk between resonances, i.e., diffusion in momentum

$\Rightarrow$ find the diffusion coefficient $D(p)$

Dynamics of weakly overlapping resonances



$$C(x_1, t_1; x_2, t_2) = \langle F(x_1, t_1) F(x_2, t_2) \rangle$$

$$D(p) = \frac{1}{2} \int_{-\infty}^{\infty} C(p\tau, \tau) d\tau$$

Scaling of $D(p)$ and Fokker-Planck equation

$$D(p) = 4\pi A^2 \int dk \int d\omega\, k^2 P(k,\omega)\delta(\omega - kp) \Rightarrow D(p) \sim \frac{D_0}{p^3}$$

spreading in momentum: Fokker-Planck eqn. for momentum density $\rho(p)$

$$\frac{\partial \rho}{\partial t} = \left(\frac{\partial}{\partial p} D(p) \frac{\partial}{\partial p}\right)\rho$$

General: $$\frac{\partial \mathrm{a}}{\partial t} = \mathrm{v}(\mathrm{a}) + \mathbf{F}(t) \quad \text{with} \quad \langle \mathbf{F}(t)\mathbf{F}(t')\rangle = 2\mathrm{B}\delta(t - t')$$

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \mathrm{a}}[\mathrm{v}(\mathrm{a}) f] + \frac{\partial}{\partial \mathrm{a}} \cdot \mathrm{B} \cdot \frac{\partial}{\partial \mathrm{a}} f$$

Implications of the scaling of $D(p)$

$$D(p) \sim p^{-3} \;\Rightarrow\; \langle p^2 \rangle \sim t^{2/5} \;\Rightarrow\; \langle x^2 \rangle \sim t^{12/5}$$

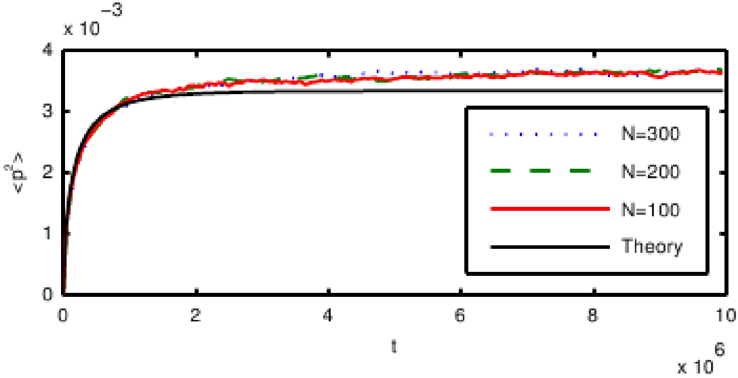
superballistic!

more realistic assumption: $P(k, \omega) = 0$ for $\omega \geq p_{\max} k$

$$D(p) = \begin{cases} 4\pi A^2 \int dk\, k^2 P(k, pk)\,, & |p| \leq p_{\max} \\ 0\,, & |p| > p_{\max} \end{cases}$$

$\Rightarrow$ ballistic behavior (established numerically!)

$\langle p^2 \rangle$ vs. t in 1d: theory vs. simulation



theory: Fokker-Planck equation

simulation: direct numerical propagation,
averaged over realizations of disorder

Behavior in $d > 1$

Chirikov resonance condition in $\boldsymbol{d > 1}$:

$$(\mathbf{k}_m - \mathbf{k}_n) \cdot \mathbf{p}_{mn}^{\mathrm{res}} = \omega_m - \omega_n$$

diffusion in the momentum is unbounded!

Asymptotically: $D_{||}(p) \sim 1/p^3$

yields a universal asymptotic expansion in momentum

$$\langle p^2 \rangle \sim t^{2/5}$$

and a superballistic expansion rate in position space

Theory vs. simulation in 2d