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Energy Partitioning of Tunneling Currents into Luttinger Liquids

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Tunneling of electrons of definite chirality into a quantum wire creates counterpropagating excitations, carrying both charge and energy. We find that the partitioning of energy is qualitatively different from that of charge. The partition ratio of energy depends on the excess energy of the tunneling electrons (controlled by the applied bias) and on the interaction strength within the wire (characterized by the Luttinger-liquid parameter κ), while the partitioning of charge is fully determined by κ. Moreover, unlike for charge currents, the partitioning of energy current should manifest itself in dc experiments on wires contacted by conventional (Fermi-liquid) leads.

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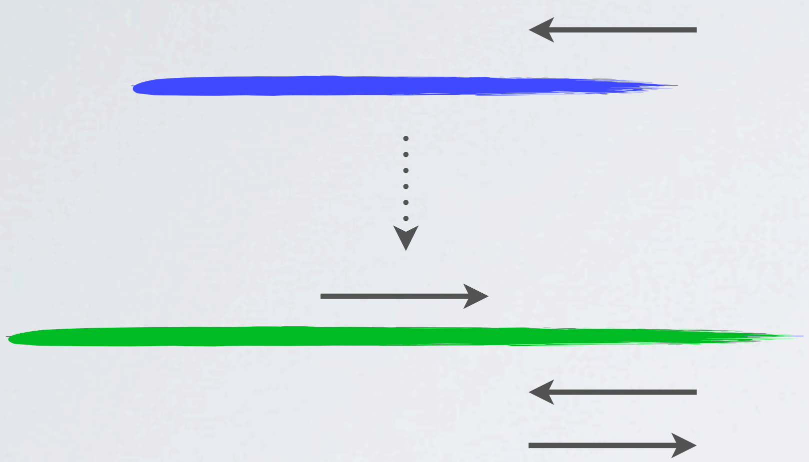
Journal club 25-10-201, by Kevin van Hoogdalem

SETUP



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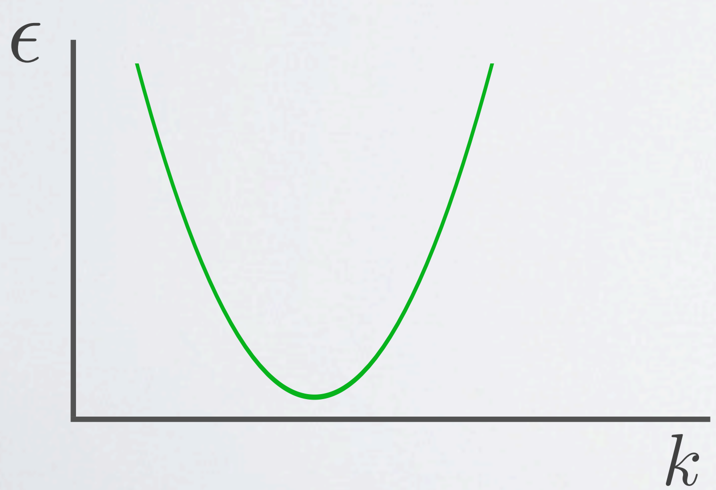

Right-going (blue) into left-going (green) only

$$\mathbf{p} \rightarrow \mathbf{p} + e\mathbf{A}$$

$$(p_x \rightarrow p_x - eyB)$$

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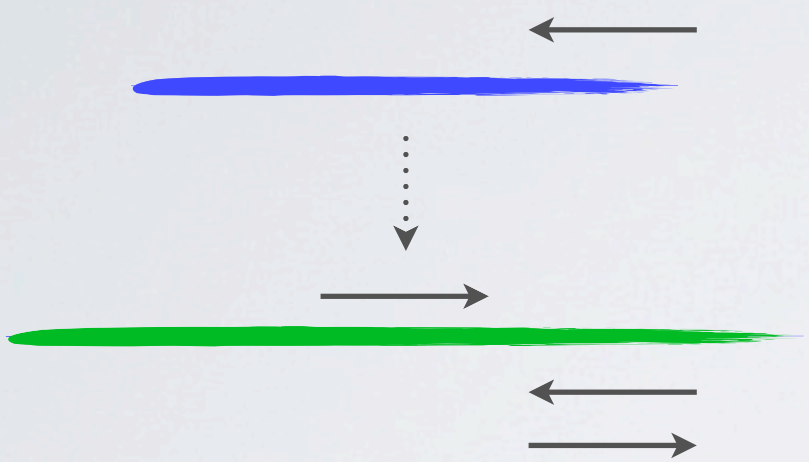

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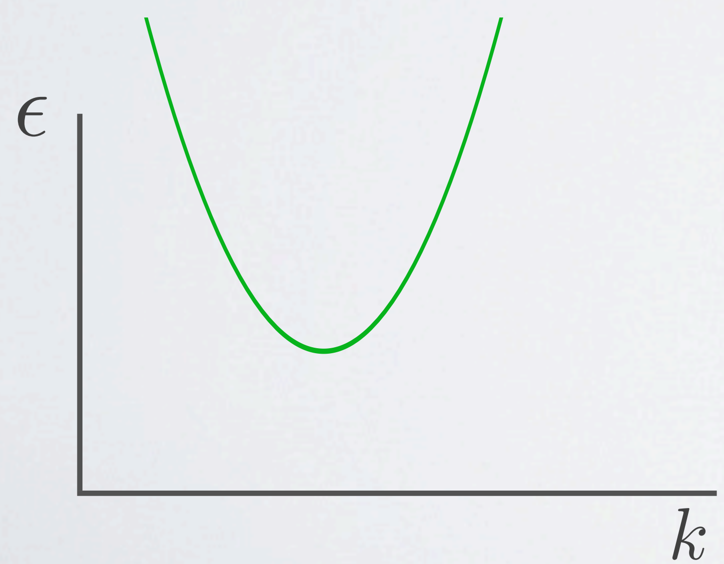

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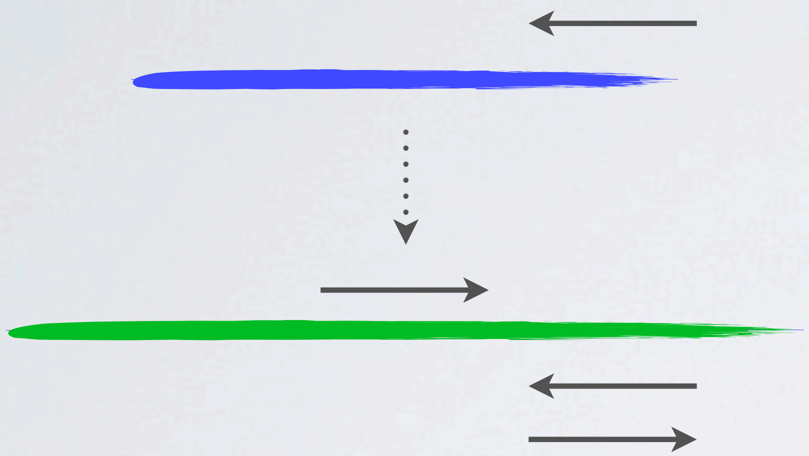

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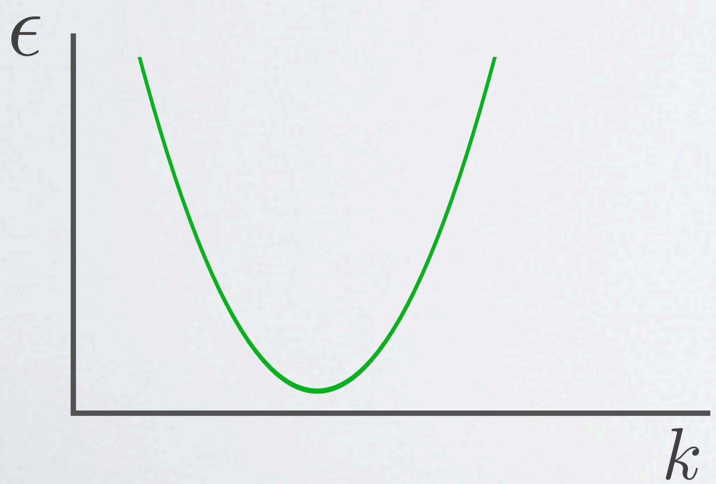

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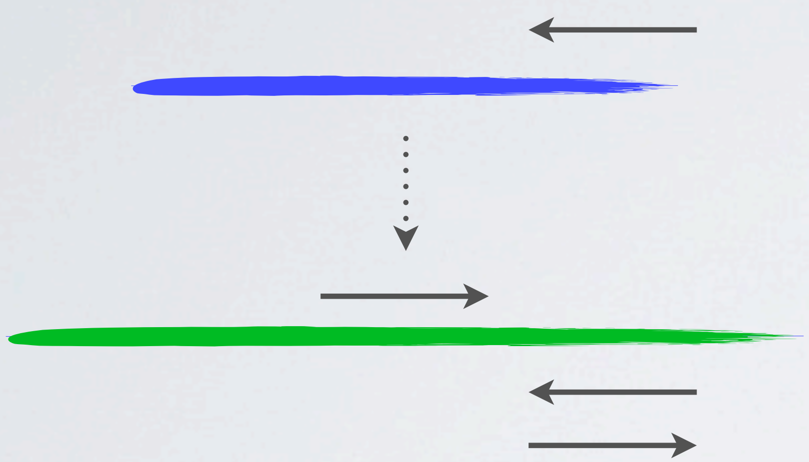

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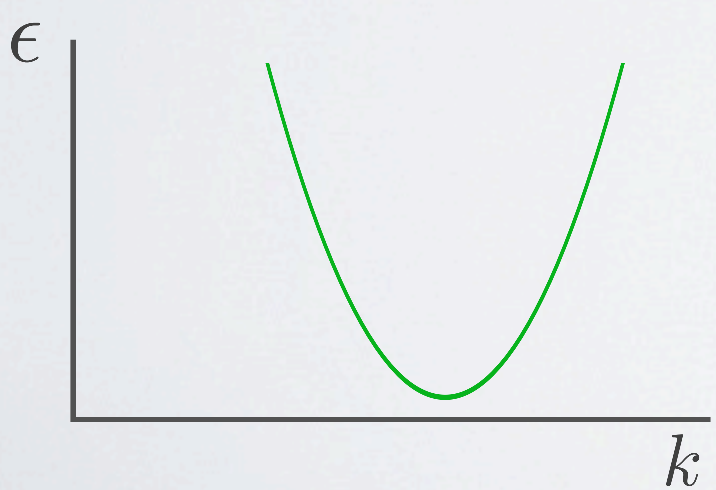

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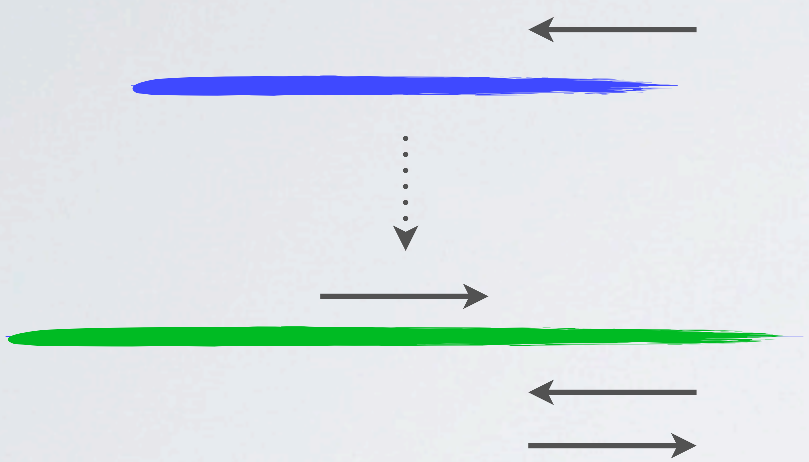

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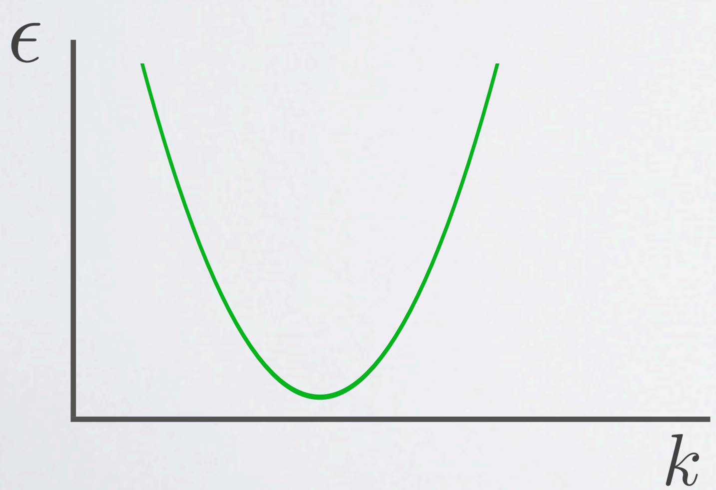

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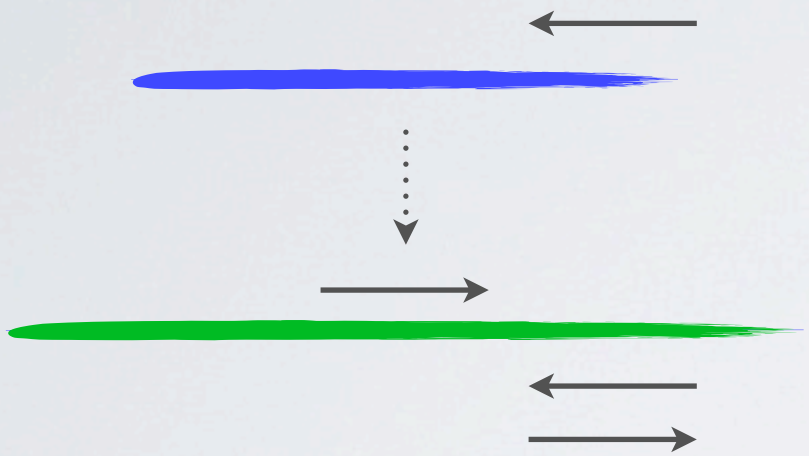

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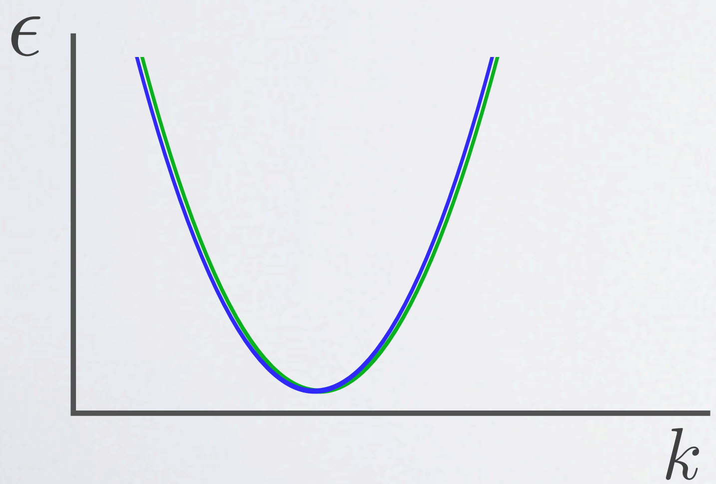

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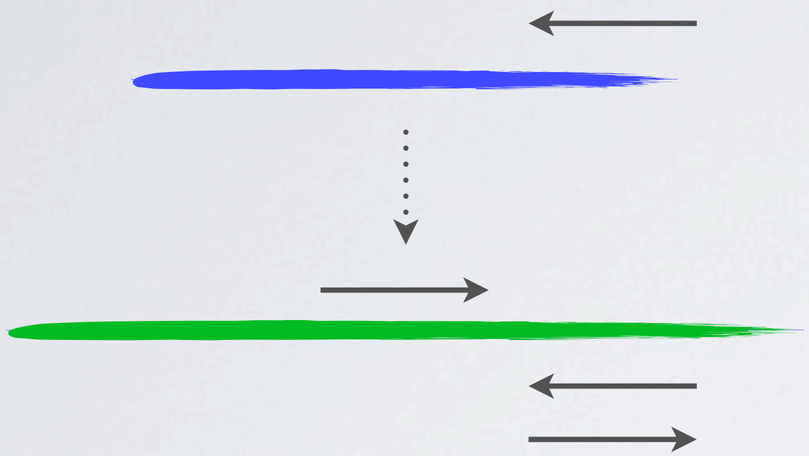

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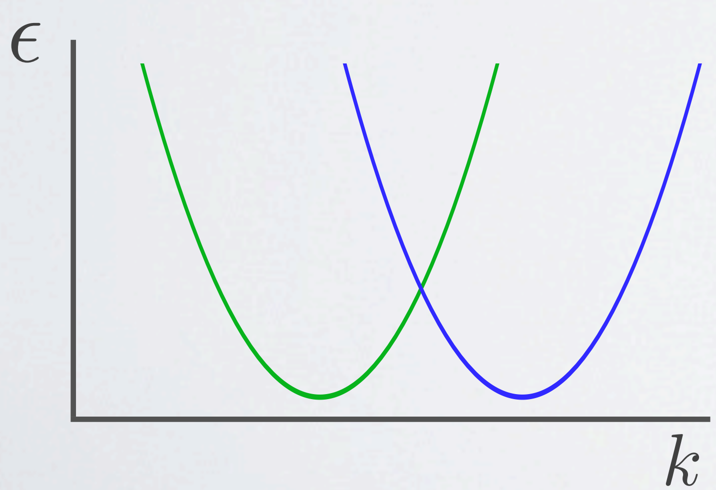

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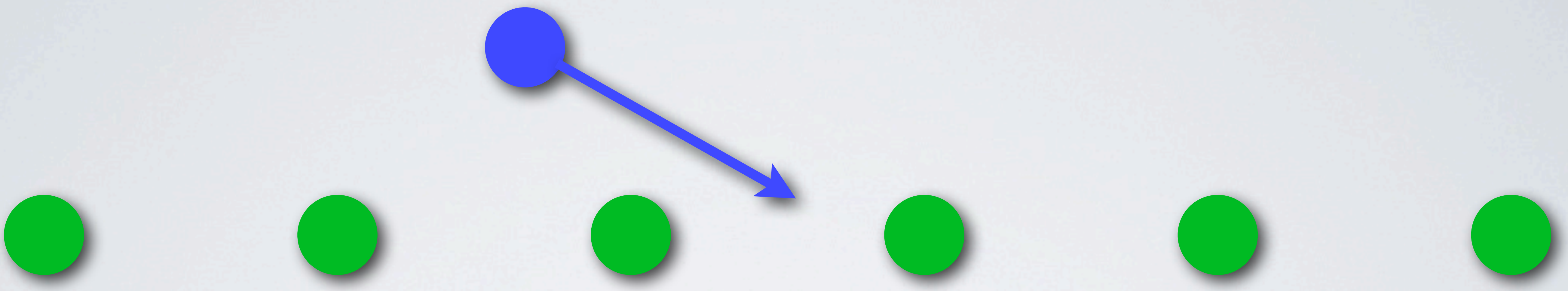

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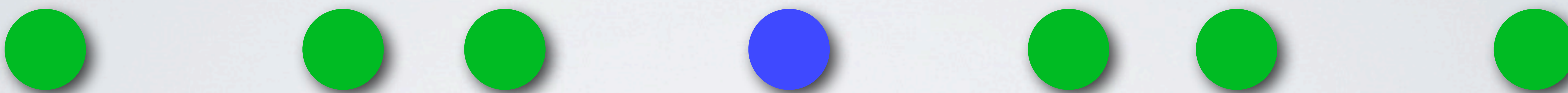

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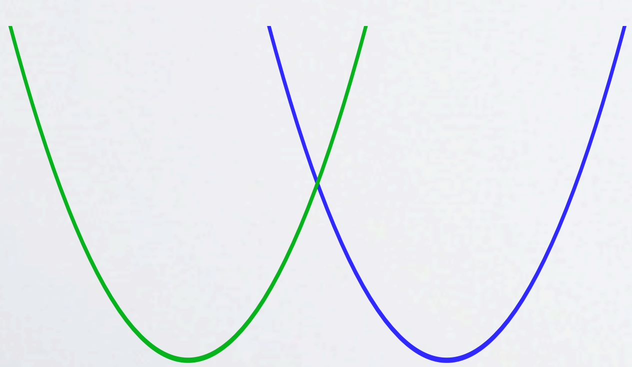

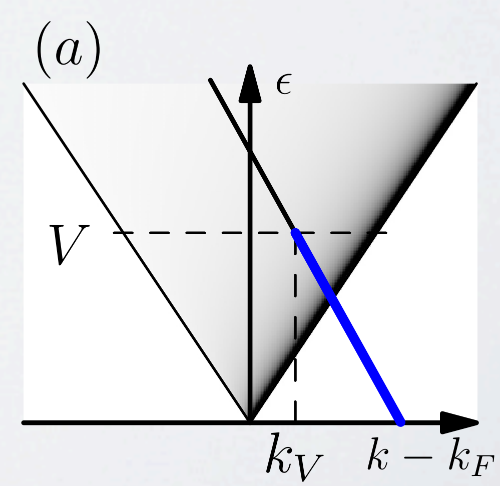

HAMILTONIAN

Description by bosonic fields

$$\phi(x) \propto \rho_+(x) + \rho_-(x) \qquad \theta(x) \propto \rho_+(x) - \rho_-(x)$$

$$\theta_\pm(x) = \theta(x) \pm \phi(x)/\kappa$$

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Hamiltonian wire

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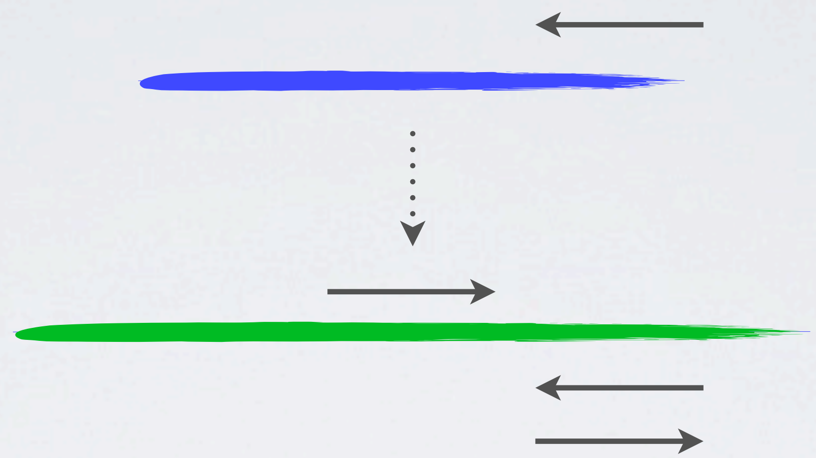
Tunneling Hamiltonian

$$H_T = t \int_S \mathrm{d}x \left[\psi_R^\dagger(x)\psi_S(x) + \mathrm{h.c.}\right]$$

PARTITION OF CHARGE CURRENT



Total charge current: $$I(x,t) = \frac{ie}{\hbar}\left[H_T, \rho_-(x) + \rho_+(x)\right]$$

Left-going current: $$I_-(x,t) = \frac{ie}{\hbar}\left[H_T, \rho_-(x)\right] = \frac{1-\kappa}{2} I(x,t)$$

Right-going current: $$I_+(x,t) = \frac{ie}{\hbar}\left[H_T, \rho_+(x)\right] = \frac{1+\kappa}{2} I(x,t)$$

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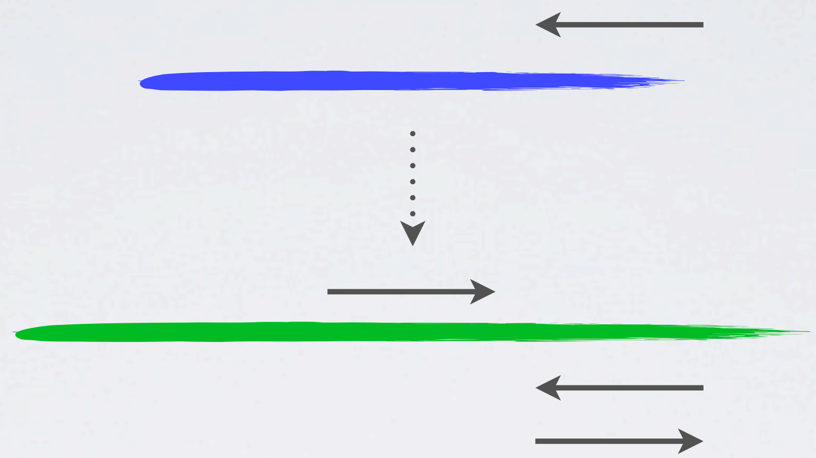


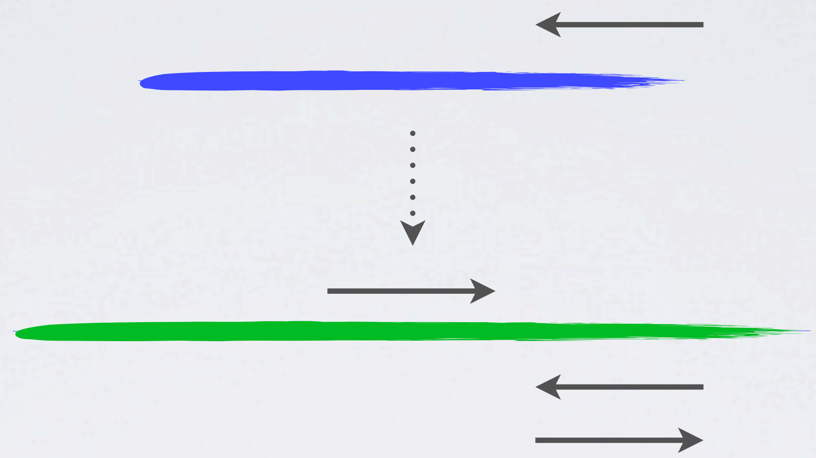
Electron coming in (Charge 1, momentum mv_F)
Eigenstates are CDWs (Charge $Q_\pm$, momentum $mc=mv_F/K$)

Conservation of charge and momentum

$$mv_F = Q_+mc - Q_-mc$$
$$Q_+ + Q_- = 1$$

Gives $Q_+=(1+K)/2$ and $Q_-=(1-K)/2$

PARTITION OF CHARGE CURRENT



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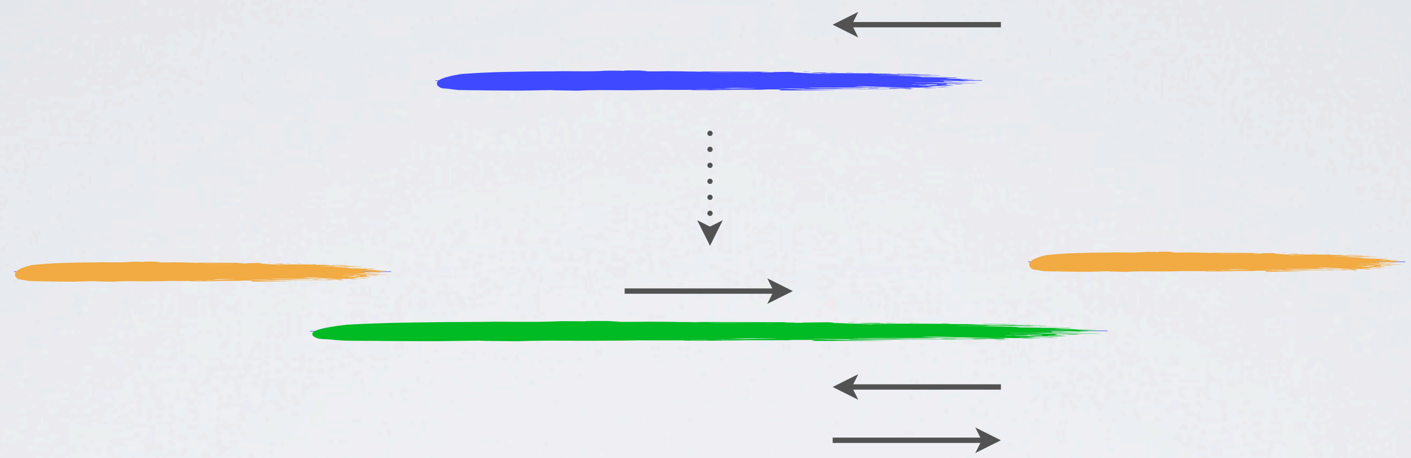

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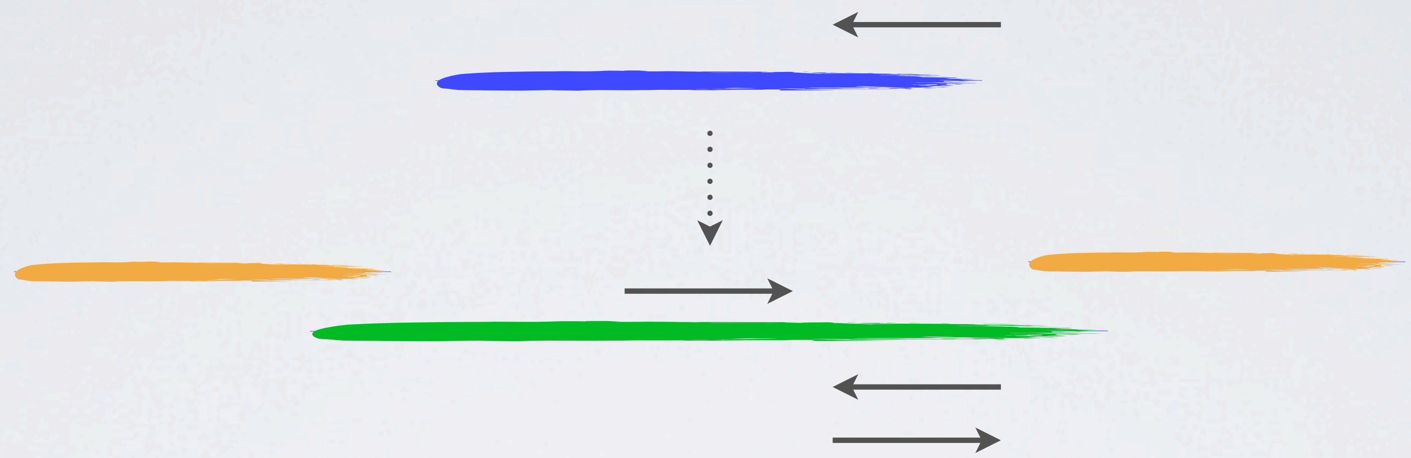

In the leads: $I_+(x,t) = I(x,t)\ ,\ I_-(x,t) = 0$

LOOK AT ENERGY CURRENTS

Definition energy current:

$$I_{\pm}^{E} = i\left[H_T, \frac{v_F}{4\pi}\int \mathrm{d}x\,(\nabla\theta_{\pm})^2\right]$$

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To leading order in tunneling

$$\langle I^E_\pm\rangle = \frac{Q^2_\pm t^2 L_S}{\kappa}\int\frac{\mathrm{d}\epsilon}{2\pi}\int\frac{\mathrm{d}k}{2\pi}\int_0^\infty \mathrm{d}\omega_q\{G^>_{+,k\mp q}(\epsilon_S-\omega_q)$$
$$\times G^<_{S,k}(\epsilon_S-eV)+G^<_{+,k\pm q}(\epsilon_S+\omega_q)G^>_{S,k}(\epsilon_S-eV)\}$$

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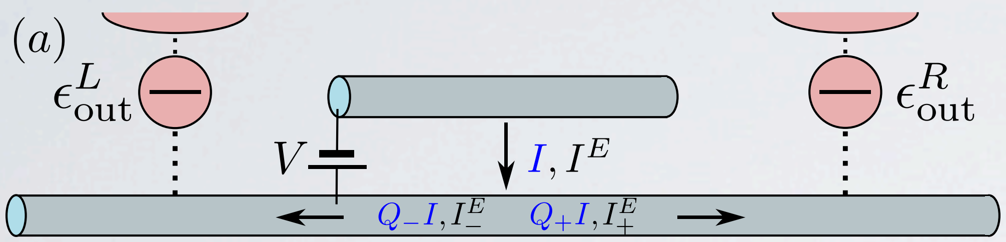
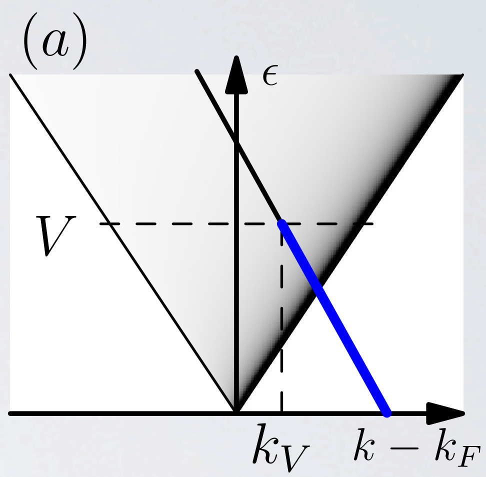
Greens functions are given by

$$G^>_k(\epsilon) = -iA(k,\epsilon)\left[1-n_F(\epsilon)\right] \qquad A_\pm \propto |\omega\mp ck|^{\varphi-1}|\omega\pm ck|^\varphi\theta(|\omega|-c|k|)$$

$$\varphi = (\kappa+\kappa^{-1}-2)/4$$

COMPARISON ENERGY AND CHARGE CURRENT





Differential conductance

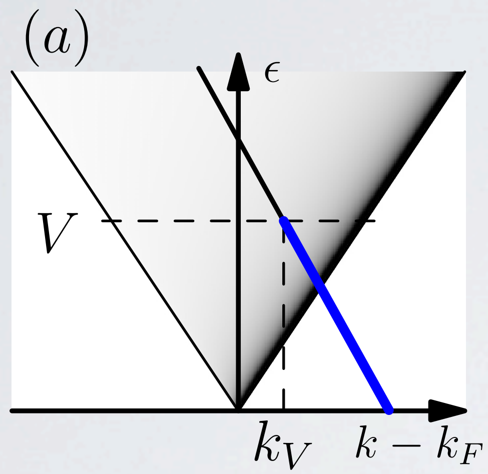
Energy current

$$\frac{d\left\langle I^E_\pm\right\rangle/dV}{d\left\langle I\right\rangle/dV} = \frac{1}{2}(eV \pm ck_V)$$

Charge current

$$\frac{d\left\langle I_\pm\right\rangle/dV}{d\left\langle I\right\rangle/dV} = Q_\pm$$

CONSERVATION ARGUMENT



Electron with energy > Fermi energy

$$\epsilon = c|k_+| + c|k_-| \qquad k_V = k_+ + k_-$$

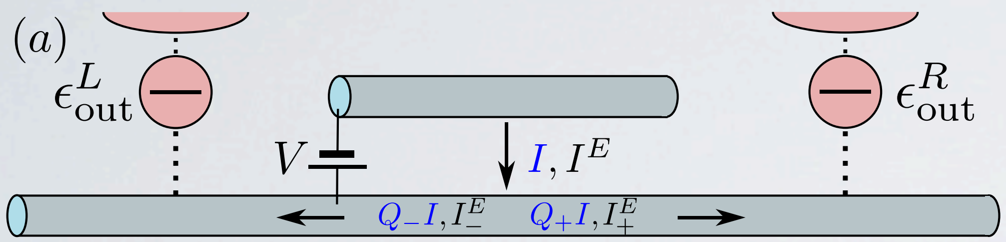
So energy in density waves

$$\epsilon_\pm = c|k_\pm| = (\epsilon \pm ck_V)/2$$

Condition for energy going to the left, charge going to the right:

$$eV \approx -ck_V \text{ and } Q_+ \approx 1$$

MEASURING ENERGY CURRENT



Energy in density waves

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Tunneling possible for: $\epsilon_\pm > \epsilon_{\rm out}^{R/L}$

With Fermi leads

For low energies: $R_E = (c - v_F)^2/(c + v_F)^2$

For high energies: Reflection exponentially suppressed

THE END