Quantum phase slips in superconducting wires with weak links

(arXiv:1108.3553v2)

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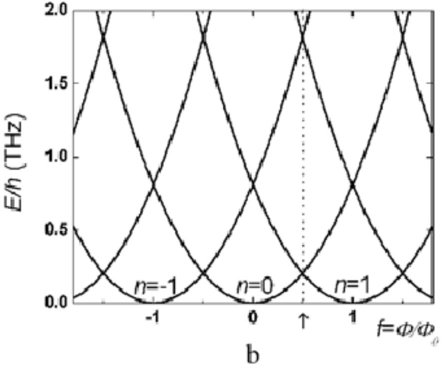
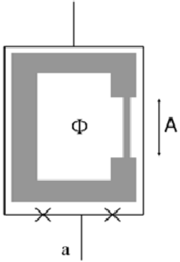
Quantum Phase Slips (QPS)

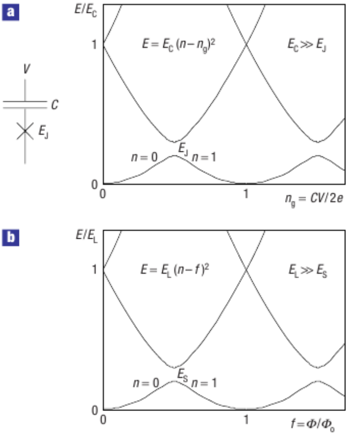


$$n\Phi_0 = \Phi - L_g I_s - L_k I_s$$
$$L_k I_s = \frac{\Phi_0}{2\pi}\gamma$$
$$\gamma = 2\pi(f - n)$$
$$E = \frac{\Phi_0^2}{2L_k}(f - n)^2$$



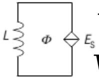
J. Mooij and C. Harmans, New J. Phys. **7**, 219 (2005)

Duality



$$\hat{H} = E_C \hat{q}^2 - E_J \cos\hat{\varphi} + \hat{H}_{env} + \hat{H}_c$$

$$\hat{H}_c = \begin{cases} \frac{\Phi_0}{2\pi}(I - \hat{I}_r)\hat{\varphi} \\ -2e(V - \hat{V}_r)\hat{q} \end{cases}$$



$$\hat{I}_r(\omega) = \frac{\hbar}{2e}(-i\omega)Y(\omega)\hat{\varphi}(\omega)$$

$$\hat{V}_r(\omega) = 2e(-i\omega)Z(\omega)\hat{q}(\omega)$$

$$\hat{H} = \frac{E_L}{(2\pi)^2}\hat{\varphi}^2 - E_S \cos 2\pi\hat{q} + \hat{H}_{env} + \hat{H}_c$$

J. Mooij and Y. Nazarov, Nat. Phys. **2**, 169 (2006)

Model

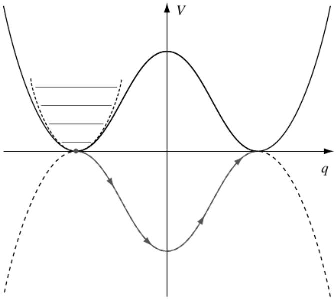
$$\mathcal{S}_c[\phi] = -\frac{1}{2}\sum_p \mathrm{Tr}\ln\left(1 + \frac{T_p}{4}(\{\hat{G}_1, \hat{G}_2\} - 2)\right)$$

$$\hat{G}_j(\tau,\tau') = e^{i\phi_j(\tau)\hat{\tau}_3/2}\,\hat{G}_0(\tau-\tau')\,e^{-i\phi_j(\tau)\hat{\tau}_3/2}$$

$$\mathcal{S}_{\mathrm{w}}[\phi] = (8\pi^2 G_Q)^{-1}\int_0^\infty d\omega\,\omega\,Y(\omega)\,|\phi(\omega)|^2$$

$$Y(\omega) = [\mathcal{L}'(\omega)/C']^{-1/2}[\tanh(\omega L_1/v_p) + \tanh(\omega L_2/v_p)]^{-1} - (L\mathcal{L}'_k\omega)^{-1}$$

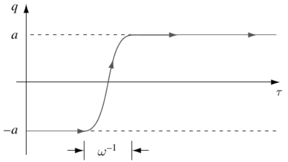
U. Eckern, G. Schön, and V. Ambegaokar, Phys. Rev. B **30**, 6419 (1984)
A. Legget, Phys. Rev. B **30**, 1208 (1984)



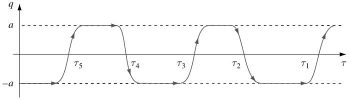
$$G(-a, a, \tau) = \int Dq \exp\left[-\frac{1}{\hbar}\int_0^\tau \left(\frac{m}{2}\dot{q}^2 + V(q)\right)\right]$$

$$-m\ddot{q} + V'(q) = 0$$

$$S_{\text{in}} = \int_{-a}^{a} dq \sqrt{2mV(q)}$$



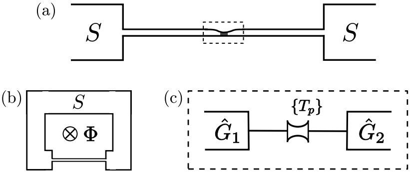
$$E_S = 2\left(\int d\tau m \dot{q}_{\text{in}}^2/2\pi\right)^{1/2} (D')^{-1/2}\, e^{-\mathcal{S}_{\text{in}}}$$



$$D' = \det{}'(\delta^2\mathcal{S}/\delta\phi^2|_{\text{in}})/\det(\delta^2\mathcal{S}/\delta\phi^2|_0)$$

R. Rajaraman, *Solitons and Instantons*

Main result



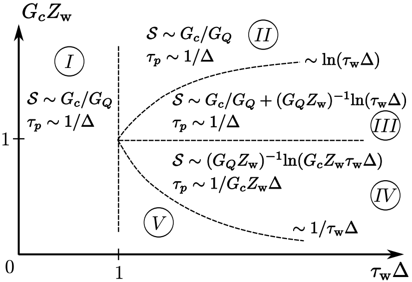
$$\mathcal{S}_c \simeq (G_c/G_Q)\,\max(1, \tau_p\Delta)$$

For $\tau_w \gg \tau_p$

$$\mathcal{S}_\mathrm{w} \simeq (G_Q Z_\mathrm{w})^{-1}\ln(\tau_\mathrm{w}/\tau_p)$$

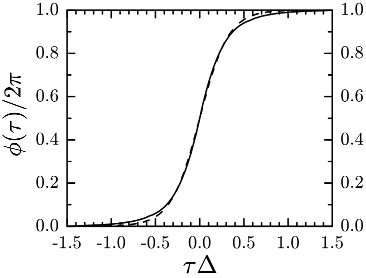
For $\tau_w \ll \tau_p$

$$\mathcal{S}_\mathrm{w} \simeq LC'/G_Q\tau_p$$



$$E_S \approx 2\Delta\sqrt{\sum_p T_p \prod_p \sqrt{1 - T_p}}$$

Tunnel limit of the weak link



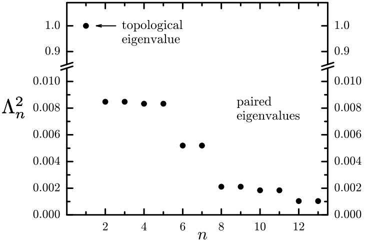
$$\mathcal{S}_{\rm t}[\phi] = \frac{G_c}{G_Q}\frac{\Delta^2}{\pi^2}\int d\tau d\tau' \times\left[\sin^2\left(\frac{\phi(\tau)-\phi(\tau')}{4}\right) K_1^2(|\tau-\tau'|\Delta) +\sin^2\left(\frac{\phi(\tau)+\phi(\tau')}{4}\right) K_0^2(|\tau-\tau'|\Delta)\right]$$

$\tau_p \gg 1/\Delta$
$\mathcal{S}_{\rm t} = \int d\tau\, E_J(1-\cos\phi)$

$\tau_p \ll 1/\Delta$
$\mathcal{S}_{\rm t} \sim (G_c/G_Q)\ln(1/\tau_p\Delta)$

$\mathcal{S}_{\rm t} \sim (G_c/G_Q)\tau_p\Delta \gg G_c/G_Q.$

Topological parts of the weak link action



$$\{\hat{G}_1, \hat{G}_2\} - 2 = -(\hat{G}_1 - \hat{G}_2)^2$$

$$\hat{\Lambda} \equiv (\hat{G}_1 - \hat{G}_2)/2$$

$$\{\hat{\Lambda}, \hat{G}_1 + \hat{G}_2\} = 0$$

$$\mathcal{S}_c[\phi] = -\frac{1}{2}\sum_{p,n} \ln(1 - T_p \Lambda_n^2).$$

$$|\,\mathrm{Tr}(\hat{\Lambda})| = |N_+ - N_-| = \lfloor |\int_{-\infty}^{\infty} \dot{\phi}(\tau)\, d\tau|/2\pi \rfloor$$

$$\mathcal{S}_{c1}(N) = -\frac{N}{2}\sum_{p} \ln(1 - T_p)$$

Phase-slip amplitude of the weak link action

$$D'E_J = \frac{\det'(H_1)}{\det'(H_0)} \qquad H_1 \equiv H_0 + \delta H$$

$$H_0(\omega) = (2E_J/\pi)\, E(i\omega/2\Delta) \qquad (\delta H)_{mm} = -(G_c/G_Q)(\textstyle\sum_n \Lambda_n^2)/2\tau_0$$

$$\ln(D'E_J) = \sum_{m=2}^{\omega_c/\delta\omega} \ln(q_m) - \left(\frac{G_c}{G_Q}\sum_n \Lambda_n^2\right)\frac{1}{4\pi}\int_{\omega_c}^{\infty} \frac{d\omega}{H_0(\omega)}$$

$$H_0(\omega) = E_J|\omega|/\pi\Delta, \quad \tau_p \ll 1/\Delta$$

$$\Downarrow$$

E_S diverges!

Coulomb interaction

$$\frac{dT_p}{d\ln E} = -\frac{T_p(1-T_p)}{\sum_p T_p}$$

$$E_S = a\left(\frac{\Delta}{2\pi}\int d\tau\,\dot{\phi}^2_{\text{in}}(\tau)\right)^{1/2}\left(\sum\nolimits_p T_p\right)^{1/2} e^{-\mathcal{S}_{\text{in}}}$$