Light-cone-like spreading of correlations in a quantum many-body system

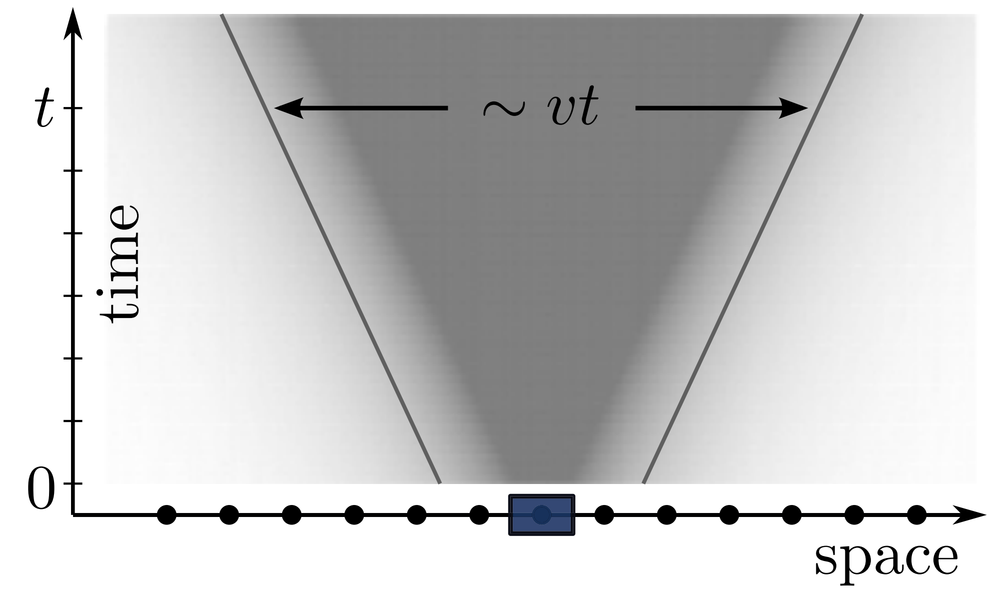
How fast can correlations spread in a quantum many-body system?

Marc Cheneau, Peter Barmettler, Dario Poletti, Manuel Endres, Peter Schauß, Takeshi Fukuhara, Christian Gross, Immanuel Bloch, Corinna Kollath, and Stefan Kuhr

arXiv:1111.0776

Journal Club 22.11.2011 - Andreas Nunnenkamp

Commun. math. Phys. 28, 251—257 (1972)



The Finite Group Velocity of Quantum Spin Systems

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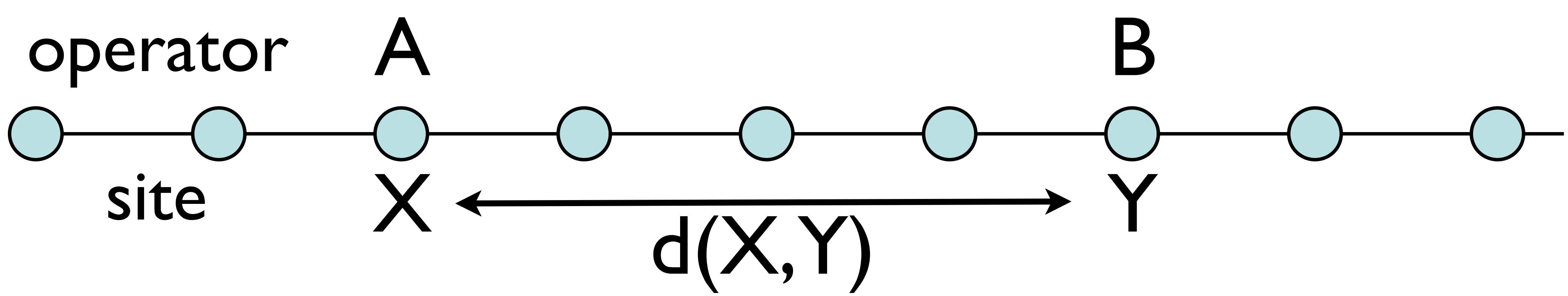
Received May 15, 1972



$$\| [\tau_t(A) , B] \| \leq C e^{-a(d(X,Y) - v|t|)}$$

light cone

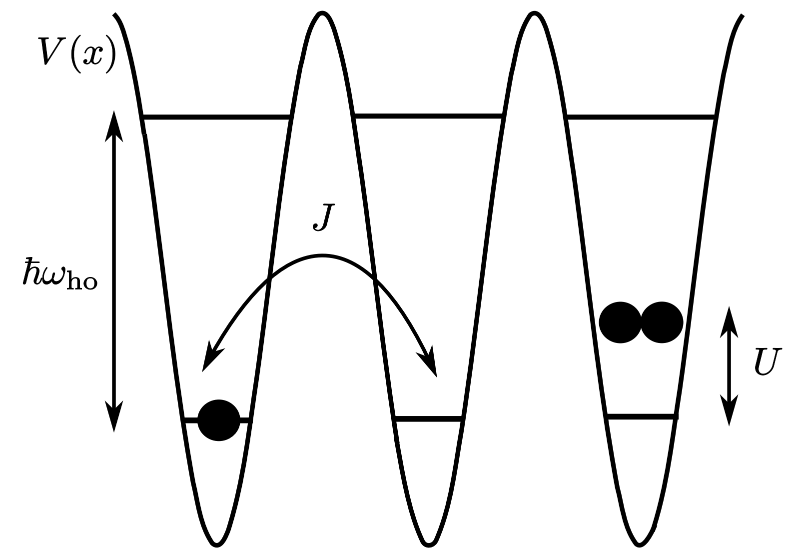
VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

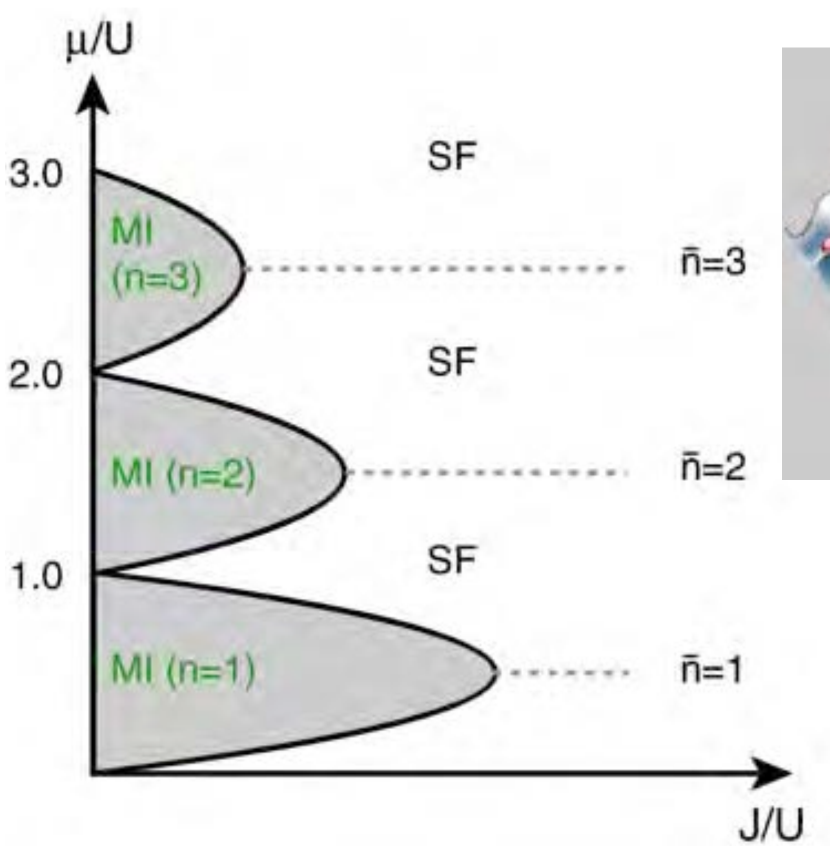
Cold Bosonic Atoms in Optical Lattices

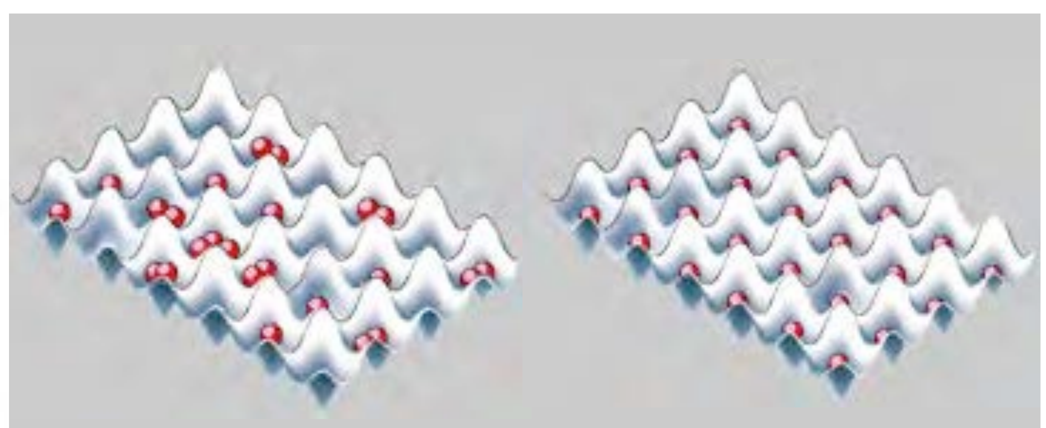
D. Jaksch,[1,2] C. Bruder,[1,3] J.I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]

Bose-Hubbard Hamiltonian

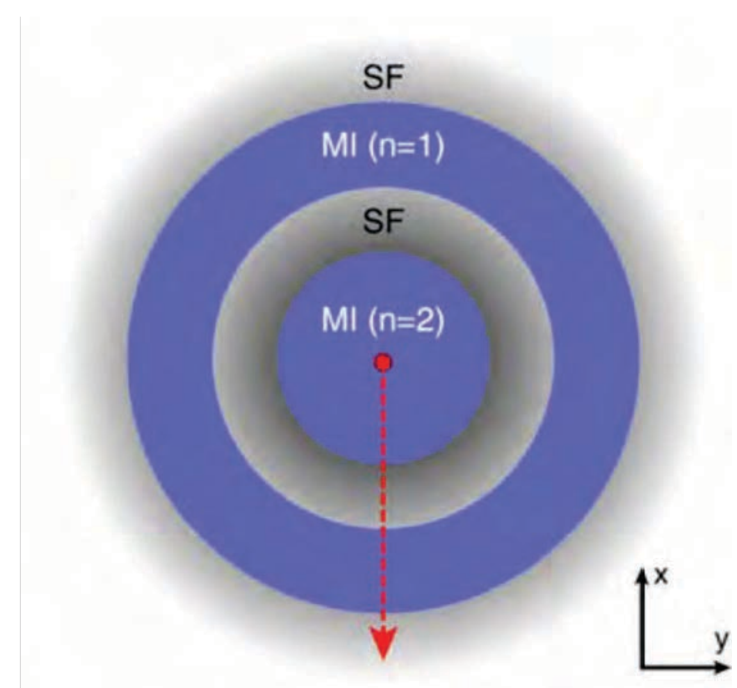
$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$







$$(U/J)_c = z \times 5.8$$

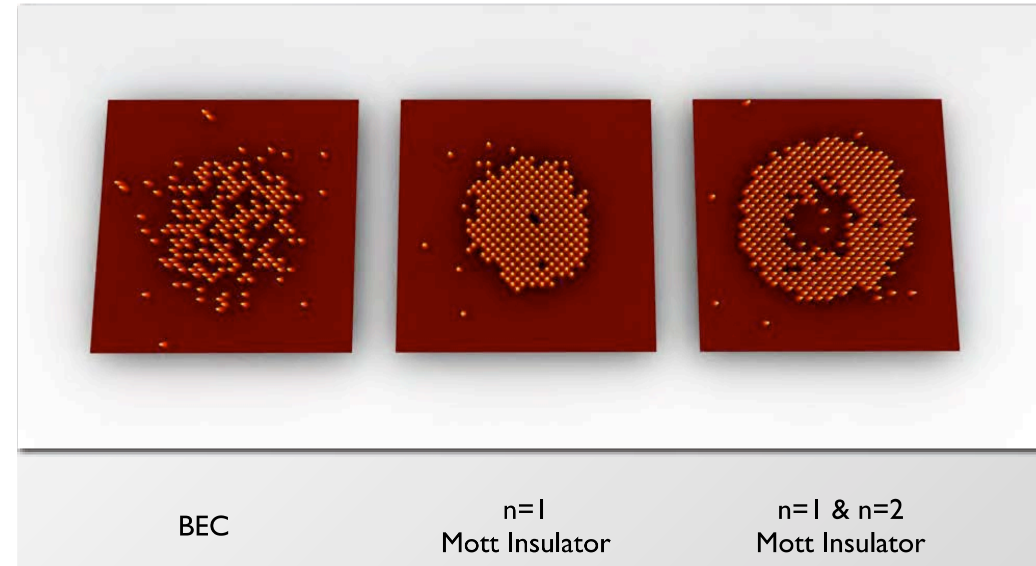


nature Vol 467|2 September 2010|doi:10.1038/nature09378

LETTERS

Single-atom-resolved fluorescence imaging of an atomic Mott insulator

Jacob F. Sherson[1]*†, Christof Weitenberg[1]*, Manuel Endres[1], Marc Cheneau[1], Immanuel Bloch[1,2] & Stefan Kuhr[1]

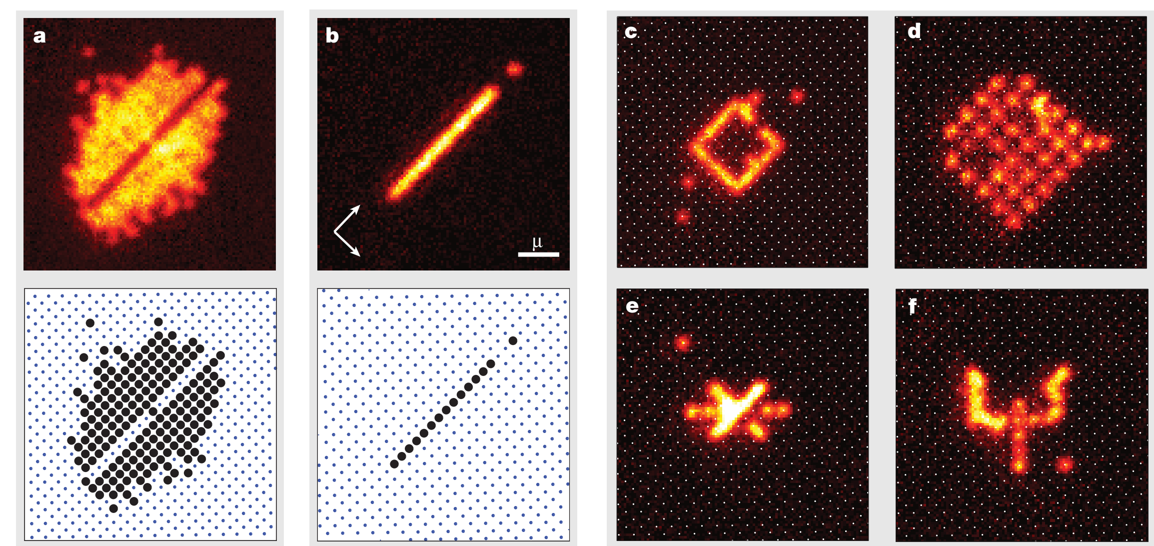


ARTICLE

17 MARCH 2011 | VOL 471 | NATURE | 319
doi:10.1038/nature09827

Single-spin addressing in an atomic Mott insulator

Christof Weitenberg[1], Manuel Endres[1], Jacob F. Sherson[1]†, Marc Cheneau[1], Peter Schauß[1], Takeshi Fukuhara[1], Immanuel Bloch[1,2] & Stefan Kuhr[1]

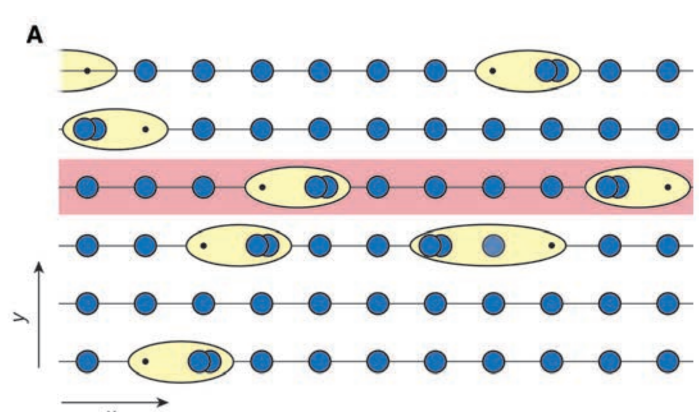


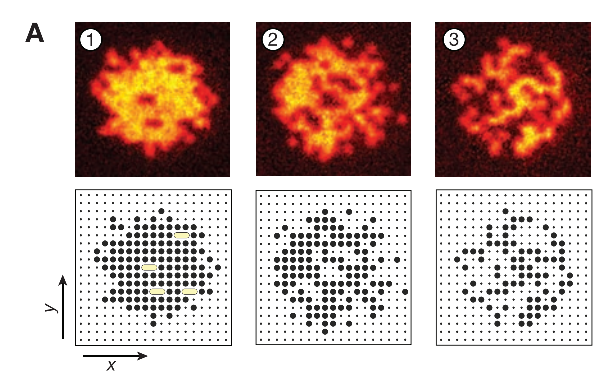
REPORTS

14 OCTOBER 2011 VOL 334 SCIENCE

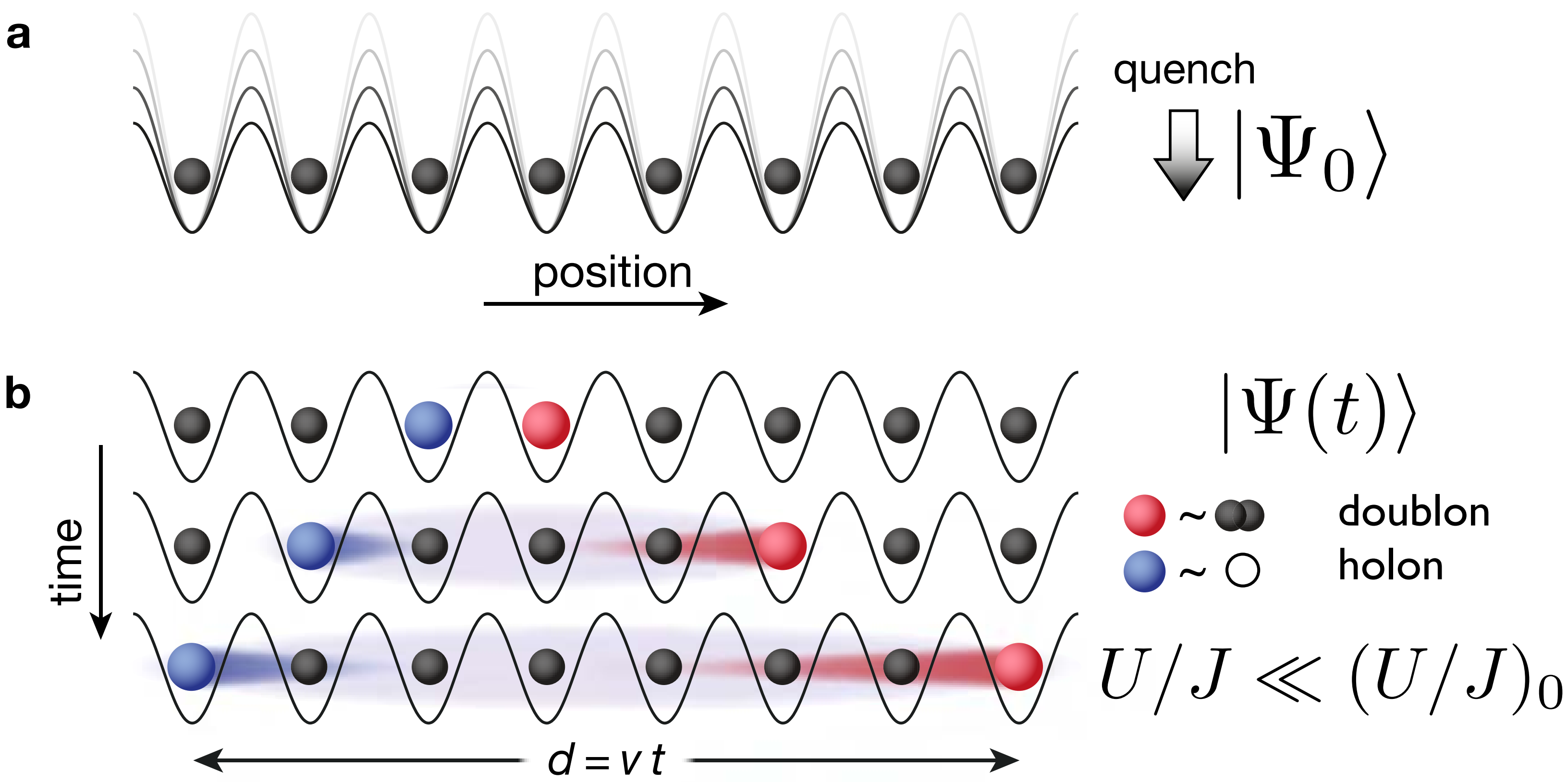
Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

M. Endres,[1]* M. Cheneau,[1] T. Fukuhara,[1] C. Weitenberg,[1] P. Schauß,[1] C. Gross,[1] L. Mazza,[1] M. C. Bañuls,[1] L. Pollet,[2] I. Bloch,[1,3] S. Kuhr[1,4]





Experimental sequence



Prepare $(U/J)_0 = 40$

$\bar{n} = 1$ Mott insulator

After the quench initial state is highly excited.

Quasi-particle model

$$\hat{H} = \sum_j \Big\{ -J\,(\hat{a}_j^\dagger\,\hat{a}_{j+1} + \mathrm{h.\,c.}) + \frac{U}{2}\hat{n}_j(\hat{n}_j - 1)\Big\}$$

Restrict Hilbert space $\hat{d}_j^\dagger|\,{}^{\circ}_{\bullet}\,\rangle_j \to |\,{}^{\bullet}_{\bullet}\,\rangle_j$ $\quad \hat{h}_j^\dagger|\,{}^{\circ}_{\bullet}\,\rangle_j \to |\,{}^{\circ}_{\circ}\,\rangle_j$

Use generalized Jordon-Wigner transformation (spin-1 to two flavors of constrained fermions)

$$\hat{H} = \sum_j \hat{\mathcal{P}}\,\Big\{ -2J\,\hat{d}_j^\dagger\,\hat{d}_{j+1} - J\,\hat{h}_{j+1}^\dagger\,\hat{h}_j - J\sqrt{2}(\hat{d}_j^\dagger\,\hat{h}_{j+1}^\dagger - \hat{h}_j\,\hat{d}_{j+1}) + \mathrm{h.\,c} + \frac{U}{2}\big(\hat{n}_{\mathrm{d},j} + \hat{n}_{\mathrm{h},j}\big)\Big\}\,\hat{\mathcal{P}}$$

with $\hat{n}_{\mathrm{d},j} = \hat{d}_j^\dagger\hat{d}_j$ and $\hat{n}_{\mathrm{h},j} = \hat{h}_j^\dagger\hat{h}_j$

$$\hat{\mathcal{P}} = \prod\nolimits_j(\hat{I} - \hat{n}_{\mathrm{d},j}\hat{n}_{\mathrm{h},j})$$

Replacing $\hat{\mathcal{P}} \to \hat{I}$ leaves us with a quadratic fermionic Hamiltonian.

Go to Fourier space and use Bogoliubov transformation

$$\hat{\gamma}_{\mathrm{d},k}^\dagger = u(k)\,\hat{d}_k^\dagger + v(k)\,\hat{h}_{-k}$$

$$\hat{\gamma}_{\mathrm{h},-k}^\dagger = u(k)\,\hat{h}_{-k}^\dagger - v(k)\,\hat{d}_k$$

$$u(k) = \cos[\theta(k)/2]$$

$$v(k) = i\sin[\theta(k)/2]$$

$$\theta(k) = \mathrm{atan}\left[\frac{\sqrt{32}J\sin(ka_{\mathrm{lat}})}{U - 6J\cos(ka_{\mathrm{lat}})}\right]$$

$$\epsilon_{\mathrm{d}}(k) = -J\cos(ka_{\mathrm{lat}}) + \frac{1}{2}\sqrt{[U - 6J\cos(ka_{\mathrm{lat}})]^2 + 32J^2\sin^2(ka_{\mathrm{lat}})}$$

$$\epsilon_{\mathrm{h}}(k) = J\cos(ka_{\mathrm{lat}}) + \frac{1}{2}\sqrt{[U - 6J\cos(ka_{\mathrm{lat}})]^2 + 32J^2\sin^2(ka_{\mathrm{lat}})}$$

This gives us the quench dynamics

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}\,|\psi_0\rangle = \prod_k \Big\{\bar{u}(k) - \bar{v}(k)\,e^{-i[\epsilon_{\mathrm{d}}(k)+\epsilon_{\mathrm{h}}(-k)]t/\hbar} \cdot \hat{\gamma}_{\mathrm{d},k}^\dagger\,\hat{\gamma}_{\mathrm{h},-k}^\dagger\Big\}|\mathrm{vac}\rangle$$

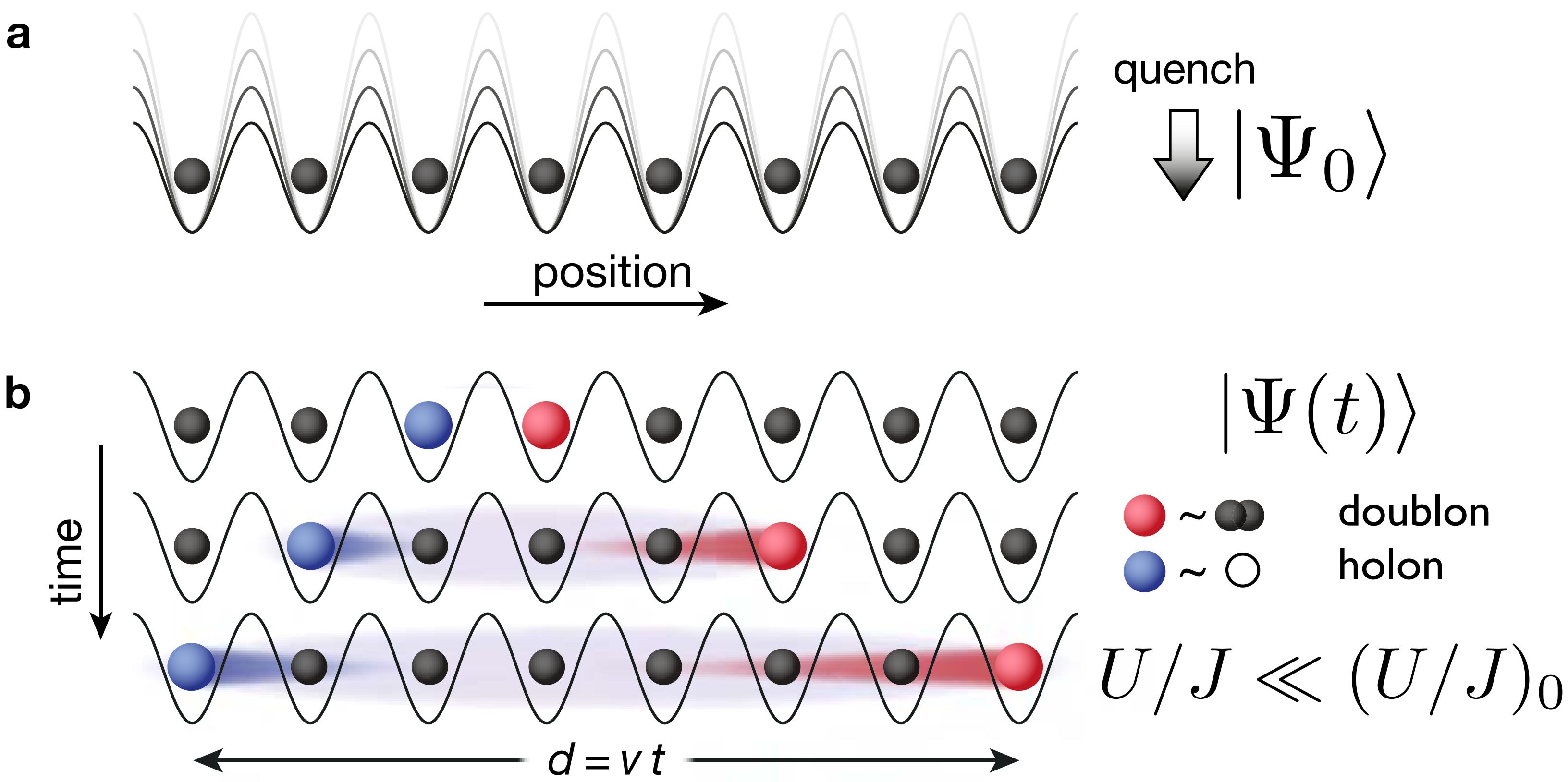
$$\bar{u}(k) = u(k)u_0(k) - v(k)v_0(k)$$

$$\bar{v}(k) = v(k)u_0(k) - u(k)v_0(k)$$

To first order this is

$$|\Psi(t)\rangle \simeq |\Psi_0\rangle + i\sqrt{8}\,\frac{J}{U}\sum_k \Big\{\sin(ka_{\mathrm{lat}}) \cdot \Big[1 - e^{-i[\epsilon_{\mathrm{d}}(k)+\epsilon_{\mathrm{h}}(-k)]t/\hbar}\Big]\,\hat{d}_k^\dagger\,\hat{h}_{-k}^\dagger\Big\}|\Psi_0\rangle$$

Experimental sequence



Prepare $(U/J)_0 = 40$

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Quasi-particle model

$$\hat{d}_j^\dagger |\substack{\circ\\ \bullet}\rangle_j \rightarrow |\substack{\bullet\\ \bullet}\rangle_j \quad \hat{h}_j^\dagger |\substack{\circ\\ \bullet}\rangle_j \rightarrow |\substack{\circ\\ \circ}\rangle_j$$

$$|\Psi(t)\rangle \simeq |\Psi_0\rangle + i\sqrt{8}\,\frac{J}{U}\sum_k \Big\{ \sin(k a_{\mathrm{lat}}) \cdot \left[1 - e^{-i[\epsilon_{\mathrm{d}}(k)+\epsilon_{\mathrm{h}}(-k)]t/\hbar}\right] \hat{d}_k^\dagger\, \hat{h}_{-k}^\dagger \Big\} |\Psi_0\rangle$$

Entangled quasi-particle pairs emerge at all sites and propagate in opposite directions.

Observable of choice

$$C_d(t) = \langle \hat{s}_j(t)\hat{s}_{j+d}(t)\rangle - \langle \hat{s}_j(t)\rangle\langle \hat{s}_{j+d}(t)\rangle$$

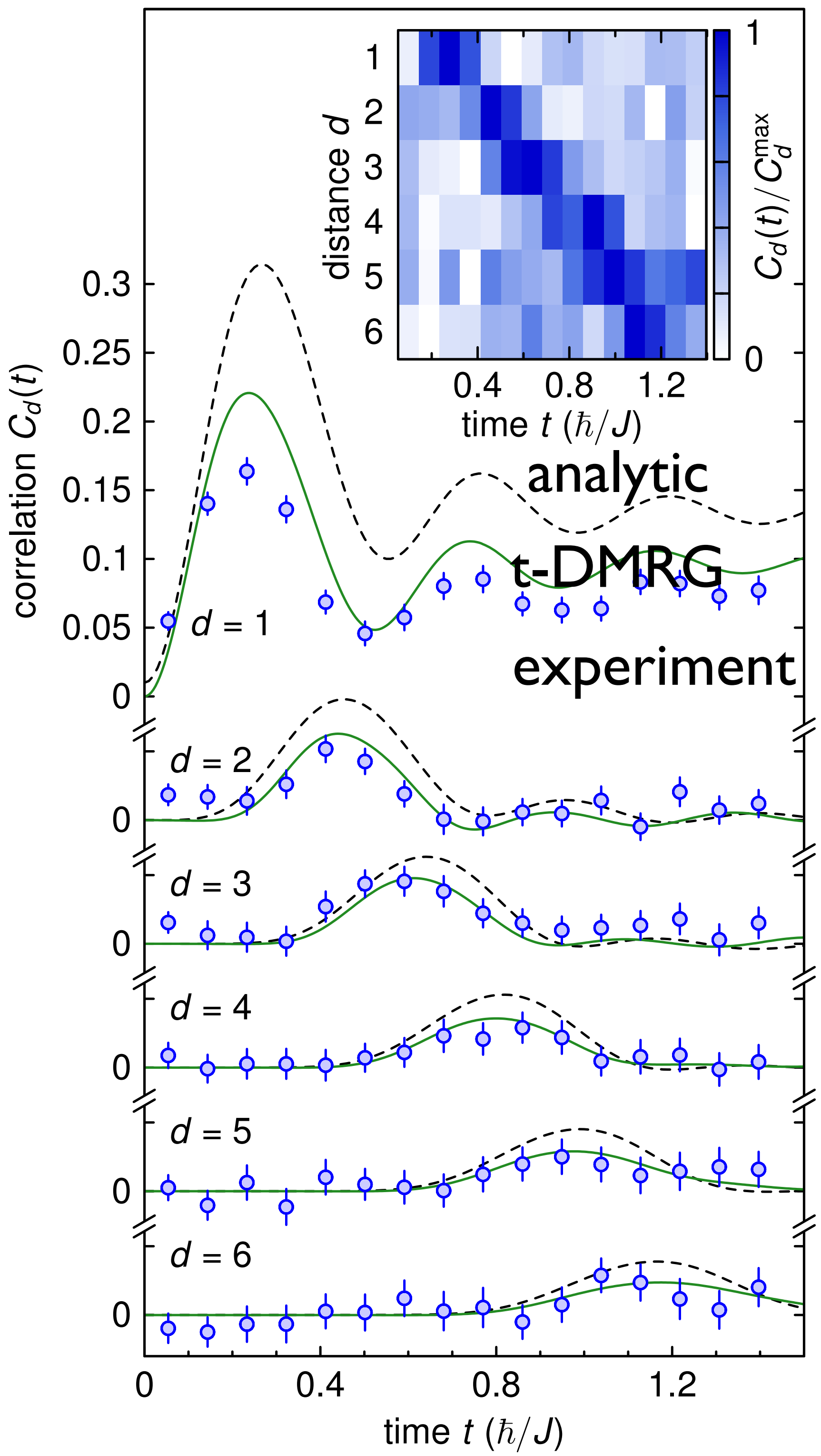
$$\hat{s}_j(t) = e^{i\pi[\hat{n}_j(t)-\bar{n}]}$$ number parity

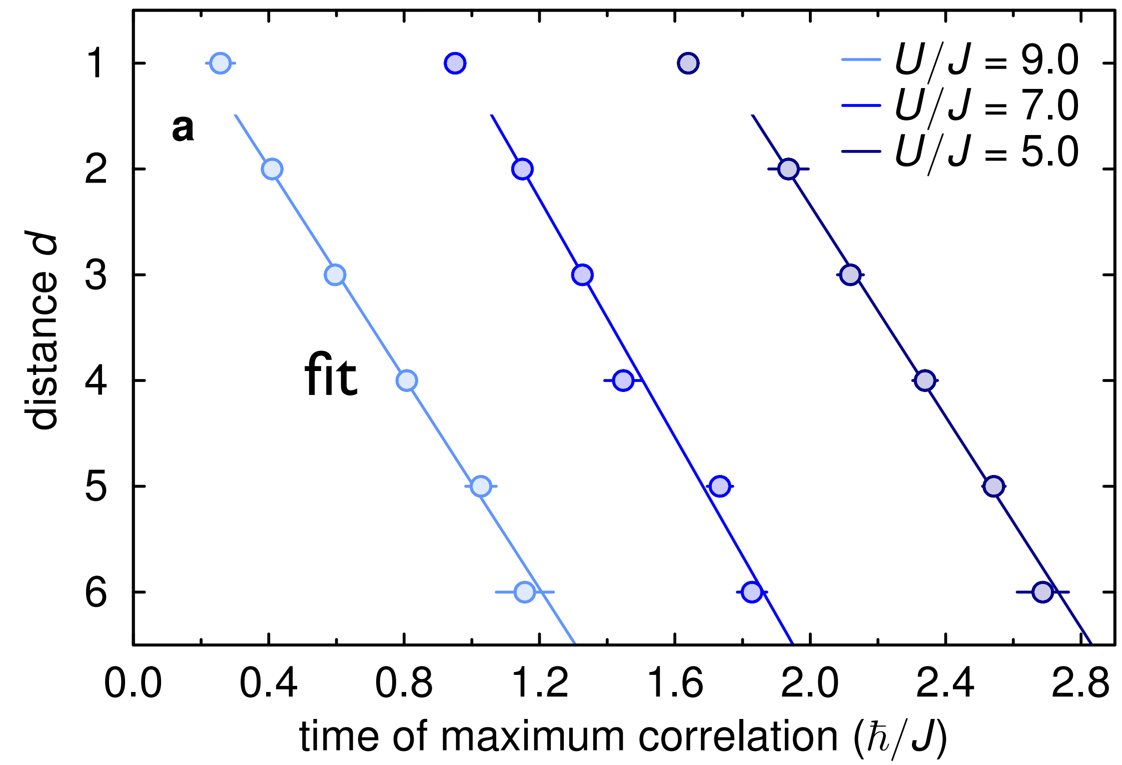
We expect a positive correlation between any pair of sites separated by a distance $d = vt$

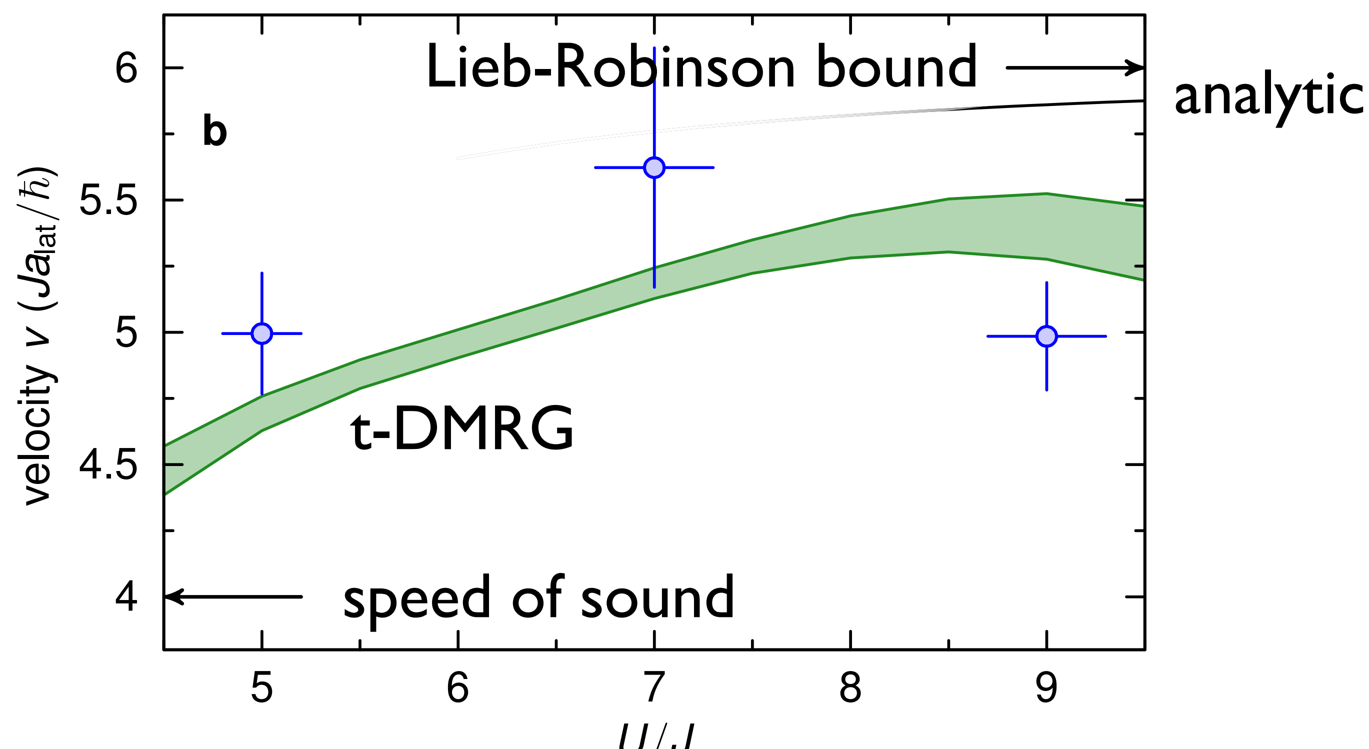
Experimental results

$$C_d(t) = \langle \hat{s}_j(t)\hat{s}_{j+d}(t)\rangle - \langle \hat{s}_j(t)\rangle\langle \hat{s}_{j+d}(t)\rangle$$

$$\hat{s}_j(t) = e^{i\pi[\hat{n}_j(t)-\bar{n}]}$$ number parity







"First experimental observation of an effective light cone for the spreading of correlations in an interacting quantum many-body system."