Journal Club, December 13, 2011

arXiv:1112.0869v1 [cond-mat.mes-hall]

Strong coupling of spin qubits to a transmission line resonator

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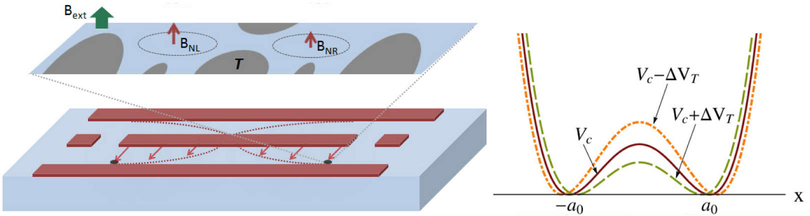
We propose a mechanism for coupling spin qubits formed in double quantum dots to a superconducting transmission line resonator. Coupling the resonator to the gate controlling the interdot tunneling creates a strong spin qubit–resonator interaction with strength of tens of MHz. This mechanism allows operating the system at a point of degeneracy where dephasing is minimized. The transmission line can serve as a shuttle allowing for two-qubit operations, including fast generation of qubit-qubit entanglement and the implementation of a controlled-phase gate.

Proposal

Challenge for spin qubits: generate non-local qubit-qubit interaction

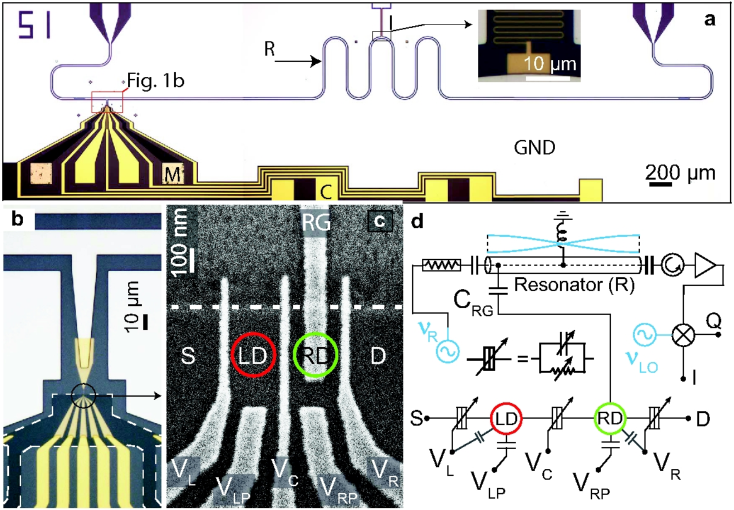


- Qubit = double quantum dot, quantum bus = s.c. resonator
- Coupling through *charge d.o.f.*: resonator ↔ interdot tunneling
- Operates at *charge degeneracy point* → minimize dephasing

Results:

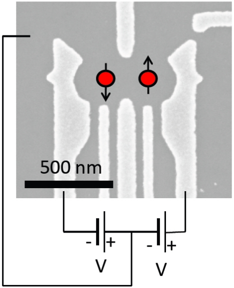
- $\mathbf{B}_{ext}$ and electric gates → transverse/longitudinal two-qubit coupling
- Blue-sideband transition → two-qubit entanglement

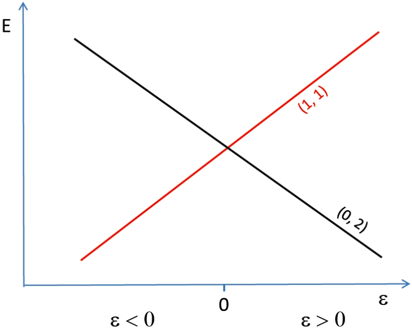
Recent similar experiment at ETH Zürich

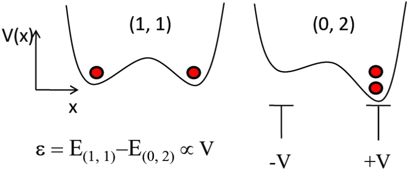


T. Frey *et al.*, Dipole coupling of a double quantum dot to a microwave resonator, arXiv:1108.5378.

Description of the double quantum dot







$\varepsilon = E_{(1,\,1)} - E_{(0,\,2)} \propto V$

(1, 1) (0, 2)

$$H = \begin{pmatrix} \varepsilon/2 & 0 \\ 0 & -\varepsilon/2 \end{pmatrix}$$

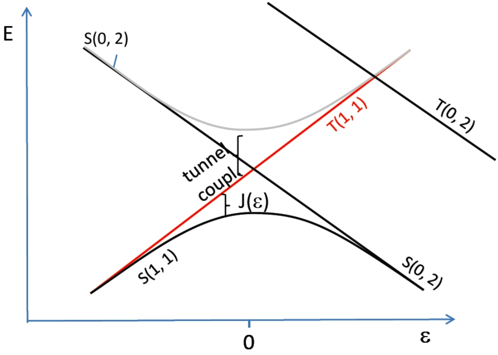
Accounting for interdot tunnel coupling,

T(1, 1) S(1, 1) S(0, 2)

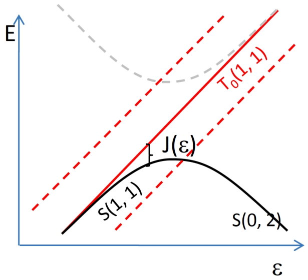
$$H=\begin{pmatrix} \varepsilon/2 & 0 & 0 \\ 0 & \varepsilon/2 & t_c \\ 0 & t_c & -\varepsilon/2 \end{pmatrix}$$

T S

$$\rightarrow \begin{pmatrix} J(\varepsilon) & 0 \\ 0 & 0 \end{pmatrix}$$



External magnetic field $\mathbf{B}_{\text{ext}}$



Qubit is encoded in

$$|S\rangle = |+\rangle \otimes \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}},$$

$$|T_0\rangle = |-\rangle \otimes \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}.$$

$|\pm\rangle$: orbital part.

Hamiltonian:

$$\boxed{H_q = \frac{J_0}{2}\tau_z + \frac{\Delta h}{2}\tau_x,}$$

$$\tau_z = |T_0\rangle\langle T_0| - |S\rangle\langle S|,$$
$$\tau_x = |T_0\rangle\langle S| + |S\rangle\langle T_0|.$$

J_0: exchange splitting,
Δh: Zeeman splitting difference.

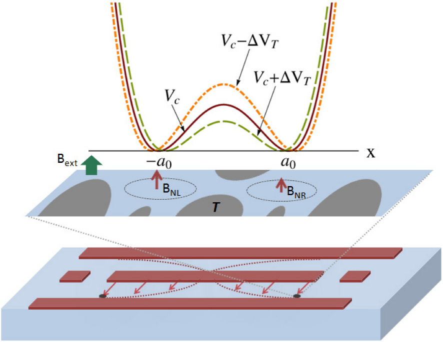
Qubit-resonator interaction

Confining potential: $$V_c(x,y) = \frac{m\omega_0^2}{2}\left[\frac{1}{4a_0^2}\left(x^2 - a_0^2\right)^2 + y^2\right]$$

Interdot tunnel gate T

$\updownarrow$

resonator voltage $V_r(a^\dagger + a)$



Exchange splitting is modified by the change of the barrier,

$$\Delta V_T(x) = eV_r \frac{x^2}{a_0^2}.$$

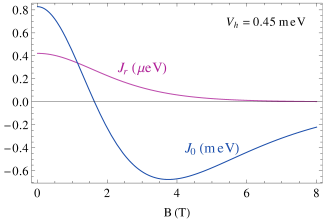
Interaction:

$$\boxed{H_c = J_r \tau_z (a^\dagger + a),} \qquad J_r = \frac{1}{2}\left[\langle -|\Delta V_T(x)|-\rangle - \langle +|\Delta V_T(x)|+\rangle\right].$$

Qubit-resonator Hamiltonian

In the qubit eigenbasis $\{|G\rangle, |E\rangle\}$,

$$H = \frac{\hbar\omega_q}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar\left(g_z\,\sigma_z + g_x\,\sigma_x\right)\left(a^\dagger + a\right),$$



$\hbar\omega_q = \sqrt{J_0^2 + \Delta h^2}$, $\quad \theta = \arctan(\Delta h / J_0)$,
$\hbar g_x = -J_r \sin\theta$, $\quad \hbar g_z = J_r \cos\theta$

Realistic parameters: $\quad \hbar\omega_0 = 4.5$ meV, $\quad \Delta h = 1\ \mu$eV.

For

- $eV_r \sim 1\mu$eV $\quad \rightarrow \quad J_r \sim 0.3\ \mu$eV ~ 70 MHZ,
- long coherence time, $T_2 \sim 10\mu$s $\quad$ (TLR, $\kappa/2\pi \sim 100$ kHZ)

$\rightarrow$ **Strong coupling regime:** $J_r/\hbar > 1/T_2, \kappa$

Blue-sideband transition: two-qubit entanglement

2 qubits + resonator:

$$H_{2q} = \hbar\omega_r a^\dagger a + \sum_{i=1,2}\left[\frac{\hbar\omega_q^{(i)}}{2}\sigma_z^{(i)} + \hbar\left(g_z^{(i)}\,\sigma_z^{(i)} + g_x^{(i)}\,\sigma_x^{(i)}\right)\left(a^\dagger + a\right)\right].$$

Driving with frequency $\omega_d = \omega_r + \omega_q^{(i)}$ and amplitude $\epsilon_d \ll \omega_r\omega_q/g_x$

$$\boxed{H_{BST}^{(i)} = \hbar\Omega_{BST}^{(i)}\left(a^\dagger\sigma_+^{(i)} + a\sigma_-^{(i)}\right), \qquad \Omega_{BST} = \frac{2\epsilon_d\, g_x\, g_z}{\omega_q\,\omega_r}.}$$

- Initialization: $|G_1, G_2\rangle \otimes |0\rangle$
- Drive qubit 1: $|E_1, G_2\rangle \otimes |0\rangle$
- Apply BST on qubit 2: $\left(|E_1, G_2\rangle \otimes |0\rangle + |E_1, E_2\rangle \otimes |1\rangle\right)/\sqrt{2}$
- Apply BST on qubit 1: $\left(|E_1, G_2\rangle + |G_1, E_2\rangle\right) \otimes |0\rangle/\sqrt{2}$

E.g. $\epsilon_d \sim 10^2 - 10^3$ MHz, $\Omega_{BST} \sim 1 - 10$ MHz.

Longitudinal coupling: CPhase gate

Strong longitudinal qubit-resonator coupling: $\Delta h^{(i)} \ll J_0^{(i)}$

$\rightarrow$ direct longitudinal qubit-qubit coupling.

Unitary transformation: $U = \exp\left[(a^\dagger - a)\sum_{i=1,2}\frac{g_z^{(i)}}{\omega_r}\sigma_z^{(i)}\right].$

Effective Hamiltonian: $H_{zz} = U\, H_{2q}\, U^\dagger$,

$$\boxed{H_{zz} = \hbar\omega_r a^\dagger a + \sum_{i=1,2}\frac{\hbar\omega_q^{(i)}}{2}\sigma_z^{(i)} - \hbar\frac{g_z^{(1)}g_z^{(2)}}{\omega_r}\sigma_z^{(1)}\sigma_z^{(2)}.}$$

Typically, $g_z^{(1)}g_z^{(2)}/\omega_r \sim 1-10$ MHz could be possible.

Transverse coupling: $\sqrt{\text{iSWAP}}$ gate

Strong transverse qubit-resonator coupling: $J_0^{(i)} \ll \Delta h^{(i)}$

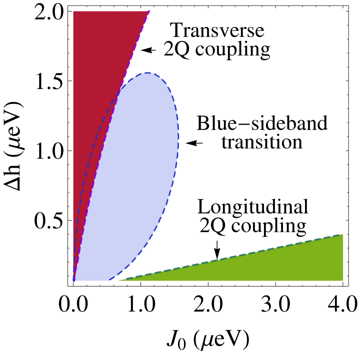
$\rightarrow$ transverse qubit-qubit interaction in the dispersive regime.

$\omega_q^{(1)} = \omega_q^{(2)}$ and $g_x^{(i)} < \omega_r - \omega_q^{(i)}$,

$$\boxed{H_{DIS} = \hbar\Omega_{DIS}\left(\sigma_+^{(1)}\sigma_-^{(2)} + \sigma_-^{(1)}\sigma_+^{(2)}\right), \qquad \Omega_{DIS} = \frac{g_x^2}{\omega_r - \omega_q}.}$$

Again, $\Omega_{DIS} \sim 1$ MHz could be within reach.

Summary



$J_r = 0.3\ \mu$eV (or 73 MHz),
$\omega_r/2\pi = 3$ GHz,
$\epsilon_d/2\pi = 450$ MHz

- $\Delta h \ll J_0$, longitudinal 2Q coupling
  $\rightarrow$ CPhase gate
- $J_0 \ll \Delta h$, transverse 2Q coupling
  $\rightarrow$ $\sqrt{\text{iSWAP}}$ gate
- $\Delta h \sim J_0$ + BST
  $\rightarrow$ fast 2Q entanglement