Completeness of quantum theory implies that wave functions are physical properties

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Journal Club

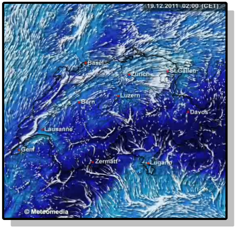
Daniel Becker
20 December 2011

Physical Property vs. State of Knowledge

Weather: Classical Deterministic System

suppose physical system is in state “white christmas”

Forecast 1:



based on:

- much, but incomplete data about initial state
- knowledge of classical mechanics
- elaborate heuristics, vast computer resources

probability of snow on christmas: 20%

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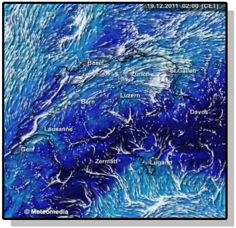

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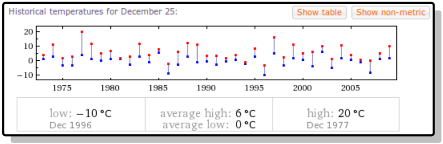
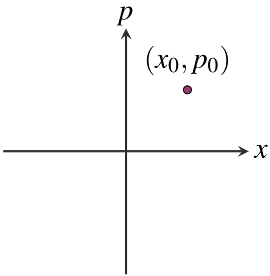


based on:

- counting of days with temperatures below $0°C$

probability of snow: $\sim 20\%$

ontic state of particle (1 D):



point in phase space

The Classical Case

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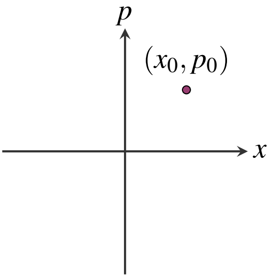

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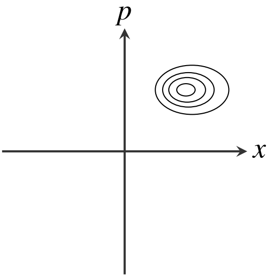

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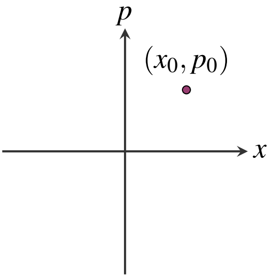

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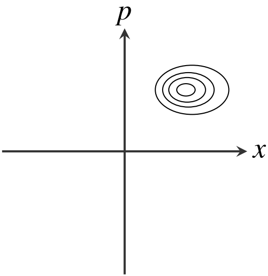
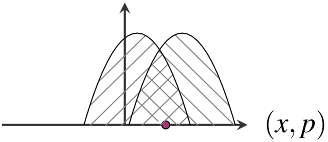


probability distribution

each ontic state possible in more than one (infinite) epistemic states:

probability density



What are Wavefunctions?

three main possibilities:

1. wavefunctions epistemic, with underlying ontic state.
   Quantum mechanics: statistical theory of ontic states
2. wavefunctions epistemic, no deeper underlying reality
3. wavefunctions ontic, i.e., describe physical state
   (many-worlds, spontaneous collaps)

- position 1 and 3 compatible with scientific realism
- position 2: anti-realism (e.g., Copenhagen approaches)

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- position 1 and 3 compatible with scientific realism
- position 2: anti-realism (e.g., Copenhagen approaches)
- theorem of paper attacks position 1:

if quantum theory correct, complete, and free
wave functions are physical properties

Bell Theorem and Local Determinism

Bell Theorem

A (contextual) hidden variable theory reproducing all predictions of quantum mechanics cannot be locally deterministic under the assumption of free choice.

in consequence, either

- psi-ontic: objective wave function not supplementable to reach local determinism
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possible escapes for epistemicists

1. abandon scientific realism
2. sacrifice locality (Lorentz invariance)
3. world could have fundamentally stochastic nature

Why be a psi-epistemicist?

classical epistemic measurement is Bayesian conditioning
$\Longrightarrow$ no change of underlying ontic state

- preserve scientific realism (hidden variable theories)
- higher explanatory power (remote steering, quantum teleportation, interference, ...)
- provides more "homogeneous" world view (no quantum-classical transition)

prominent advocats: Albert Einstein, Niels Bohr, Rob Spekkens, Anton Zeilinger (quantum information), ...

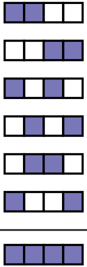
epistemic states $\Longrightarrow$ states of incomplete knowledge

Explanatory Power of Epistemic Position

toy model (Spekkens, PRA 75, 032110 (2007))

maximal knowledge: for every system, at every time, knowledge possessed about ontic state equals knowledge lacked

two-level system
(Qubit)



$\Leftrightarrow |0\rangle$

$\Leftrightarrow |1\rangle$

$\Leftrightarrow |+\rangle$

$\Leftrightarrow |-\rangle$

$\Leftrightarrow |+\boldsymbol{i}\rangle$

$\Leftrightarrow |-\boldsymbol{i}\rangle$

$\Leftrightarrow \hat{\mathbb{1}}/2$

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transformations on Bloch sphere
$\Leftrightarrow$ permutations of ontic states

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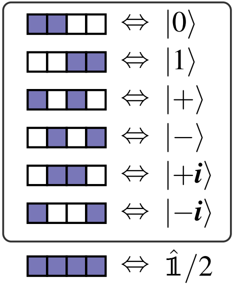

transformations on Bloch sphere
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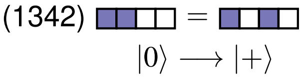

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transformations on Bloch sphere $\Leftrightarrow$ permutations of ontic states

(1342) ■■□□ = ■□■□

$$|0\rangle \longrightarrow |+\rangle$$

interference:

1. prep. $|0\rangle$
2. prep. $|1\rangle$
3. prep. $|+\rangle$

measure $(|+\rangle, |-\rangle)$ → (1/2, 1/2), (1/2, 1/2), (1, 0)

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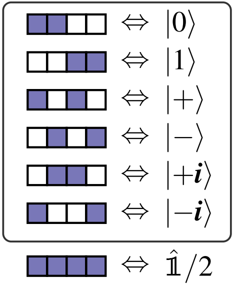

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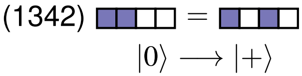

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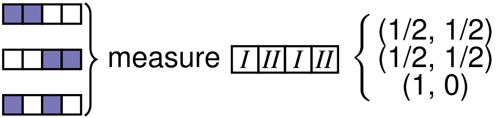


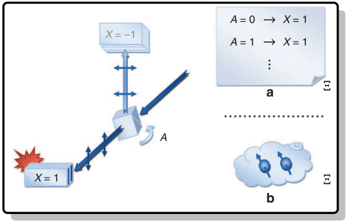
Claim: Wavefunction is Physical Property

assumptions

1. QM: quantum mechanics correct (empircally adequate)
2. FR: freedom of choice (measurement independent of any prexisting values)

in schematic experimental setup:

- measurement setting A
- measurement outcome X
- Ξ: additional information provided by extended theory



Colbeck and Renner, Nat. Comm. **2**, 411 (2011)

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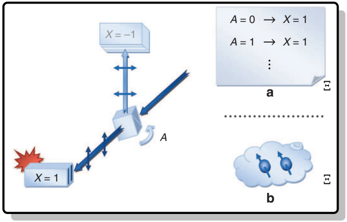

theorem: completeness of quantum mechanics (CPL)

Ξ cannot increase knowledge given by A and wavefunction ψ: markov chain $\Xi \leftrightarrow (A, \psi) \leftrightarrow X$

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Sketched Proof of CPL - Part I

ingredients:

- bipartite measurement of (spacelike) A and B with outcomes X and Y
- additional information Ξ (static) accessed by measurement C with outcome Z

cond. prob.

$$P_{Q|R} = P_{QR}/P_R$$

CPL

$$\forall_{acx} : P_{Z|acx} = P_{Z|ac}$$

FR

$$P_{A|BCYZ} = P_A$$

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FR implies
Ξ is non-signalling:

$$P_{YZ|ABC} = P_{YZ|BC}$$
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$$P_{YZ|ABC} \times P_{A|BC} = P_{A|BCYZ} \times P_{YZ|BC}$$

Sketched Proof Part II

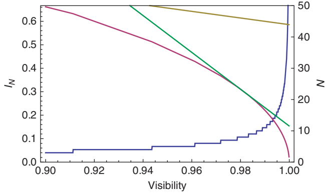
correlations of bipartite measurement with $2N$ inputs:

$$I_N := P(X = Y \mid A = 0, B = 2N-1) + \sum_{\substack{a,b \\ |a-b|=1}} P(X \neq Y \mid A = a, B = b).$$

under non-signalling conditions:

$$\frac{1}{2} \sum_z |P_{Z|abcx}(z) - P_{Z|abc}(z)| \leq I_N \propto 1/N$$

measurement scheme



maximally entangled states:

$$\{|\theta_+^j\rangle, |\theta_-^j\rangle\} = \left\{\cos\frac{\theta^j}{2}|0\rangle + \sin\frac{\theta^j}{2}|1\rangle,\ \sin\frac{\theta^j}{2}|0\rangle - \cos\frac{\theta^j}{2}|1\rangle\right\}.$$

Final Result

- from CPL $\Xi \leftrightarrow (\psi, A) \leftrightarrow X$ follows

$$P_{X|\Xi=\xi,A=a} = P_{X|\Xi=\xi,\Psi=\psi,A=a} = P_{X|\Psi=\psi,A=a}$$

  for all ψ, ξ, a with $P_{\Psi,\Xi,A}(\psi, \xi, a) > 0$
- free choice: $P_{\Psi,\Xi,A}(\psi, \xi, a) = P_{\Psi,\Xi}(\psi, \xi) \times P_A(a)$, hence

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ontic nature of wavefunction

For each $\Xi = \xi$ value of $\Psi = \psi$ is uniquely determined.