Quantum Magnetomechanics with Levitating Superconducting Microspheres

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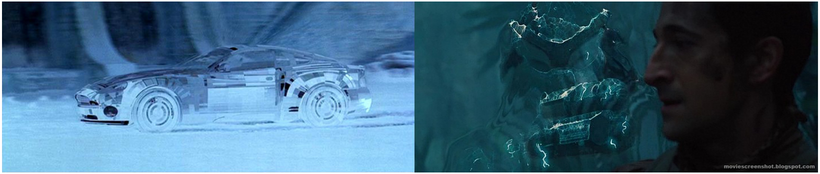
Material- and geometry-independent multishell cloaking device
&
Demonstration of temporal cloaking



Motivation:

- Combine the technology of magnetic mictrotraps and superconducting qubits to bring relatively large objects to the quantum regime.
- Preparation of quantum superposition states, which aim at the very fundamental goal of testing the validity of quantum mechanics when large masses are involved.
- Unclamped setup: improve isolation of the mechanical motion from the environment.

Optical vs magnetostatic levitation

Limitations of optically levitating mechanical oscillators:
Decoherence due to

- scattering of photons: position localization decoherence.
- absorption of photons: increase of the bulk temperature of the object.
  → emission of black body radiation.

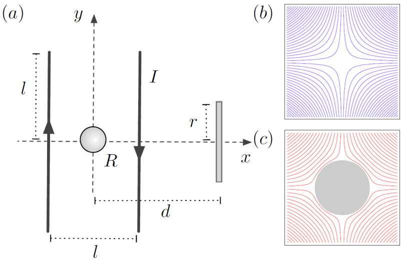
Advantages of magnetostatic levitation:

- Also no clamping losses.
- Circumvention of the limitations above due to the absence of photons.

→ Extremely high mechanical quality factor.

Setup

- Superconducting Pb sphere in the Meissner state with μm radius and mass of $\sim 10^{14}$ amu.
- Quadrupole trap created by two circular coils in an anti-Helmholtz configuration.
- Pickup coil of
  (a) a LC oscillator or
  (b) a flux qubit.



$$\hat{V}_{\text{trap}} = M[\omega_t^2\hat{x}^2 + \omega_\perp^2(\hat{y}^2 + \hat{z}^2)]/2 \tag{1}$$

Quantum magnetomechanical coupling

The quantum magnetomechanical coupling for both cases depends on the dimensionless parameter $\eta \equiv x_{\text{zp}} \Phi'_{\text{ext}} / \Phi_0$, where

- $x_{\text{zp}} = \sqrt{\hbar/(2M\omega_t)}$ is the mechanical zero point motion,
- $\Phi_0 = \pi\hbar/e$ is the flux quantum and
- $\Phi'_{\text{ext}}(0)$ is the derivative with respect to the axial motion of the center of mass evaluated at its equilibrium position of the flux threading the pick-up coil.

  $\Phi'_{\text{ext}}(0)$ can also be evaluated analytically and its approximated expression for $R \ll l, r$ is given by

$$\Phi'_{\text{ext}}(0) \approx 2.7\mu_0 (I/l^2) R^3 r^2 / (d^2 + r^2)^{3/2}.$$

Hamiltonian

(a) LC oscillator

The Hamiltonian is given by

$$\hat{H}_{\mathrm{LC}} = [\hat{\Phi} - \Phi_{\mathrm{ext}}(\hat{x})]^2/(2L) + \hat{Q}^2/(2C) \tag{2}$$

with $[\hat{\Phi}, \hat{Q}] = i\hbar$. By expanding $\Phi_{\mathrm{ext}}(\hat{x})$ linearly in $\hat{x}$, one obtains the linear Hamiltonian:

$$\hat{H}_{\mathrm{LC}} = \hbar\omega_{\mathrm{LC}}\hat{a}^\dagger\hat{a} + \hbar g_{\mathrm{LC}}(\hat{a}^\dagger + \hat{a})(\hat{b}^\dagger + \hat{b}) \tag{3}$$

with $\hat{x} = x_{\mathrm{zp}}(\hat{b}^\dagger + \hat{b})$, $\hat{\Phi} = \Phi_{\mathrm{zp}}(\hat{a}^\dagger + \hat{a})$, $\Phi_{\mathrm{zp}} = \sqrt{\hbar/(2C\omega_{\mathrm{LC}})}$,
$\omega_{\mathrm{LC}} = 1/\sqrt{LC}$
and the magnetomechanical coupling rate $g_{\mathrm{LC}} = \epsilon_{\mathrm{LC}}\eta$ with
$\epsilon_{\mathrm{LC}} = \Phi_0\Phi_{\mathrm{zp}}/(\hbar L)$.

Hamiltonian

(b) Flux qubit

For a three junction flux qubit with persistent currents of amplitude I_p flowing clockwise or counterclockwise. The Hamiltonian in the basis of the persistent current states reads

$$\hat{H}_s = -\hbar\tilde{\epsilon}\hat{\sigma}_z/2 - \hbar\Delta\hat{\sigma}_x/2 \tag{4}$$

with bias $\tilde{\epsilon}$ and tunneling amplitude Δ.
One obtains the Quantum magnetomechanical Hamiltonian by expanding $\tilde{\epsilon}(\hat{x}) \approx \tilde{\epsilon}(0) + \tilde{\epsilon}'(0)\hat{x}$:

$$\frac{\hat{H}_{\mathrm{MM}}}{\hbar} = \omega_t \hat{b}^\dagger \hat{b} - \frac{\epsilon}{2}\hat{\sigma}_z - \frac{\Delta}{2}\hat{\sigma}_x - g_0\hat{\sigma}_z(\hat{b}^\dagger + \hat{b}) \tag{5}$$

with $\epsilon = \tilde{\epsilon}(0)$, $g_0 = \epsilon_0\eta$.

Sources of decoherence

(i) Clamping losses and

(ii) scattering of photons are absent.

(iii) Damping created by the background gas is negligible at sufficiently low pressure.

(iv) Decoherence due to black body radiation is even more negligible due to cryogenic bulk temperatures.

(v) Internal vibrational modes are decoupled to the center-of-mass motion for micrometer-sized objects.

(vi) Heating due to fluctuations in the trap frequency is negligible due to the nearly perfect stability of the intensity currents circulating in the superconducting magnetic microtrap.

(vii) The magnetomechanical coupling to microtrap coils is negligible due to the low inductance, otherwise we could use them instead of adding an additional quantum circuit.

(viii) The absence of vortices in superconductors in the Meissner state also prevents damping due to their incoherent motion.

(ix) Currents induced in superconductors in the Meissner state do not lead to decoherence since they are reversible.

(x) Within trapping frequencies in the MHz regime, the superconductor can be considered to act instantaneously to external fields.

(xi) And finally, we remark that the flux qubit present in the setup can always be decoupled from the center-of-mass motion by tuning the bias ϵ to zero.

$\Rightarrow$ At sufficiently low pressure and for superconductors in the Meissner state, the center of mass of micrometer-sized metallic spheres is effectively decoupled from the environment.

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Ground state cooling of the mechanical motion

by driving the flux qubit

An applied ac flux with frequency ω_d (in the GHz regime) and amplitude Ω (in the kHz-MHz regime) drives the flux qubit. In this case, the total Hamiltonian reads $\hat{H}_t = \hat{H}_{\text{MM}} + \hat{H}_{\text{drive}}$, where $\hat{H}_{\text{drive}} = \Omega\cos(\omega_d t)\hat{\sigma}_z$. The dynamics of the qubit and the mechanical oscillator is given by the master equation $\dot{\rho} = -i/\hbar[\hat{H}'_{\text{t}}, \hat{\rho}] + \mathcal{L}_0[\hat{\rho}] + \mathcal{L}_\varphi[\hat{\rho}]$.

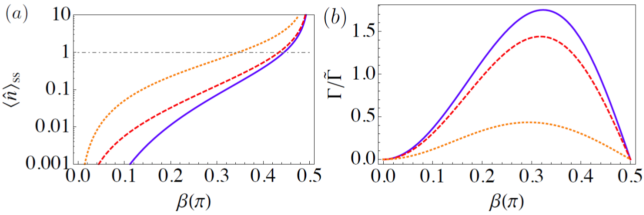
After performing a RWA, transforming to the diagonal basis of the qubit, moving to the interaction picture and performing a second RWA, one arrives at the master equation

$$\dot{\rho} = \frac{i}{\hbar}\left[\tilde{g}(\hat{\sigma}_- \hat{b}^\dagger + \text{H.c.}), \hat{\rho}\right] + \mathcal{L}_\Gamma[\hat{\rho}]. \tag{6}$$

Typically $\tilde{g} \ll \Gamma_{\downarrow(\uparrow)}, \Gamma^*_\varphi$, which allows to adiabatically eliminate the qubit. This leads to an effective master equation describing the mechanical oscillator density matrix that can be used to obtain a dynamical equation for the mean phonon number occupation.

Ground state cooling of the mechanical motion

can be achieved in a wide range of β



(a) The steady state phonon number occupation $\langle\hat{n}\rangle_{\rm ss}$ of the mechanical oscillator and

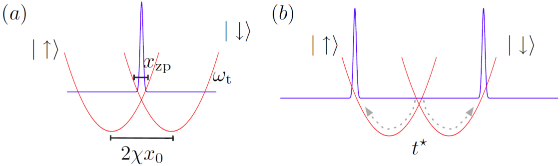
(b) the cooling rate Γ over $\tilde{\Gamma} = \tilde{g}^2\cos^2(\alpha)/\Gamma_0$ are plotted as a function of $\beta = \arctan\left(\frac{\Omega\sin(\arctan(\Delta/\epsilon))}{\omega_d - \sqrt{\epsilon^2+\Delta^2}}\right)$ for $\Gamma_\varphi/\Gamma_0 = 0$ (solid blue line), $\Gamma_\varphi/\Gamma_0 = 0.1$ (dashed red line) and $\Gamma_\varphi/\Gamma_0 = 1$ (dotted orange line).

Entangled state

The Hamiltonian written in the form

$$\hat{H}'_{\mathrm{MM}} = \frac{\hbar\omega_s}{2}\hat{\sigma}_z + \hat{T}^\dagger(\chi\hat{\sigma}_z)\hat{H}_m\hat{T}(\chi\hat{\sigma}_z)$$

points out that the center of the harmonic trap depends on the spin state of the qubit.



(a) Joint state of qubit $|+\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ and the mechanical oscillator in the ground state $|0\rangle$ at time $t = 0$ and recovered at $t = 2t^*$.

(b) Entangled state $|\Psi_s\rangle = \frac{1}{\sqrt{2}}\left[\hat{T}(-2\chi)|\uparrow, 0\rangle + \hat{T}(2\chi)|\downarrow, 0\rangle\right]$ at time $t = t^*$.

Set of parameters

that fulfill the required conditions.

Pb microsphere:

- density $\rho = 11360 Kg/m^3$,
- penetration depth $\lambda = 30.5nm$,
- coherence length $\xi = 96nm$,
- critical temperature $T_\mathrm{C} = 7.2K$,
- critical field $B_\mathrm{C} = 0.08T$,
- radius $R = 2\mu m$.

Magnetic trap with AH coils:

- radius $l = 30\mu m$,
- current $I = 20A$,
- $\omega_t = 2\pi \times 39kHz$.

Pickup coil:

- radius $l = 28\mu m$,
- placed at $d = 20\mu m$,

Flux qubit:

- $\epsilon_0 = \Delta = 2\pi \times 20GHz$,
- $T_1 = T_2 = 10\mu s$,
- $\omega_t = 2\pi \times 39kHz$,
- $g_0 = 2\pi \times 1.3kHz$.

LC oscillator:

- $L = 1pF$,
- $C = 0.1nH$,
- $g_\mathrm{LC} = 2\pi \times 96kHz$.

Conclusion

They have shown that micrometer-sized superconducting metallic spheres containing $\sim 10^{14}$ atoms can be cooled down to the ground state and prepared into superposition states.

They propose to merge the technology of the growing fields of magnetic trapping of atoms and superconducting qubits to bring massive objects into the quantum regime.