Handout

Resistively detected nuclear magnetic resonance:

Electron spins in the ν = 5/2 quantum Hall state are fully polarized

17 Jan 2012

NMR probing of the spin polarization of the ν = 5/2 quantum Hall state
M. Stern, B. A. Piot, Y. Vardi, V. Umansky, P. Plochocka,
D. K. Maude, and I. Bar-Joseph

18 Jan 2012

Unraveling the spin polarization of the ν = 5/2 fractional quantum Hall state
L. Tiemann, G. Gamez, N. Kumada, and K. Muraki

Srecan Rodjendan!

Happy Birthday

Vladimir!!

Nature Physics **8**, 54 (2012)

Coherent Control of Three-Spin States in a Triple Quantum Dot

L. Gaudreau,[1,2] G. Granger,[1] A. Kam,[1] G. C. Aers,[1]
S. A. Studenikin,[1] P. Zawadzki,[1] M. Pioro-Ladrière,[2]
Z. R. Wasilewski,[1] and A. S. Sachrajda[1]

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[2]Département de physique, Université de Sherbrooke; Sherbrooke, Québec, Canada

Journal Club, 24 January 2012

Motivation

Quantum computer

… Qubit Qubit Qubit Qubit …

Motivation

<u>Quantum computer</u>

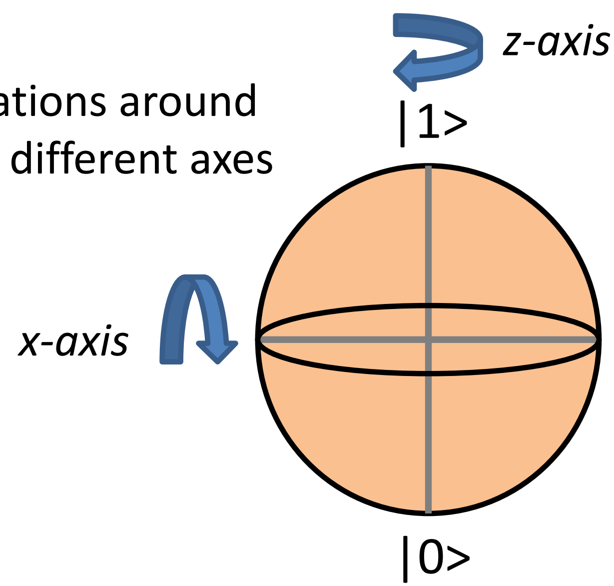
Basic requirements for universal set of quantum gates:

... ⟷ Qubit ⟷ Qubit ⟷ Qubit ⟷ Qubit ⟷ ...

Arbitrary two-qubit gate
(CNOT, SWAP, ...)

For each qubit:
Full control over the Bloch sphere

Rotations around
two different axes



Motivation

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Basic requirements for universal set of quantum gates:

... ↔ Qubit ↔ Qubit ↔ Qubit ↔ Qubit ↔ ...

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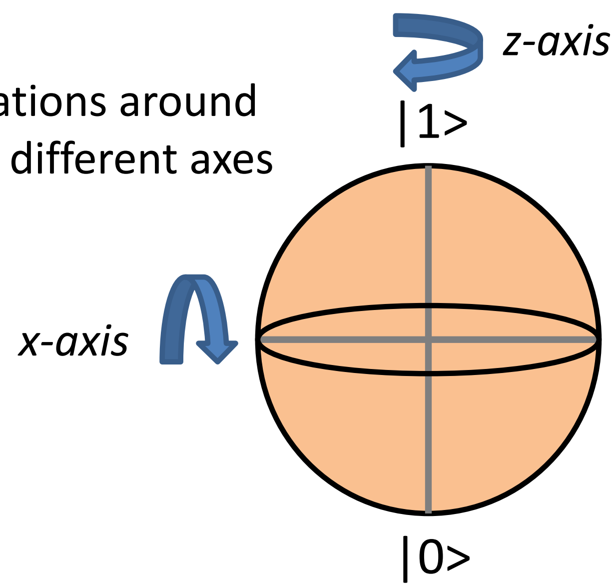
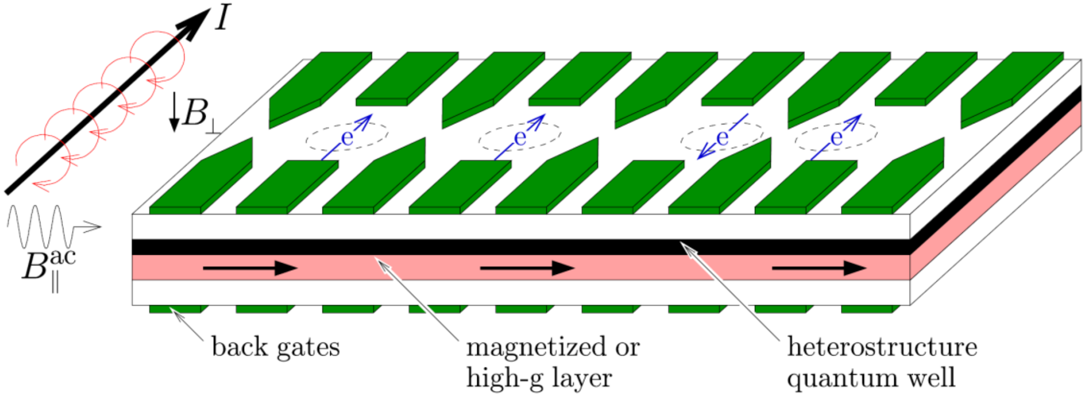


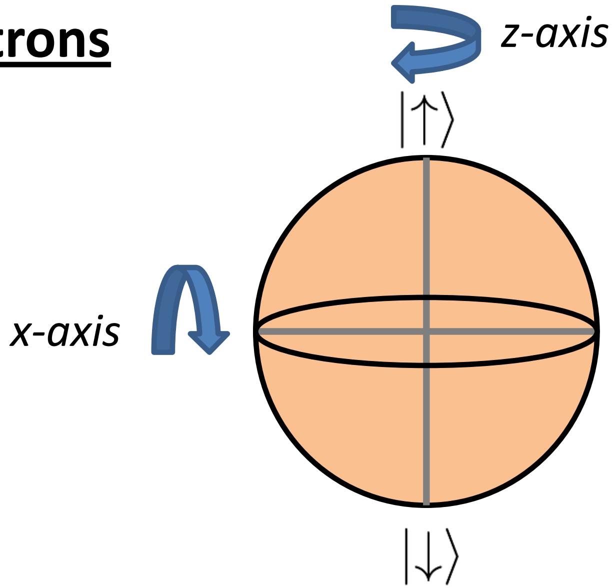
How to implement the qubits?

Motivation

Spin states of SINGLE electrons



D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998)



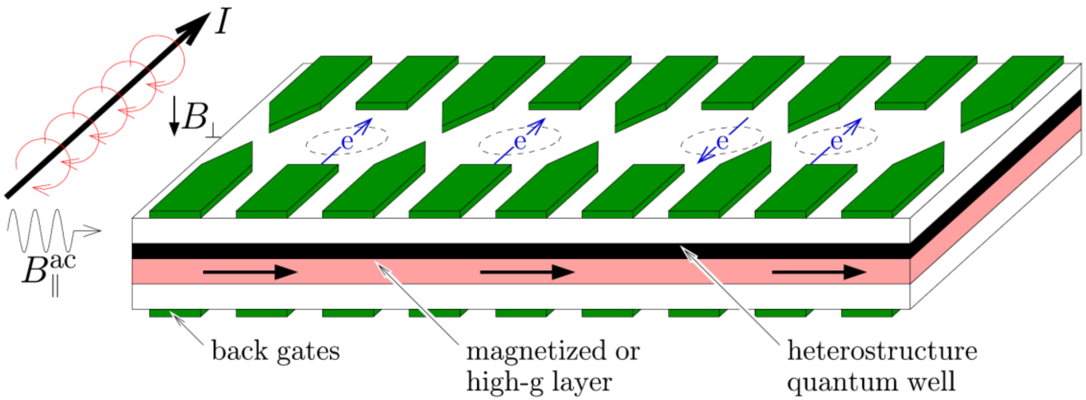
Two-qubit operations via the isotropic (Heisenberg) exchange interaction

$$H_{\mathrm{ex}}(t) = J(t)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

J. R. Petta *et al.*, Science (2005):
SWAP operation within 350 ps

Motivation

Spin states of SINGLE electrons



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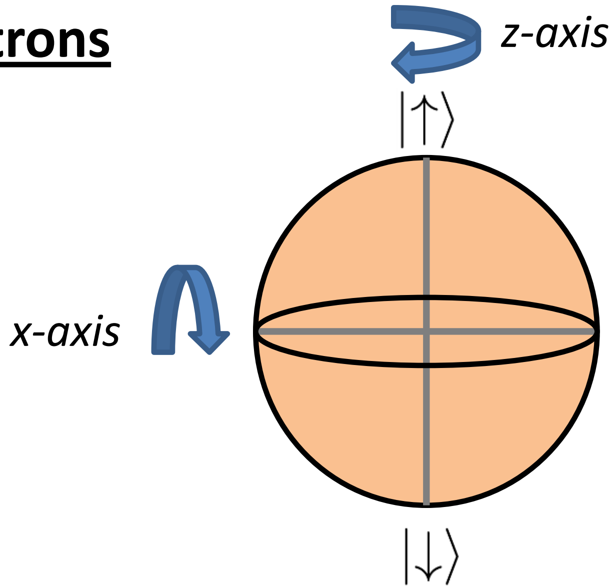

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$$H_{ex}(t) = J(t)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

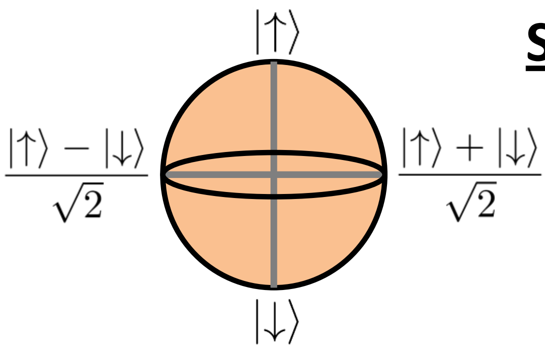
J. R. Petta *et al.*, Science (2005): SWAP operation within 350 ps

Rotation about *z* axis: Static magnetic field

Rotation about *x* axis: Oscillating (effective) magnetic field ESR, EDSR

Currently about 20 ns for a rotation of π

Motivation



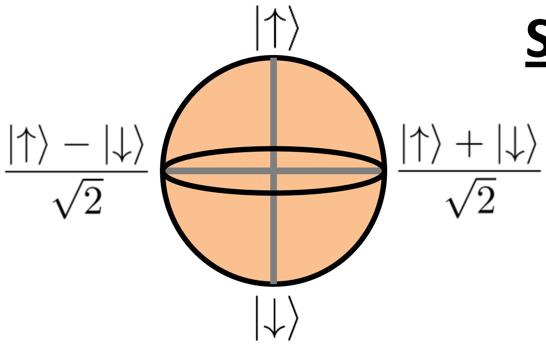
Spin states of SINGLE electrons

z-rotation: (static) magnetic field

x-rotation: ESR, EDSR, ...

Two-qubit gate: **exchange interaction**

Motivation



<u>Spin states of SINGLE electrons</u>

z-rotation: (static) magnetic field
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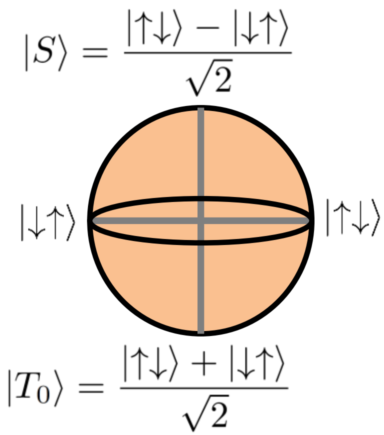
Two-qubit gate: **exchange interaction**

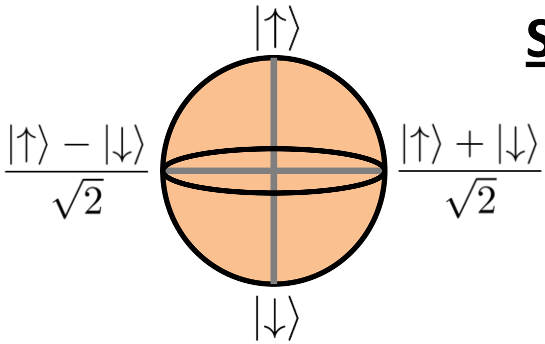


<u>Spin states of TWO electrons</u>

z-rotation: **exchange interaction**
x-rotation: magnetic field gradient

Two-qubit gate: Coulomb interaction (CPHASE), ...

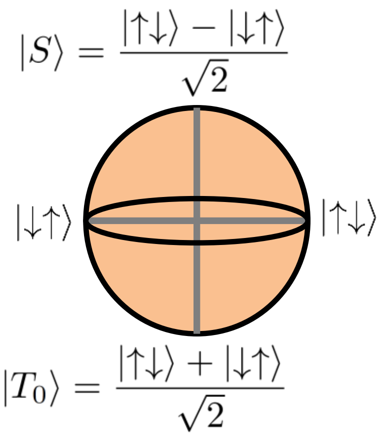
Motivation



Spin states of SINGLE electrons

z-rotation: (static) magnetic field
x-rotation: ESR, EDSR, ...

Two-qubit gate: **exchange interaction**



Spin states of TWO electrons

z-rotation: **exchange interaction**
x-rotation: magnetic field gradient

Two-qubit gate: Coulomb interaction (CPHASE), ...

Spin states of THREE electrons

Exchange only!

D. P. DiVincenzo *et al.*, Nature **408**, 339 (2000)

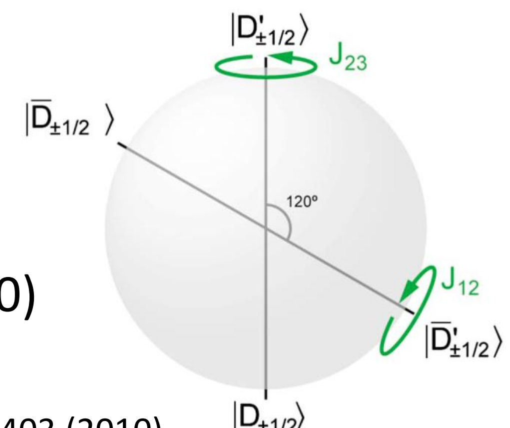


Figure from Laird *et al.*, PRB **82**, 075403 (2010)

Motivation

<u>Further motivation:</u>

Three-spin states useful for quantum error correction

Fundamental research,
multipartite entanglement in the solid state

<u>Spin states of THREE electrons</u>

Exchange only!

D. P. DiVincenzo *et al.*, Nature **408**, 339 (2000)

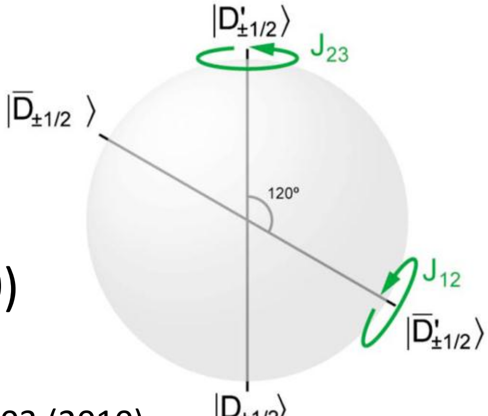
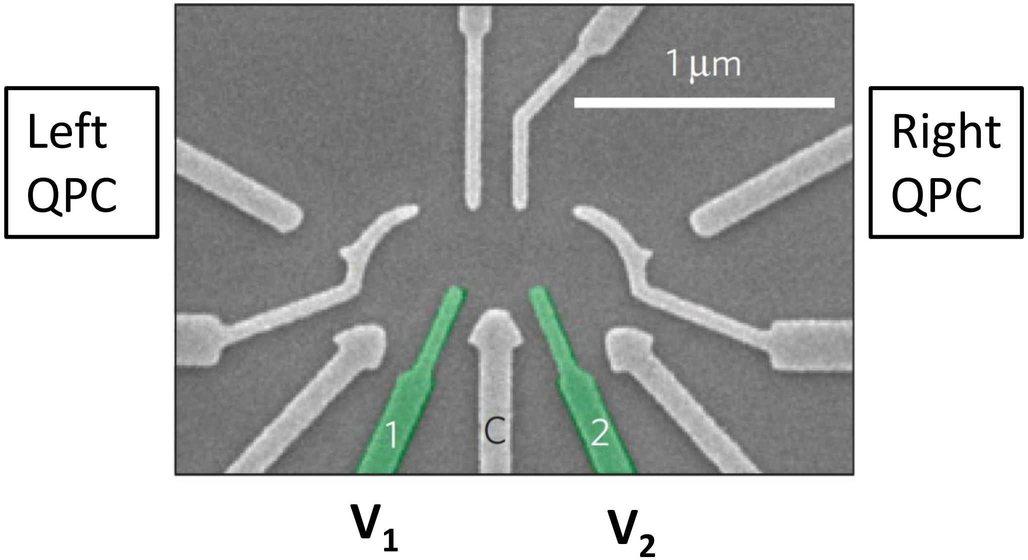


Figure from Laird *et al.*, PRB **82**, 075403 (2010)

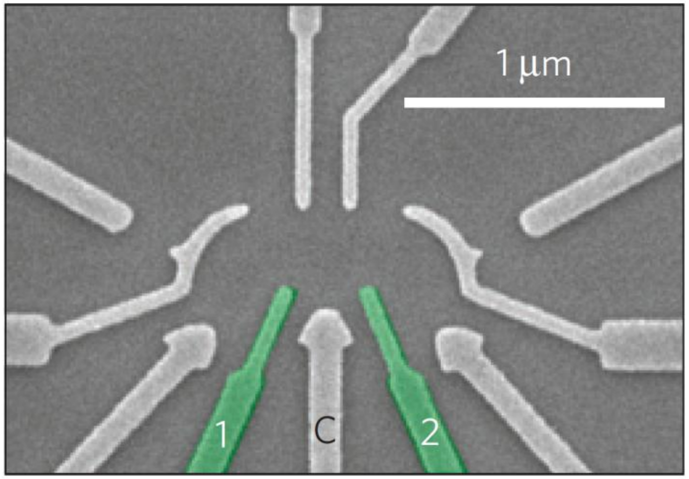
Setup



2DEG within GaAs/AlGaAs heterostructure

Setup

Left QPC

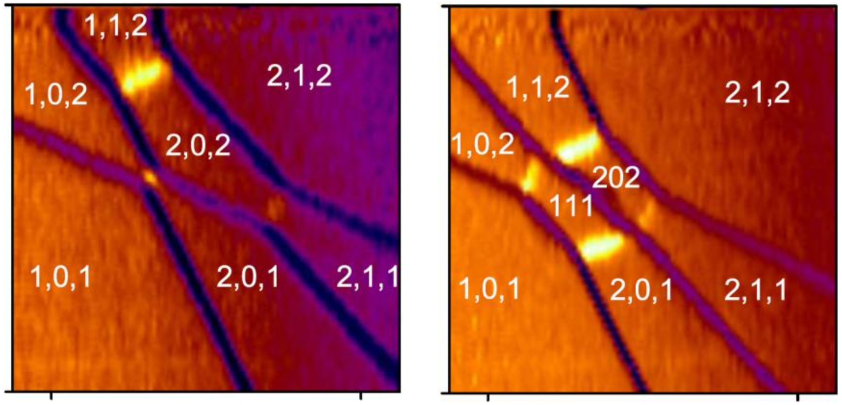


Right QPC

2DEG within GaAs/AlGaAs heterostructure

$\mathbf{V_1}$ $\mathbf{V_2}$

Width of (111) region can be tuned via the voltage on gate C



Granger *et al.*, PRB **82**, 075304 (2010)

(111) configuration

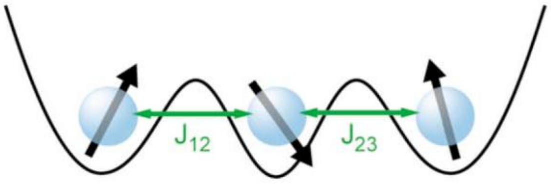


Figure from
Laird *et al.*, PRB **82**, 075403 (2010)

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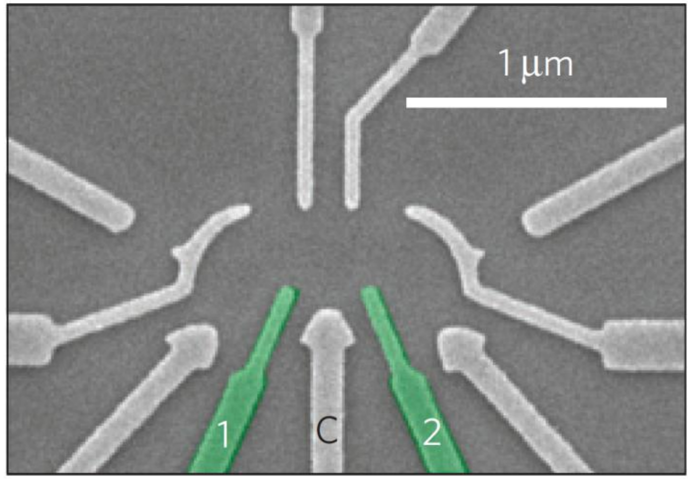

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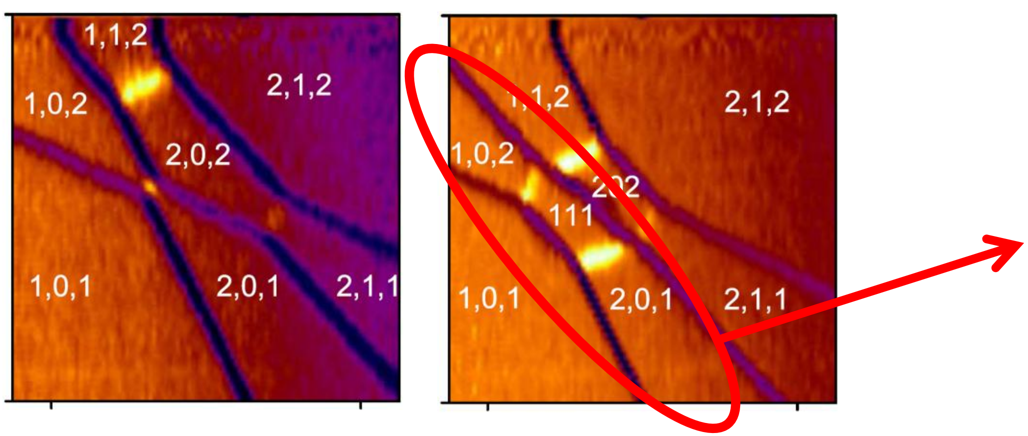
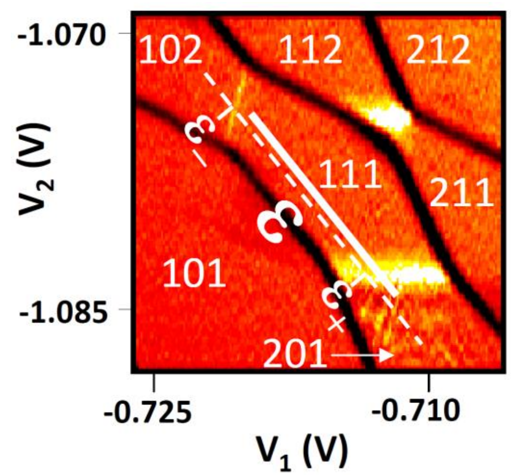


Operation in the **(102)-(111)-(201) regime**, where **ε denotes the detuning**



Granger *et al.*, PRB **82**, 075304 (2010)

Three-Spin Eigenstates

Hamiltonian:

$$H = J_{12}\left(\mathbf{S}_1 \cdot \mathbf{S}_2 - \frac{1}{4}\right) + J_{23}\left(\mathbf{S}_2 \cdot \mathbf{S}_3 - \frac{1}{4}\right) - E_Z(S_1^z + S_2^z + S_3^z)$$

Notation:

$J_{12} = J_{LC}$

$J_{23} = J_{RC}$

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Quadruplet:

$$|Q_{+3/2}\rangle = |\uparrow\uparrow\uparrow\rangle$$

$$|Q_{+1/2}\rangle = \frac{1}{\sqrt{3}}(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)$$

$$|Q_{-1/2}\rangle = \frac{1}{\sqrt{3}}(|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle)$$

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$$E_{Q_{S_z}} = -E_Z S_z$$

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$$\Omega = \sqrt{J_{12}^2 + J_{23}^2 - J_{12}J_{23}}$$

Doublet T_0:

$$|\Delta_{+1/2}\rangle = \frac{1}{\sqrt{4\Omega^2 + 2\Omega(J_{12} - 2J_{23})}}((J_{12} - J_{23} + \Omega)|\uparrow\uparrow\downarrow\rangle + (J_{23} - \Omega)|\uparrow\downarrow\uparrow\rangle - J_{12}|\downarrow\uparrow\uparrow\rangle)$$
$$|\Delta_{-1/2}\rangle = \frac{1}{\sqrt{4\Omega^2 + 2\Omega(J_{12} - 2J_{23})}}((J_{12} - J_{23} + \Omega)|\downarrow\downarrow\uparrow\rangle + (J_{23} - \Omega)|\downarrow\uparrow\downarrow\rangle - J_{12}|\uparrow\downarrow\downarrow\rangle)$$

$$E_{\Delta_{S_z}} = -(J_{12} + J_{23} - \Omega)/2 - E_Z S_z$$

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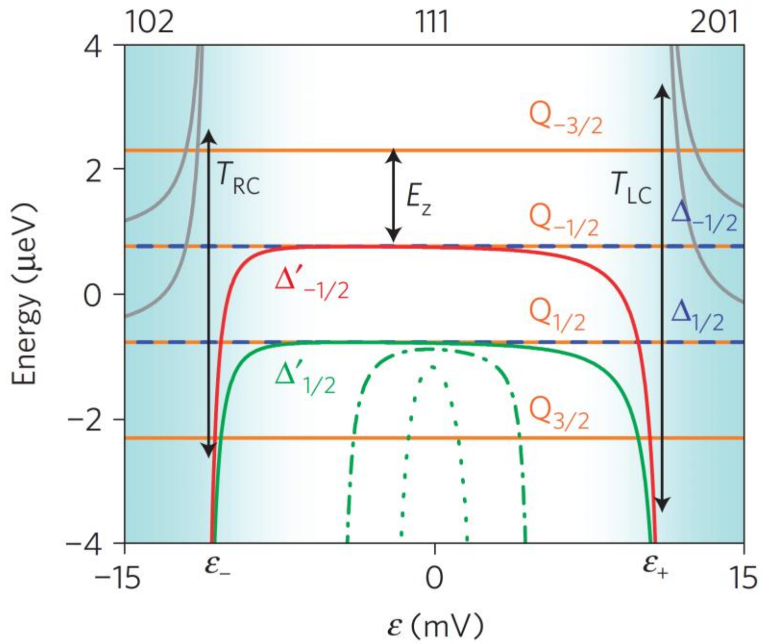
Doublet S:

$$|\Delta'_{+1/2}\rangle = \frac{1}{\sqrt{4\Omega^2 + 2\Omega(2J_{23} - J_{12})}}((-J_{12} + J_{23} + \Omega)|\uparrow\uparrow\downarrow\rangle - (J_{23} + \Omega)|\uparrow\downarrow\uparrow\rangle + J_{12}|\downarrow\uparrow\uparrow\rangle)$$
$$|\Delta'_{-1/2}\rangle = \frac{1}{\sqrt{4\Omega^2 + 2\Omega(2J_{23} - J_{12})}}((-J_{12} + J_{23} + \Omega)|\downarrow\downarrow\uparrow\rangle - (J_{23} + \Omega)|\downarrow\uparrow\downarrow\rangle + J_{12}|\uparrow\downarrow\downarrow\rangle)$$

$$E_{\Delta'_{S_z}} = -(J_{12} + J_{23} + \Omega)/2 - E_Z S_z$$

Three-Spin Spectrum

Calculated spectrum for a wide (111) region:



Notation:
$J_{12} = J_{LC}$
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$$\frac{J_{LC}}{\tilde{\alpha}_{LC}} = (\epsilon - \epsilon_+)/2 + \sqrt{[(\epsilon - \epsilon_+)/2]^2 + \left(\frac{T_{LC}}{\tilde{\alpha}_{LC}}\right)^2}$$

$$\frac{J_{RC}}{\tilde{\alpha}_{RC}} = (\epsilon_- - \epsilon)/2 + \sqrt{[(\epsilon_- - \epsilon)/2]^2 + \left(\frac{T_{RC}}{\tilde{\alpha}_{RC}}\right)^2}$$

$$T_{LC}(\epsilon) = \begin{cases} T_{LC}\exp[C_{LC}(\epsilon - \epsilon_+)], & \epsilon < \epsilon_+ \\ T_{LC}, & \epsilon \geq \epsilon_+ \end{cases}$$

$$T_{RC}(\epsilon) = \begin{cases} T_{RC}\exp[C_{RC}(\epsilon_- - \epsilon)], & \epsilon > \epsilon_- \\ T_{RC}, & \epsilon \leq \epsilon_- \end{cases}$$

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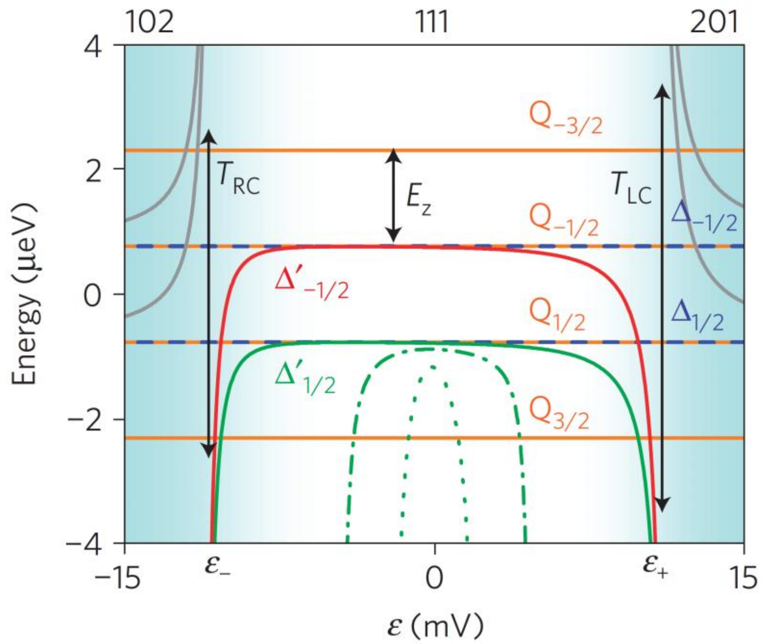

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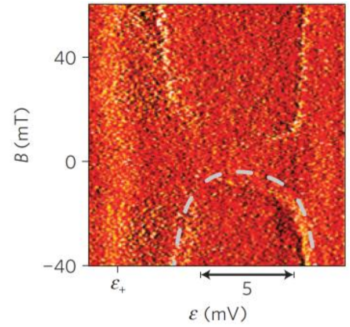
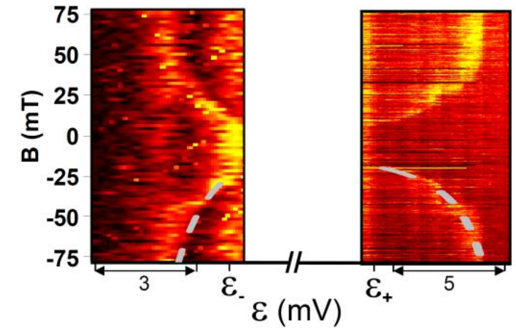
Parameters from fits to experiment:

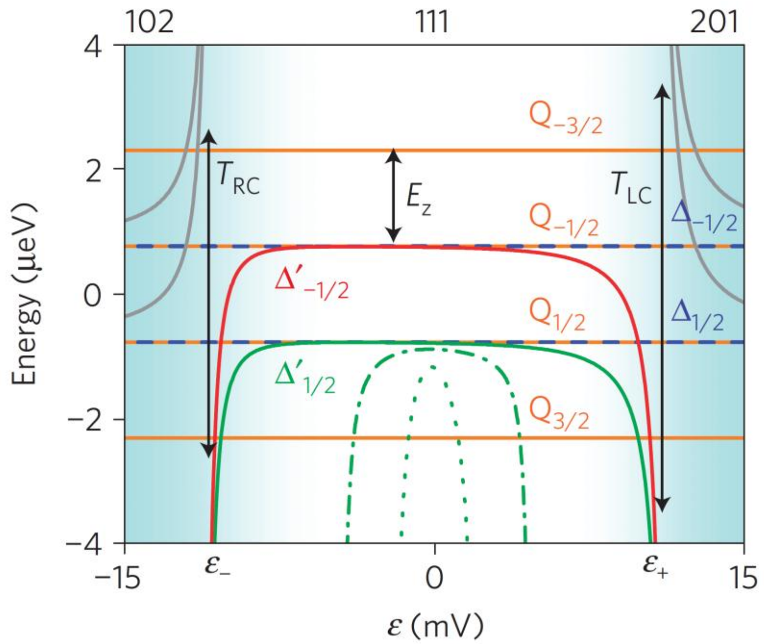
(Examples)





Three-Spin Spectrum

Calculated spectrum for a wide (111) region:



Notation:

$J_{12} = J_{LC}$

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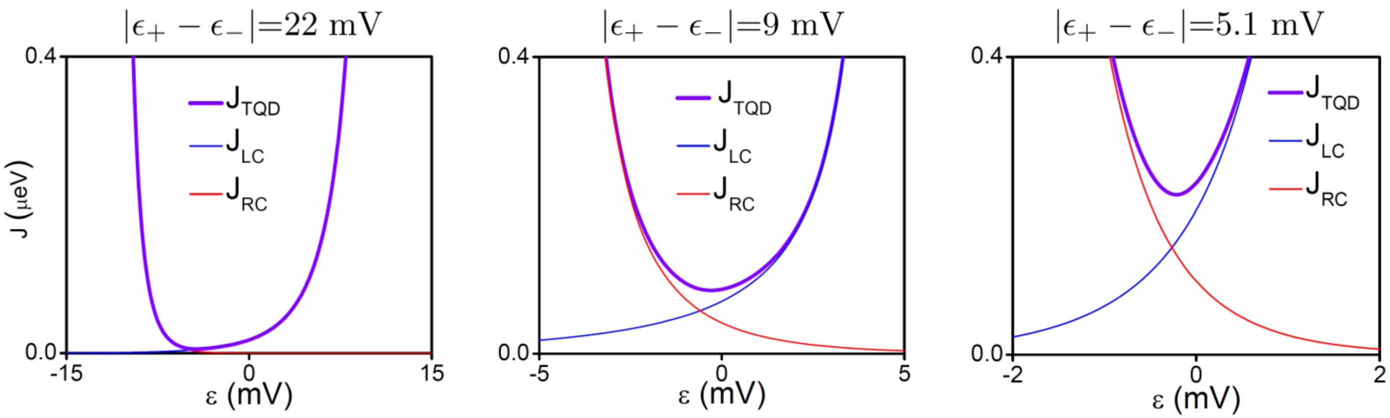
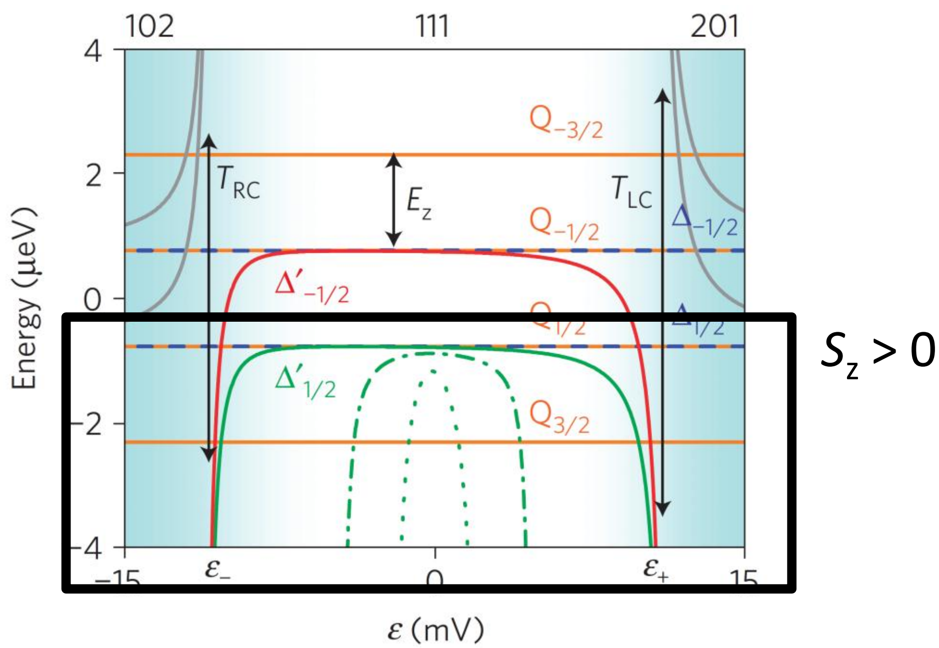
Resulting J_{LC} and J_{RC} for different (111) regions:



When (111) is wide, one spin is always decoupled

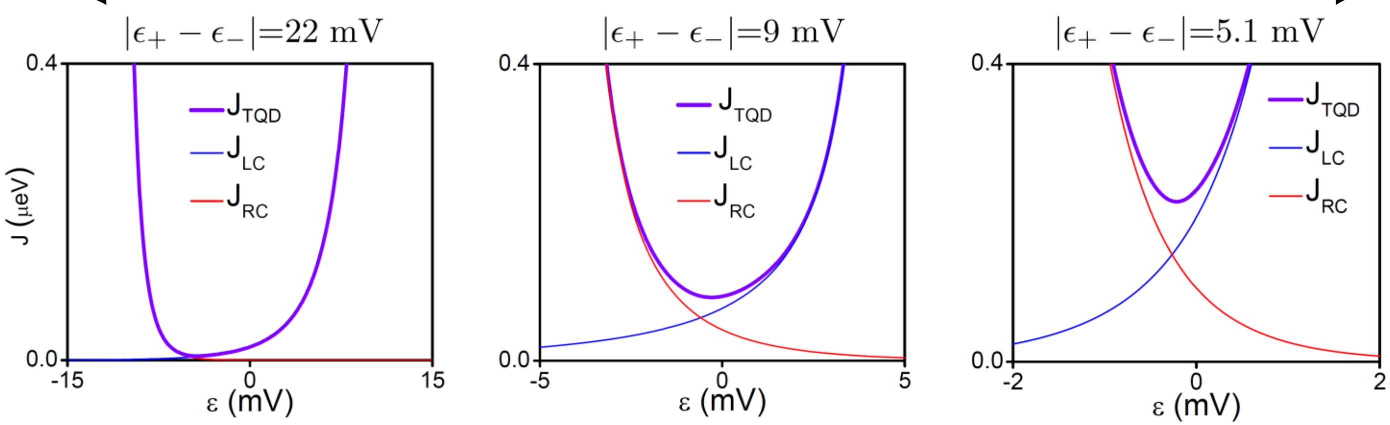
When (111) is narrow, both J_{LC} and J_{RC} are finite

Three-Spin Spectrum



Experiment Part A

Experiment Part B



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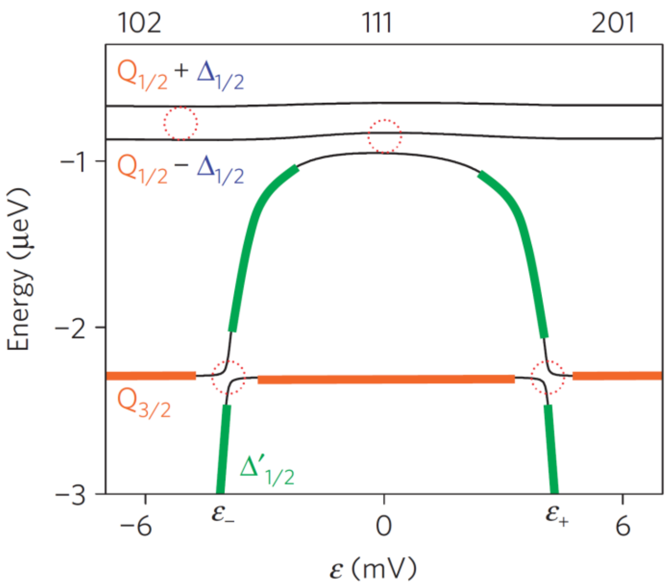
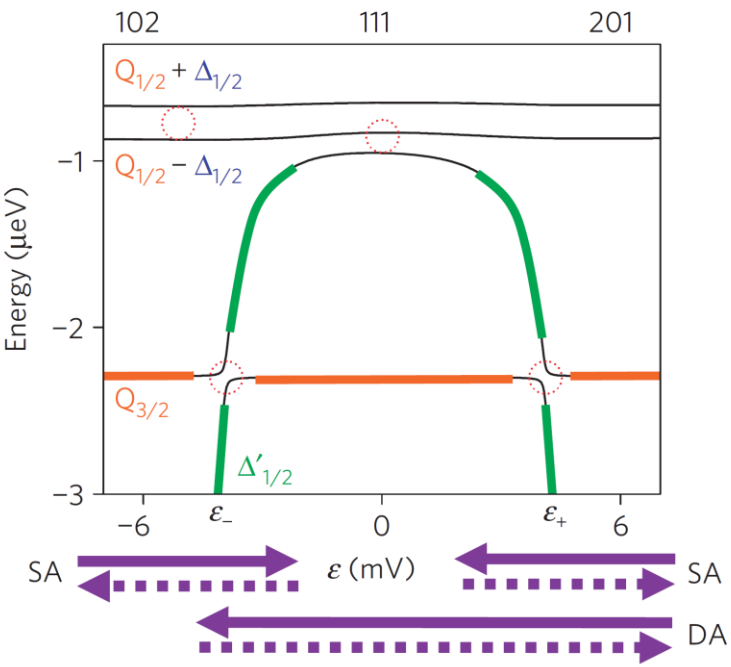
Spectrum with Hyperfine Interaction

Hyperfine interaction with underlying nuclear spin bath leads to anti-crossings



Experiment

Hyperfine interaction with underlying nuclear spin bath leads to anti-crossings



Experiment:

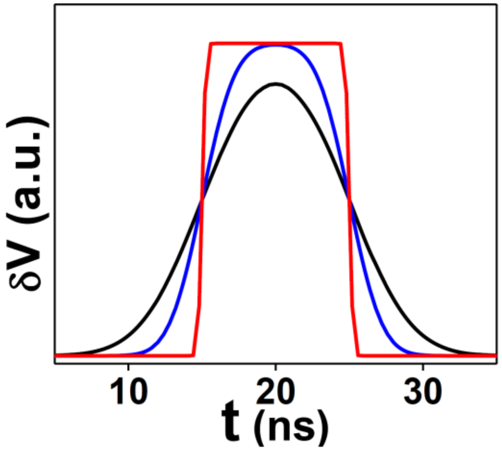
1. Start in the (102) [or (201)] configuration, where the system is in state $|\Delta'_{1/2}>$
2. Apply a voltage pulse to the gates, which increases [or decreases] ε for a short time
   SA: a single anti-crossing is passed (PART A)
   DA: all anti-crossings are passed (PART B)
3. As a function of pulse duration τ, measure probability of $|\Delta'_{1/2}>$ via the QPC
   $|\Delta'_{1/2}>$ ⟶ (102) [or (201)]
   other states ⟶ (111), because of Pauli exclusion

Pulse Shape

Rectangular pulses of duration τ (typically < 25 ns)

↓ Low-pass filter

Pulses with finite rise time (typically around 6 ns)



Theoretical description:
Convolution of rectangular pulse (duration τ) with Gaussian

$$\frac{1}{\sqrt{2\pi}s}e^{-t^2/2s^2}$$

Calculated pulse shapes for τ = 10 ns after Gaussian convolution, leading to rise times of 6.6 ns, 3.5 ns, and 0.4 ns

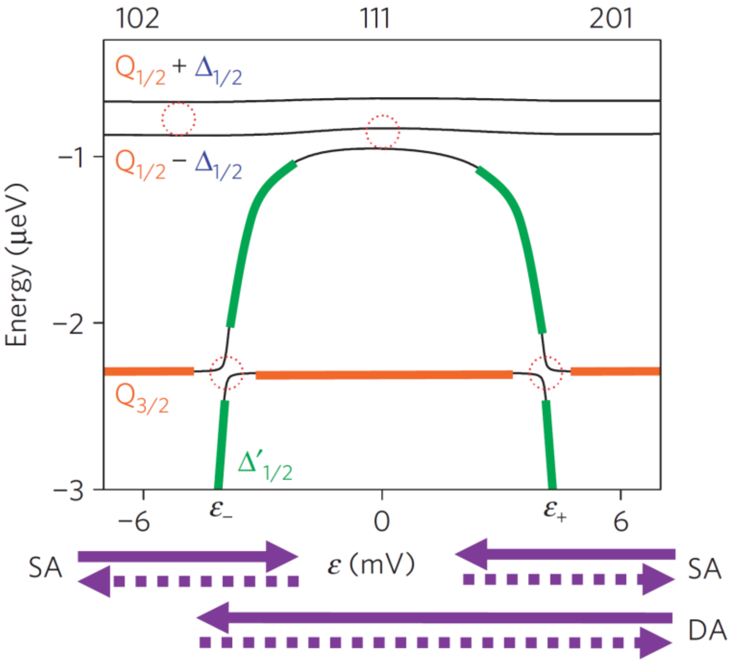
Pulse Shape

Figure	$\lvert\epsilon_+ - \epsilon_-\rvert$ (mV)	$(\delta V_1, \delta V_2)$ (mV)	Duration τ (ns)	Period T_m (μs)	Rise time (ns)	Filtered	Numerically convoluted
M1d	9.0	(-8.8,11)	16	2	6.6	Yes	No
M2a	27	(4.0,-1.7)	1-16	2	6.6	Yes	No
M2c	41.5	(-4.11,7)	1-16	2	6.6	Yes	No
M2b, S5a	50	(4.0,-1.7)	0-25	5	6.6	Yes	No
M2d, S5b	27	(-3.75,6.6)	0-25	5	5.3	Yes	No
M3a	5	(-5.4,6)	16	2	6.6	No	Yes
M3c,d, M4b (40 mT)	5.6	(-5,4.6)	0-25	2	6.6	No	Yes
M4b (5 mT)	3.9	(-5.4.6)	0-25	2	6.6	No	Yes
M4b (25 mT)	5.1	(-5.4.6)	0-25	2	6.6	No	Yes
M4b (60 mT)	4.6	(-5.4.6)	0-25	2	6.6	No	Yes
S3a, S4b (left)	50	(4.0,-1.7)	100	5	6.6	Yes	No
S3b, S4b (right)	27	(-3.75,6.6)	100	5	3.3	Yes	No
S6 (top)	24	(4.0,-1.7)	0-25	5	6.6	Yes	No
S7	34	(-3.75,6.6)	-	10	0.4	No	No
S8a,b	9	(-8,10)	16	2	6.6	Yes	No
S8c	9	(-8,10)	1-16	2	6.6	Yes	No
S9	9	δV_1=-0.8δV_2	10	2	6.6	Yes	No

TABLE I: Pulse details for the experiments. In the Figure column, an M signifies Main text and an S, Suppl. Info.

Experiment

Hyperfine interaction with underlying nuclear spin bath leads to anti-crossings

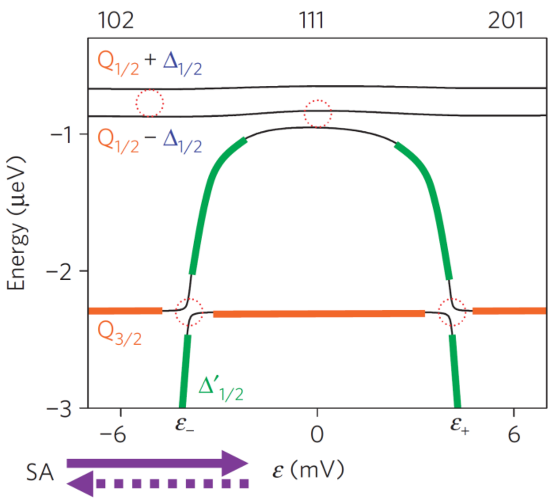


Experiment:

1. Start in the (102) [or (201)] configuration, where the system is in state $|\Delta'_{1/2}\rangle$
2. Apply a voltage pulse to the gates, which increases [or decreases] ε for a short time
   SA: a single anti-crossing is passed (PART A)
   DA: all anti-crossings are passed (PART B)
3. As a function of pulse duration τ, measure probability of $|\Delta'_{1/2}\rangle$ via the QPC
   $|\Delta'_{1/2}\rangle$ ⟶ (102) [or (201)]
   other states ⟶ (111), because of Pauli exclusion

Part A – Wide (111)

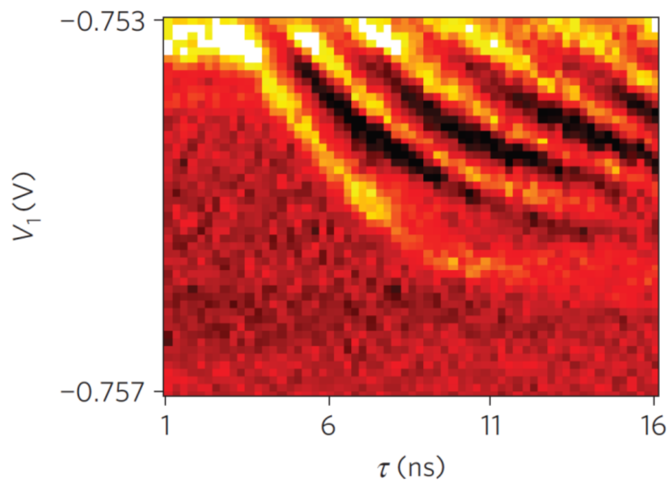
SA Pulse
(102) ↔ (111)



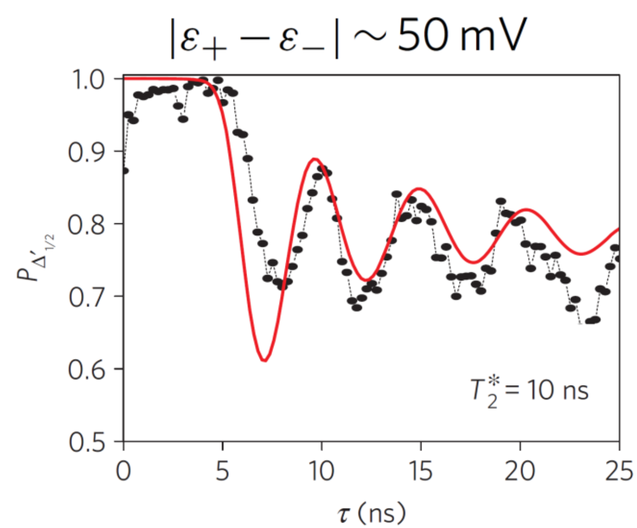
$J_{LC} = 0$
Left spin always decoupled

Results:

B = 60 mT



Pulse reaches further into (111) region



Part A – Wide (111)

SA Pulse
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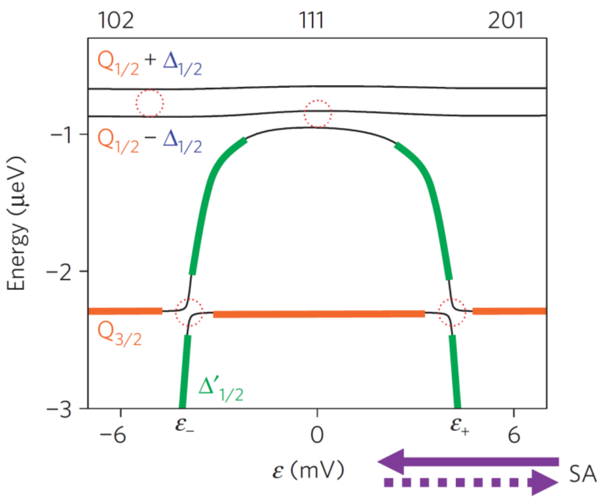
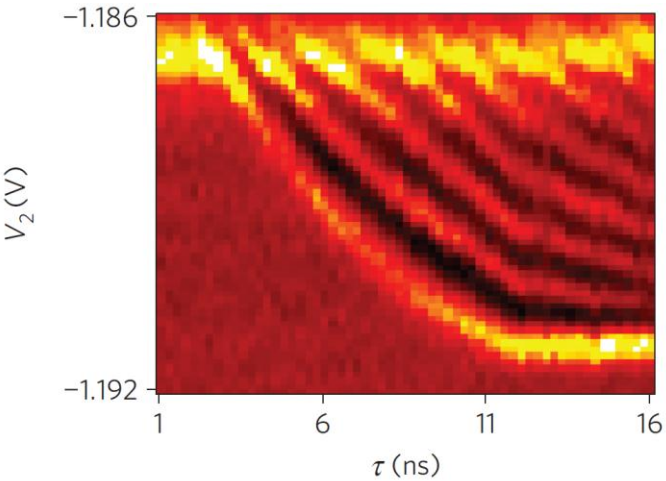


$J_{RC} = 0$
Right spin always decoupled

Results:

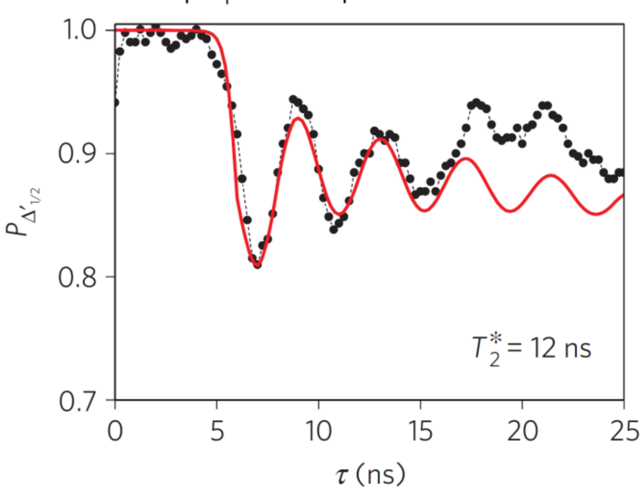
B = 60 mT

$|\varepsilon_+ - \varepsilon_-| = 41.5$ mV



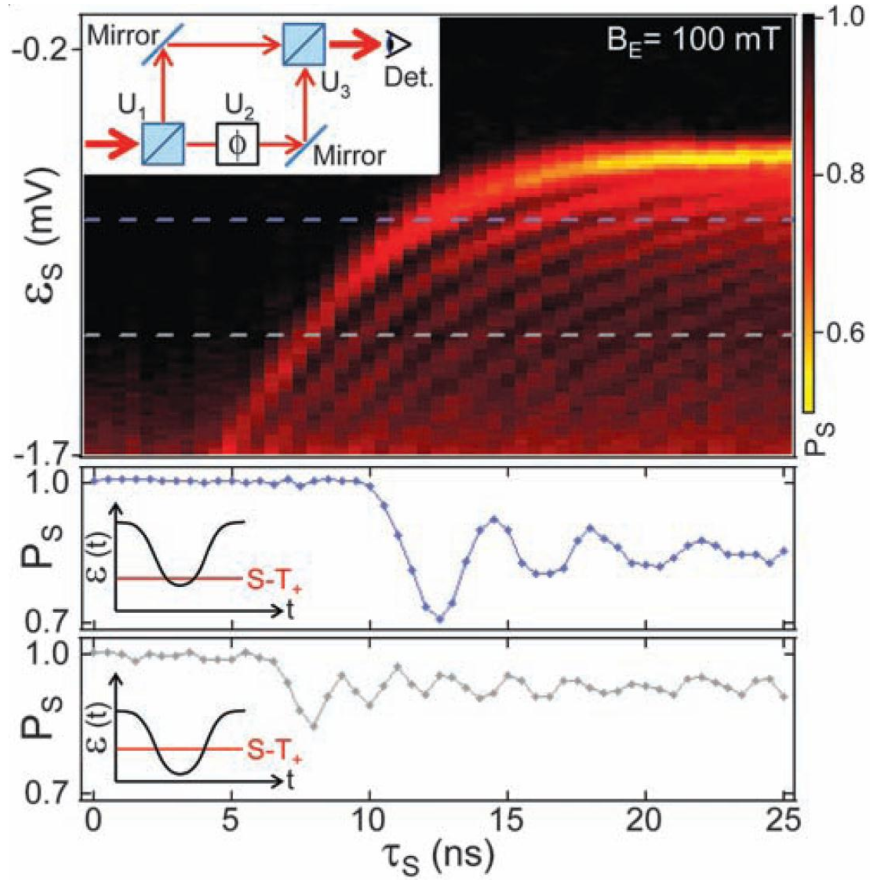
Pulse reaches further into (111) region

$|\varepsilon_+ - \varepsilon_-| = 27$ mV



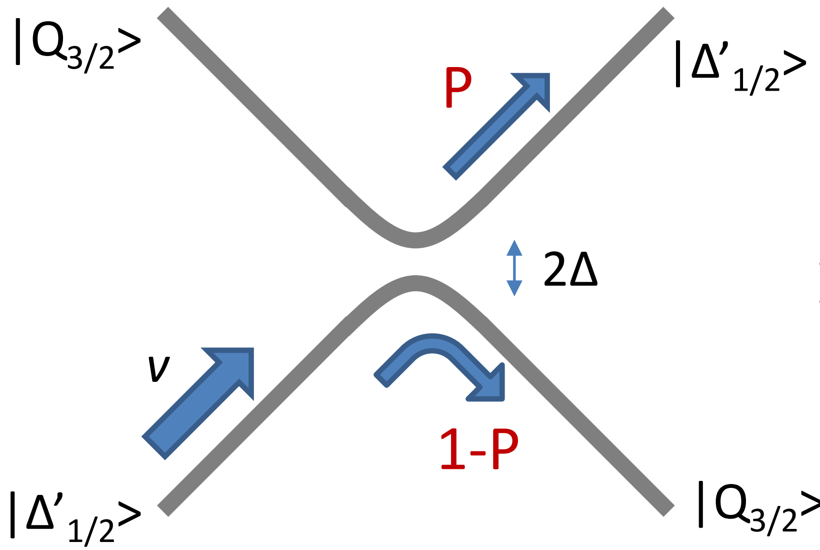
Comparison: Experiment in Double Quantum Dot

For wide (111), the results resemble those from experiments on two-spin states in a double quantum dot



J. R. Petta, H. Lu, and A. C. Gossard, Science **327**, 669 (2010)

Why Oscillations? Qualitative Explanation



$$P_{\mathrm{LZS}} = e^{-2\pi\Delta^2/(\hbar v)}$$

When tuning back, also the phase is important!

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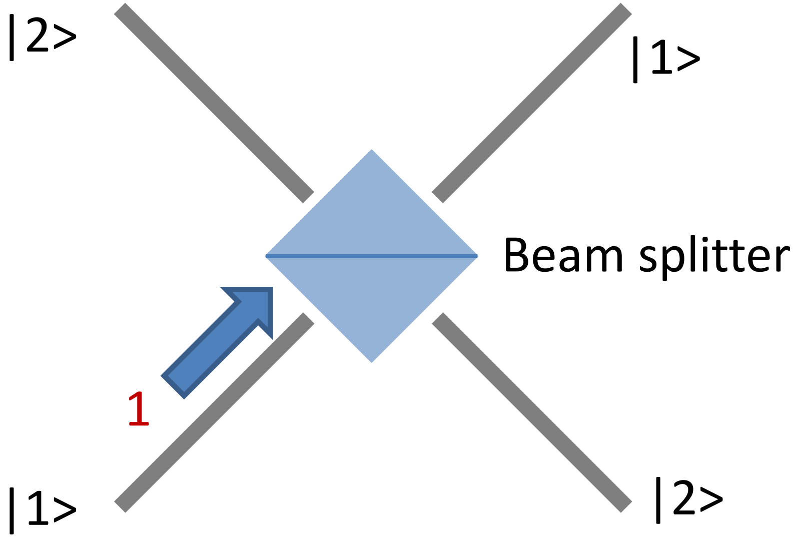

Simple representation:

$$\begin{pmatrix} t & r \\ -r & t \end{pmatrix} \quad \begin{matrix} t, r > 0 \\ t^2 + r^2 = 1 \end{matrix}$$

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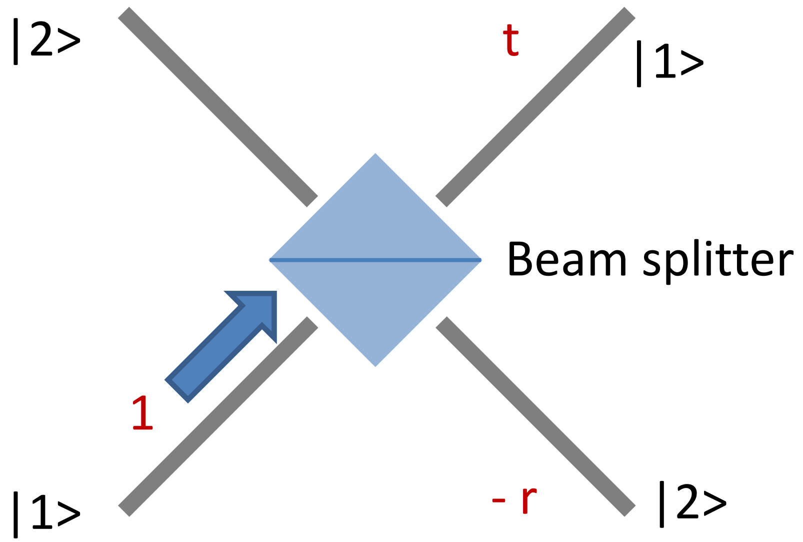

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a) Pass through anti-crossing $\begin{pmatrix} t & r \\ -r & t \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} t \\ -r \end{pmatrix}$

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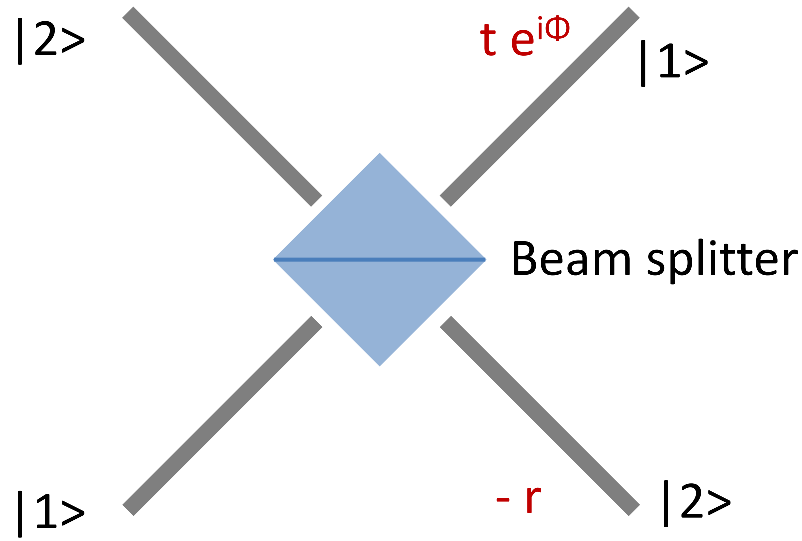

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Due to energy difference between $|\Delta'_{1/2}\rangle$ and $|Q_{3/2}\rangle$

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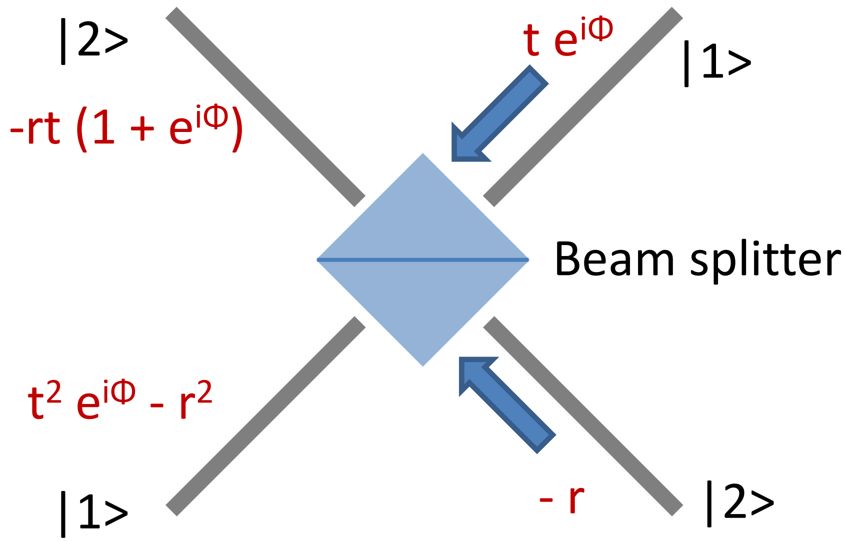

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Due to energy difference between $|\Delta'_{1/2}\rangle$ and $|Q_{3/2}\rangle$

c) Tune back through anti-crossing $\begin{pmatrix} t & r \\ -r & t \end{pmatrix} \begin{pmatrix} te^{i\phi} \\ -r \end{pmatrix} = \begin{pmatrix} t^2 e^{i\phi} - r^2 \\ -rt(1 + e^{i\phi}) \end{pmatrix}$

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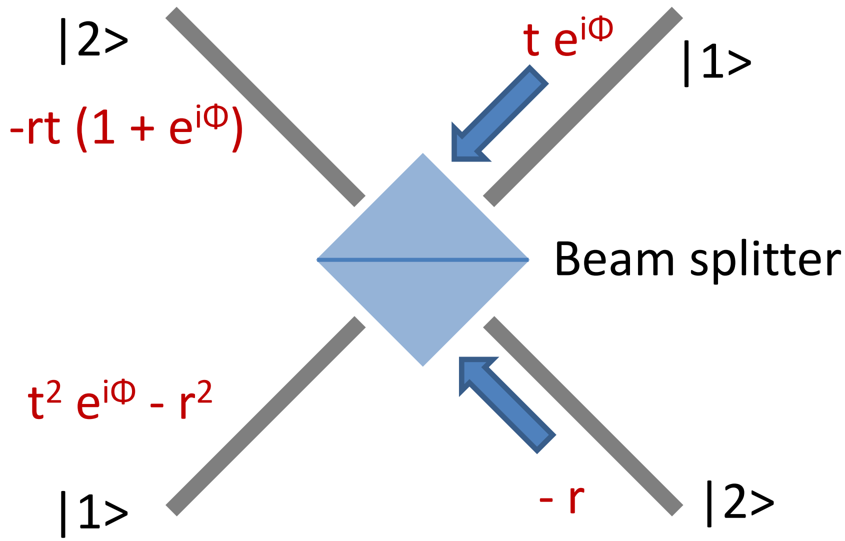

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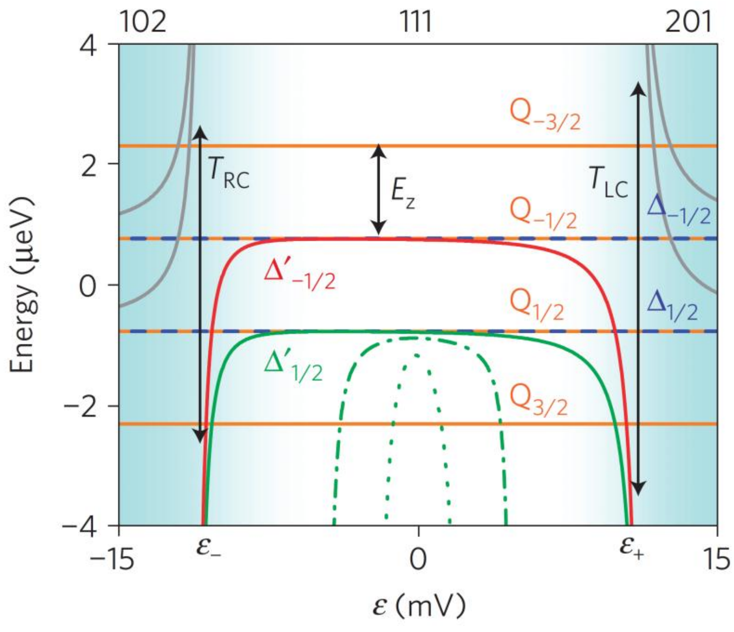
⟶ **"Landau-Zener-Stückelberg oscillations" as function of pulse duration**

Recap

So far:

Only SA pulses for wide (111)
Resembles previous experiments in double quantum dots

Also DA?
Not of interest, because more than two states are involved
(thus not a good system for qubits)



Calculated spectrum for a wide (111)

Part B – Narrow (111)

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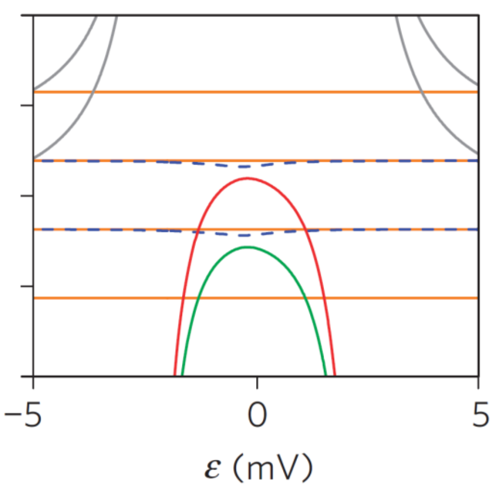
Now:

SA/DA pulses for narrow (111), where all spins are coupled



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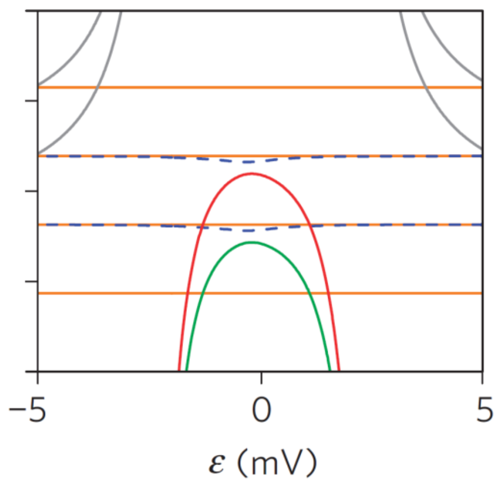


Calculated spectrum for a narrow (111)

NEW Experiment!
What does one expect??

→ SIMULATION!

Simulation

Brief outline:

Pulse shape ⟶ ε(t) ⟶ J(t) ⟶ eigenenergies(t)

Hamiltonian with parameter for hyperfine coupling

$$H = \begin{pmatrix} E_{\mathrm{Q}_{3/2}} & \Gamma_{\Delta',Q_{3/2}} \\ \Gamma^*_{\Delta',Q_{3/2}} & E_{\Delta'_{1/2}} \end{pmatrix}$$

Inclusion of other states:

$$H = \begin{pmatrix} E_{\mathrm{Q}_{1/2}} & \Gamma_{\Delta,Q_{1/2}} & 0 & \Gamma_{\Delta',Q_{1/2}} \\ \Gamma^*_{\Delta,Q_{1/2}} & E_{\Delta_{1/2}} & 0 & 0 \\ 0 & 0 & E_{\mathrm{Q}_{3/2}} & \Gamma_{\Delta',Q_{3/2}} \\ \Gamma^*_{\Delta',Q_{1/2}} & 0 & \Gamma^*_{\Delta',Q_{3/2}} & E_{\Delta'_{1/2}} \end{pmatrix}$$

Master equation for the density matrix

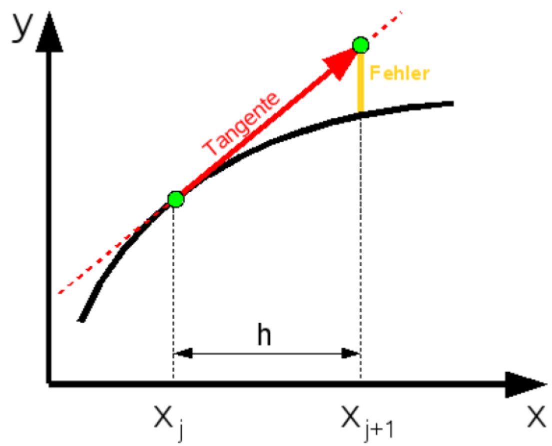
$$\frac{d\rho}{dt} = i\left[\rho, H/\hbar\right]$$ (+ decoherence)

Resulting differential equations are
solved **numerically via the Runge-Kutta method**

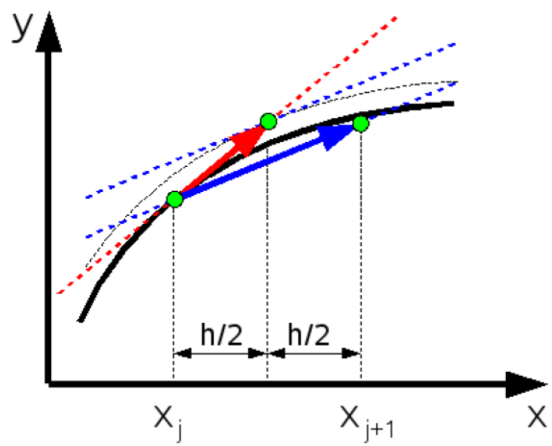
Runge-Kutta Method

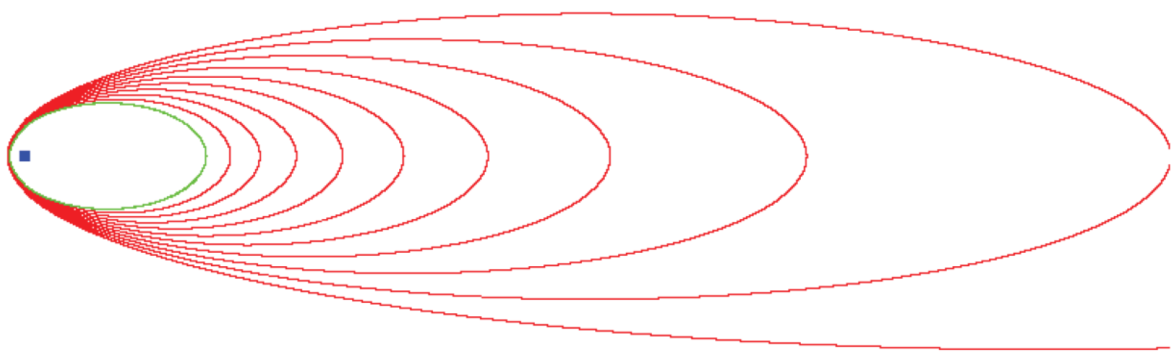
Excerpt from lecture notes on “Computational Physics” by Haye Hinrichsen

Euler



Runge-Kutta (2nd order)





“Runge-Kutta method”: Usually means the extension to 4th order

Parameters for Simulation

Figure	$\lvert\epsilon_+ - \epsilon_-\rvert$ (mV)	$J_{\mathrm{TQD}}^{\mathrm{min}}$ (μeV)	$\frac{J_{LC}+J_{RC}}{2}$ (μeV)	$\tilde{\alpha}_{RC}$ ($\frac{\mu\mathrm{eV}}{\mathrm{mV}}$)	T_{RC} (μeV)	C_{RC} ($\frac{1}{\mathrm{mV}}$)	$\tilde{\alpha}_{LC}$ ($\frac{\mu\mathrm{eV}}{\mathrm{mV}}$)	T_{LC} (μeV)	C_{LC} ($\frac{1}{\mathrm{mV}}$)	$\Gamma_{\Delta',\mathrm{Q}_{3/2}}$ (μeV)	$\Gamma_{\Delta',\mathrm{Q}_{1/2}}$ (μeV)	$\Gamma_{\Delta,\mathrm{Q}_{1/2}}$ (μeV)
M1d, S2b	9.0	0.116	0.0751	62.5	8.20	0.1627	38.0	5.28	0.061	-	-	-
M1c, S8d,e,f, S9b	9.0	0.116	0.0751	62.5	8.20	0.1627	38.0	5.28	0.061	0.2	0.2	0.2
M4a (5 mT)	3.9	0.628	0.418	57.8	15.8	0.4995	39.0	13.4	0.380	-	-	-
M4a (25 mT), S2c	5.1	0.309	0.191	57.8	15.8	0.4995	39.0	13.4	0.380	-	-	-
M4a (40 mT)	5.6	0.229	0.140	57.8	15.8	0.4995	39.0	13.4	0.380	-	-	-
M4a (60 mT)	4.6	0.394	0.262	57.8	15.8	0.4995	39.0	13.4	0.380	-	-	-
M4c (5 mT)	3.9	0.628	0.418	57.8	15.8	0.4995	39.0	13.4	0.380	0.2	0.2	0.2
M3b, M4c (25 mT)	5.1	0.309	0.191	57.8	15.8	0.4995	39.0	13.4	0.380	0.2	0.2	0.2
M3c,d, M4c (40 mT)	5.6	0.229	0.140	57.8	15.8	0.4995	39.0	13.4	0.380	0.2	0.2	0.2
M4c (60 mT)	4.6	0.394	0.262	57.8	15.8	0.4995	39.0	13.4	0.380	0.2	0.2	0.2
S2a, S4a	22	0.0057	0.0037	54.0	9.39	0.3414	40.0	9.96	0.1154	-	-	-
S4b(left)	~50	-	-	42.5	10.0	0.0	-	-	-	-	-	-
M2b, S5a	~50	-	-	42.5	10.0	0.0	-	-	-	0.15	0.0	0.0
S4b(right)	27	-	-	-	-	-	35.9	5.89	0.0	-	-	-
M2d, S5b(mid & bottom)	27	-	-	-	-	-	35.9	9.96	0.1154	0.12	0.0	0.0
S5b(top)	27	-	-	-	-	-	35.9	9.96	0.1154	0.17	0.0	0.0
S6 (bottom)	24	-	-	42.5	9.39	0.3414	-	-	-	0.2	0.0	0.0

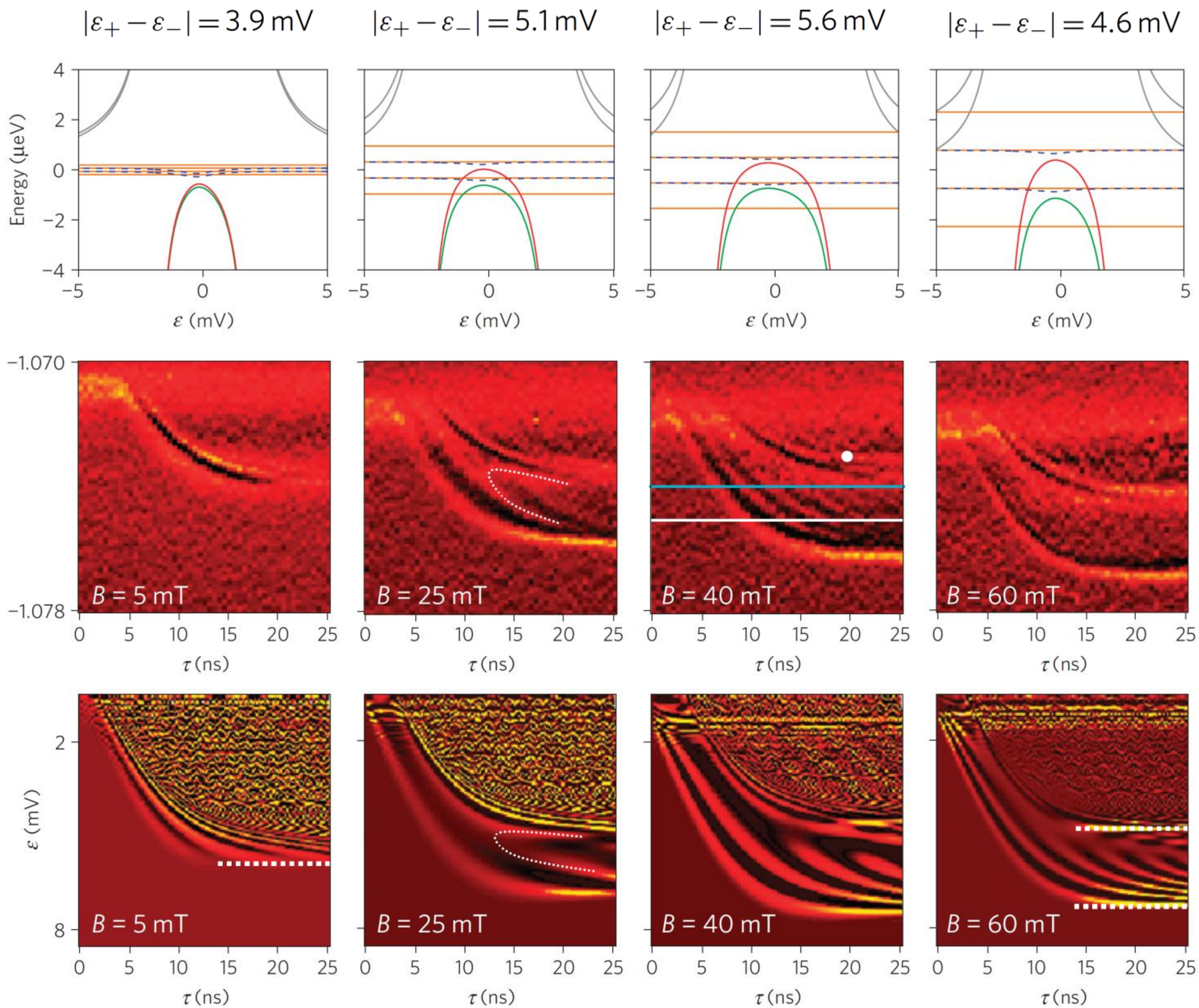
Pulse starts from (201)

Results – Narrow (111)

Calculated spectrum

Experiment

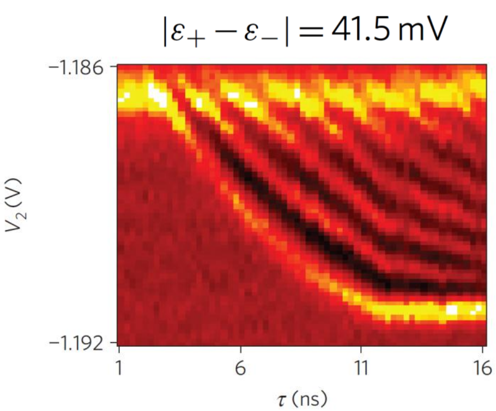
Simulation (no decoherence)

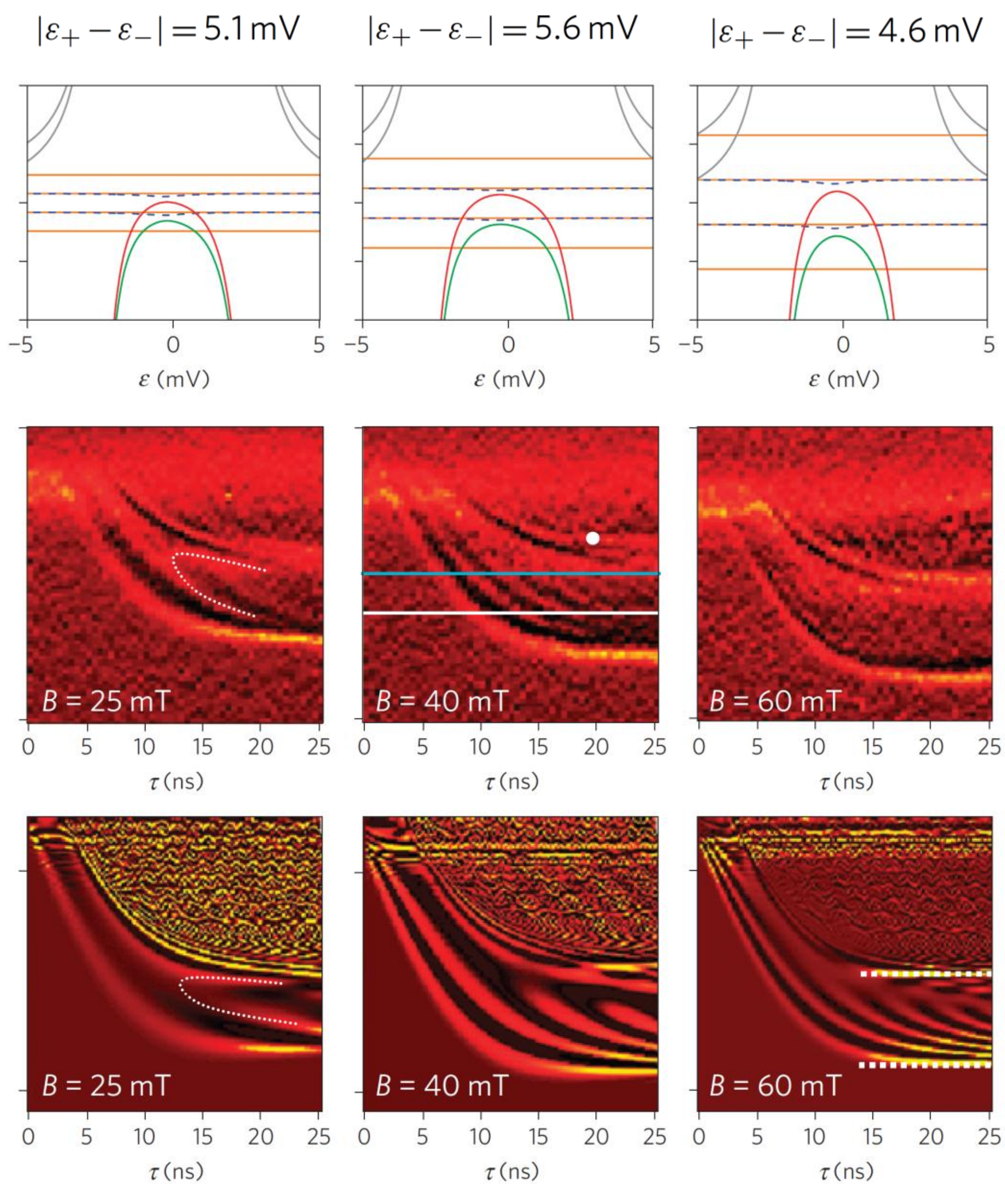


Pulse starts from (201)

Results – Narrow (111)

Comparison to effective two-spin result





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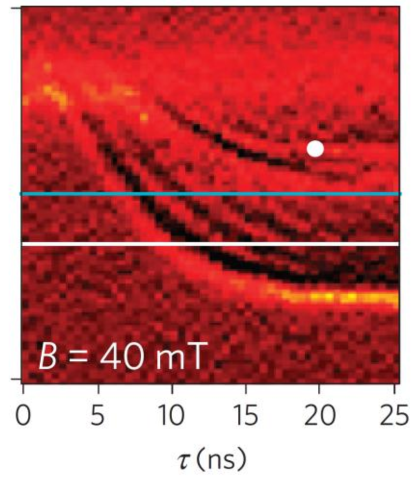
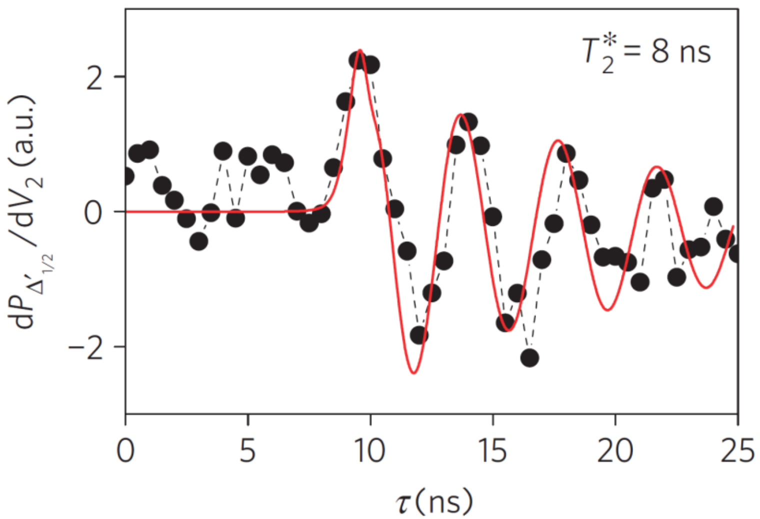
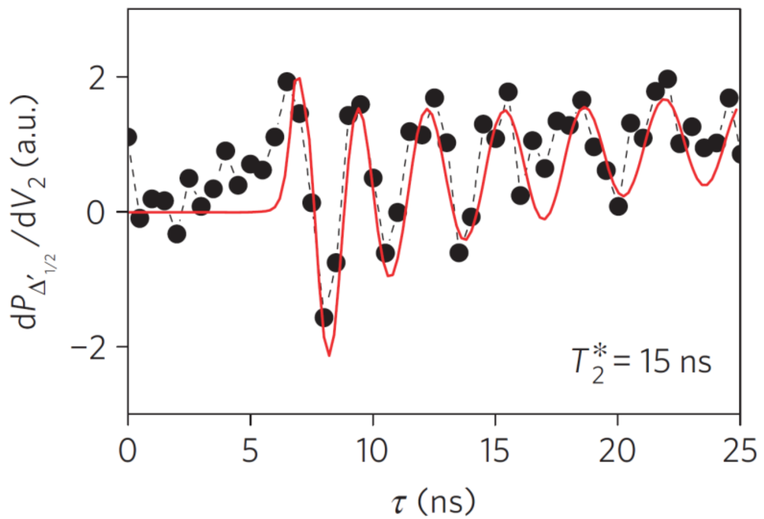


white line

blue line





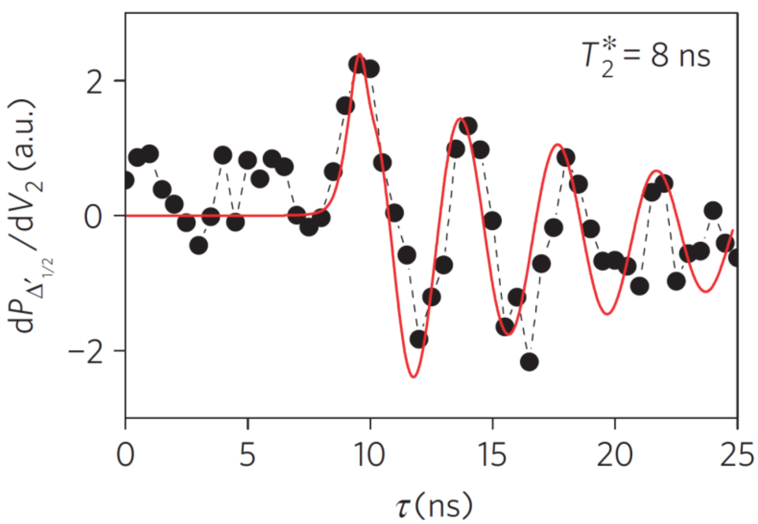
The fits use B = 60 mT (not B = 40 mT) due to dynamic nuclear polarization

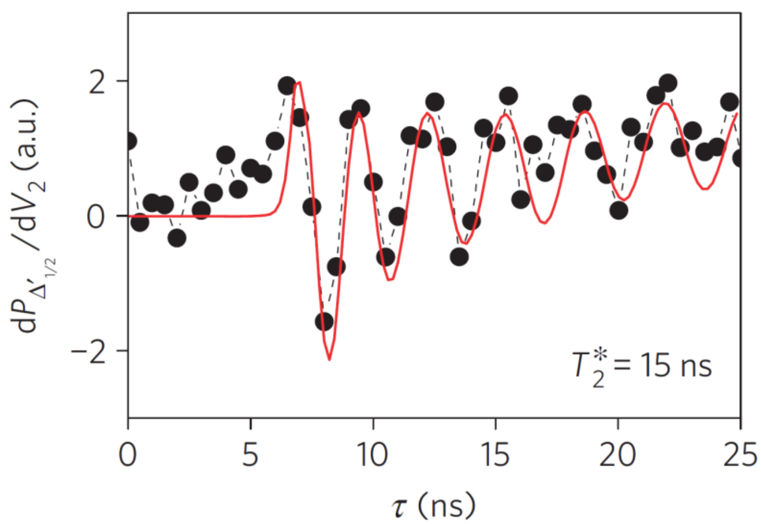
Pulse starts from (201)

Results – Narrow (111)

Dephasing times similar to those of two-spin states

Fluctuations in the underlying nuclear spin bath
seem to be the dominant mechanism for dephasing





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- Demonstrated coherent evolution for a three-spin qubit

Drawback: The generated qubit is **not** a proper exchange-only qubit (basis states differ in both the total spin and S_z)

Comment: Exchange-Only Qubit

$$H_{\text{ex}}(t) = J(t)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

The Heisenberg interaction commutes
with the total spin and its projection on the z axis

→ It can only rotate among states with the same quantum numbers S, S_z

Suitable bases for an exchange-only qubit with three spins:
$|\Delta_{1/2}\rangle$, $|\Delta'_{1/2}\rangle$ or $|\Delta_{-1/2}\rangle$, $|\Delta'_{-1/2}\rangle$

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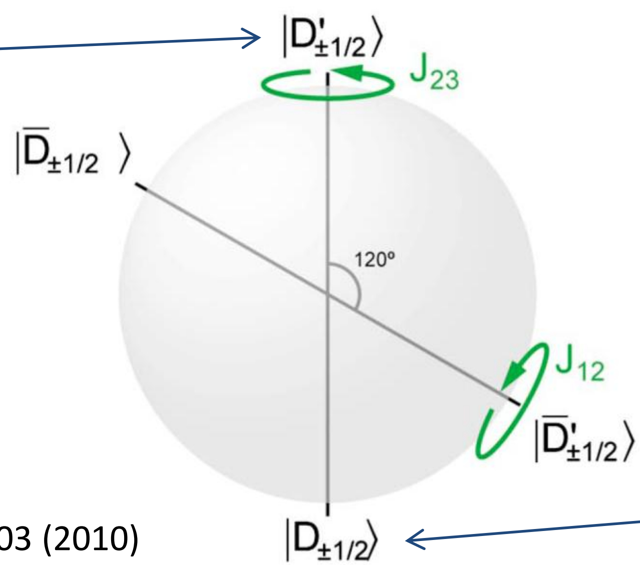
$|\Delta'_{\pm 1/2}\rangle$, $J_{12} = 0$

$|\Delta_{\pm 1/2}\rangle$, $J_{23} = 0$



$|\Delta'_{\pm 1/2}\rangle$, $J_{23} = 0$

$|\Delta_{\pm 1/2}\rangle$, $J_{12} = 0$

Figure from
Laird *et al.*, PRB **82**, 075403 (2010)

In contrast, Gaudreau *et al.* work with the qubit basis $|Q_{3/2}\rangle$, $|\Delta'_{1/2}\rangle$

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"This is good news for the future: now nothing is holding us back from building an all-electrically controlled quantum chip made up of large numbers of electron spins"
Frank Koppens, news & views