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ARTICLES

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Fermionic transport and out-of-equilibrium dynamics in a homogeneous Hubbard model with ultracold atoms

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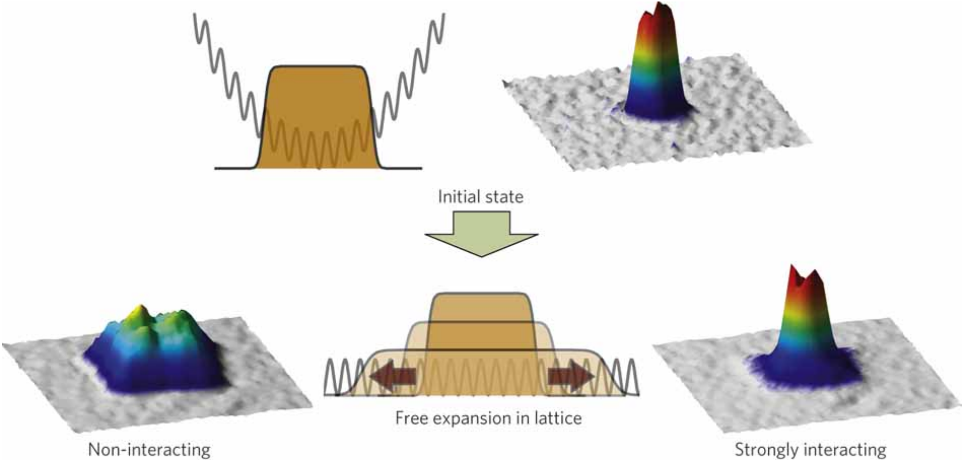
Transport properties are among the defining characteristics of many important phases in condensed-matter physics. In the presence of strong correlations they are difficult to predict, even for model systems such as the Hubbard model. In real materials, additional complications arise owing to impurities, lattice defects or multi-band effects. Ultracold atoms in contrast offer the possibility to study transport and out-of-equilibrium phenomena in a clean and well-controlled environment and can therefore act as a quantum simulator for condensed-matter systems. Here we studied the expansion of an initially confined fermionic quantum gas in the lowest band of a homogeneous optical lattice. For non-interacting atoms, we observe ballistic transport, but even small interactions render the expansion almost bimodal, with a dramatically reduced expansion velocity. The dynamics is independent of the sign of the interaction, revealing a novel, dynamic symmetry of the Hubbard model.

Overview

- Study transport in a clean and well-controlled environment: No impurities, no lattice defects and no phonons.
- Study transport in strongly correlated, out-of equilibrium systems
- Create a quantum simulator for condensed-matter systems
- Transport in a homogeneous Hubbard model: two component Fermi gas:

$$\hat{H} = -J \sum_{<i,j>,\sigma} \hat{a}^{\dagger}_{\sigma} \hat{a}_{\sigma} + U \sum_{i} \hat{n}_{i\downarrow} \hat{n}_{i\uparrow}$$



Initial state
Free expansion in lattice
Non-interacting
Strongly interacting

Tuning the Scattering Length

Interaction between two atoms:

$$V(r_1, r_2) = \frac{4\pi\hbar^2}{M} a_s \delta(r_1 - r_2)$$

$$a_s(B) = a_{bg}\left[1 - \frac{\Delta B}{B - B_0}\right]$$

a_{bg} is the off-resonant scattering length and ΔB and B_0 describe the width and position of the resonance.

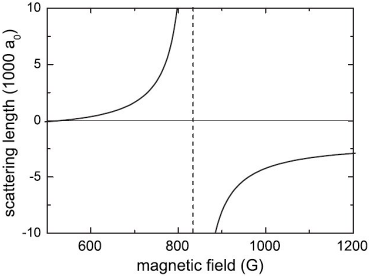


FIG. 2. Magnetic field dependence of the scattering length between the two lowest magnetic substates of ^{6}Li with a Feshbach resonance at B_0=834 G and a zero crossing at $B_0+\Delta B$ =534 G. The background scattering length $a_{bg}=-1405a_B$ is exceptionally large in this case (a_B the Bohr radius).

Figure: Rev.Mod. **80**, pp.892

Magnetic Feshbach Resonances

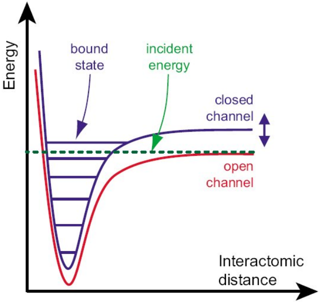
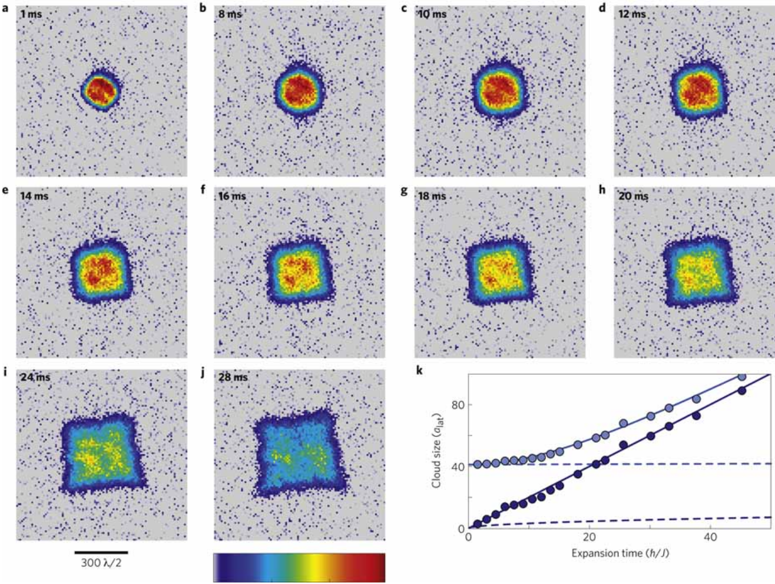


FIG. 3. (Color online) The two-channel model for a Feshbach resonance. Atoms prepared in the open channel, corresponding to the interaction potential $V_{\mathrm{op}}(r)$, undergo a collision at low incident energy. In the course of the collision, the open channel is coupled to the closed channel $V_{\mathrm{cl}}(r)$. When a bound state of the closed channel has an energy close to zero, a scattering resonance occurs. The position of the closed channel can be tuned with respect to the open one, e.g., by varying the magnetic field B.

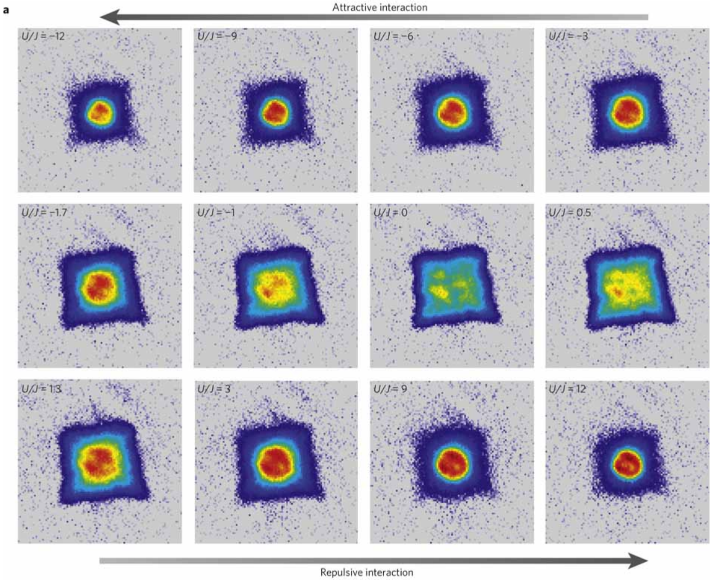
Figure: Rev.Mod. **80**, pp.893

The two channels have a different magnetic moment: You can change the relative distance ΔE between them.

Expansion of non-interacting fermions



Expansion of interacting fermions



Observations

- Already small interactions cause a drastic reduction of mass transport
- For strong interactions the core of the atomic cloud does not expand, but shrinks
- Bimodal system: Like melting a melting ball of ice
- Only the magnitude but not the sign of the interaction matters

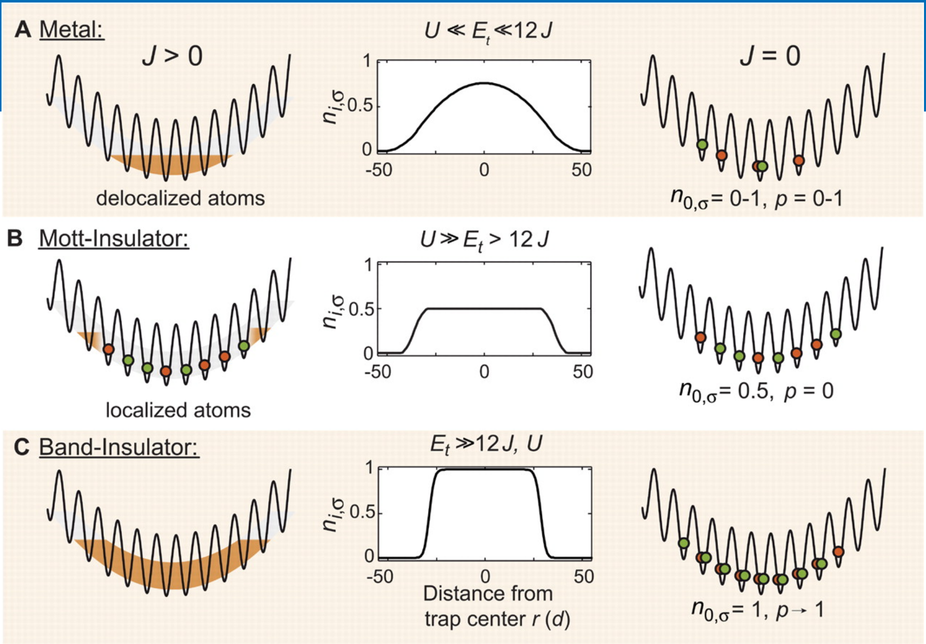


Figure: From Science 322 / 1520

Expansion of non-interacting particles

$$\hat{H} = -J \sum_{<i,j>,\sigma} \hat{a}^{\dagger}_{\sigma} \hat{a}_{\sigma}$$

$$\Rightarrow \quad \hat{H} = \sum_{\mathbf{q},\sigma} \epsilon_{\mathbf{q}} \hat{a}^{\dagger}_{\mathbf{q}\sigma} \hat{a}_{\mathbf{q}\sigma}$$

with
$\epsilon_{\mathbf{q}} = -2J \sum_i \cos(q_i a_{lat})$

Use group velocity
$\mathbf{v_q} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{q}}}{\partial \mathbf{q}}$

$$\Rightarrow v_{exp} = \sqrt{\langle \mathbf{v}_{\mathbf{q}}^2 \rangle} = \sqrt{2d} \frac{J}{\hbar} a_{lat}$$

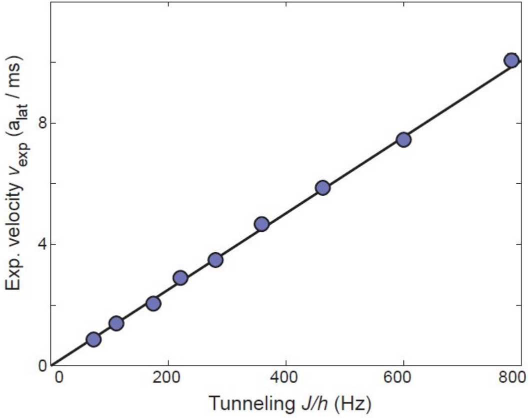


FIG. S6. Mean expansion velocity for different tunnelling J. The solid line shows the quantum-mechanical prediction for 2D: $v_{\mathrm{exp}} = 2\,\frac{J}{\hbar} a_{lat}$. Statistical fit errors are comparable to symbol size.

$U \leftrightarrow -U$ symmetry

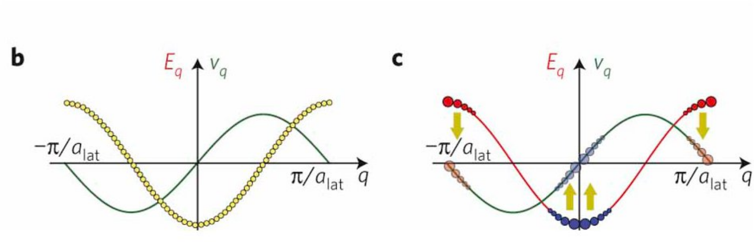


Figure: (b) U=0 (initial state); (c) momentum distribution after some time when $U \neq 0$

- Initially: atoms are at rest
- During the expansion interaction energy is converted into kinetic energy

$U \leftrightarrow -U$ symmetry

- The sign of kinetic energy of the Hubbard model

$$\epsilon_{\mathbf{q}} = -2J \sum_i \cos(q_i a_{lat})$$

can be changed by shifting all momenta $\mathbf{q} \rightarrow \mathbf{q} + \tilde{\pi}/\mathbf{a_{lat}}$

- Initial state and observable are time reversal invariant and invariant under the shift of the momenta

$$\Rightarrow \qquad U \leftrightarrow -U \text{ symmetry}$$

Dublon protection

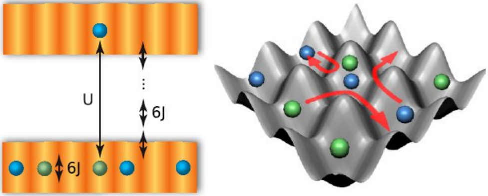


FIG. 1 (color online). Stability of highly excited states in the single-band Hubbard model. Doubly occupied lattice sites are protected against decay by the on-site interaction energy U. The average kinetic energy of a single particle in a periodic potential is half the bandwidth $6J$. Thus the relaxation of excitations requires several scattering partners to maintain energy conservation.

Figure: PRL 104, 080401

Boltzmann equation

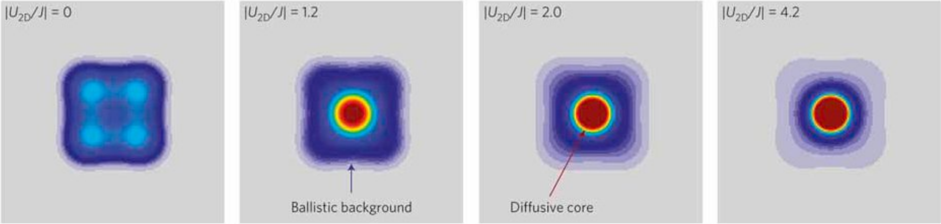
Use the quasi-classical momentum distribution $f_{\mathbf{q}}$

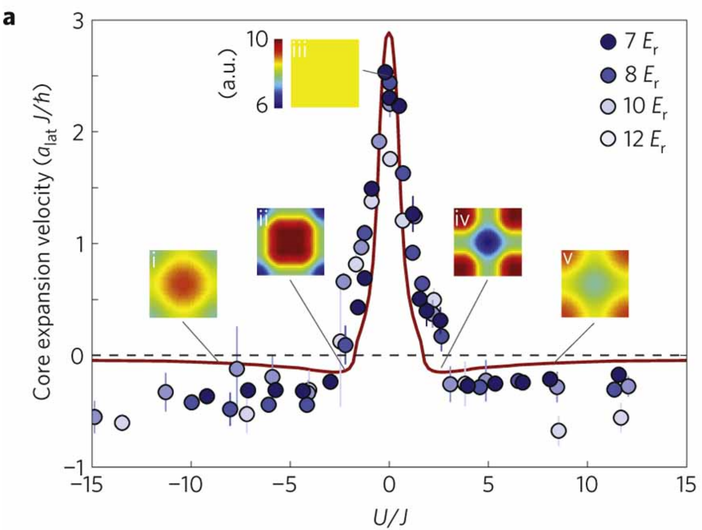
$$\partial_t f_{\mathbf{q}} + \mathbf{v}_{\mathbf{q}} \nabla_{\mathbf{r}} f_{\mathbf{q}} + \mathbf{F}(\mathbf{r}) \nabla_{\mathbf{q}} f_{\mathbf{q}} = -\frac{1}{\tau(\mathbf{n})} \left(f_{\mathbf{q}} - f_{\mathbf{q}}^0(\mathbf{n}) \right)$$

where $\tau(\mathbf{n})$ is the transport scattering time, $f_{\mathbf{q}}^0$ the equilibrium Fermi distribution.

- The mass transport is driven by density gradients (not external potentials)
- $\tau(\mathbf{n})$ is determined from microscopic calculations.

Simulated density distribution

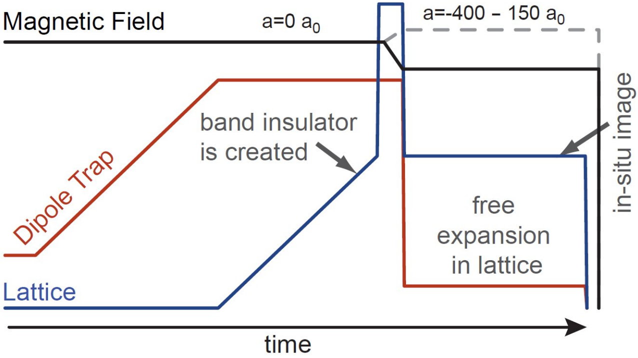



a
Core expansion velocity (a_{lat} $J/\hbar$)
U/J
7 E_r
8 E_r
10 E_r
12 E_r
(a.u.)
10
6
i
ii
iii
iv
v
3
2
1
0
−1
−15
−10
−5
0
5
10
15

Conclusions

- Study non-equilibrium dynamics with full control of almost all parameters.
- The Dynamics is independent of the sign of the interactions due to the symmetry of the kinetic energy.
- Outlook: Study dynamics in disordered lattices.
- Outlook: Bose/Fermi mixture to study ohmic transport.

Experimental sequence



Diffusion equation

Define $\vec{n} = (n, e)$ where $n(\vec{r}, t)$ is the local density and $e(\vec{r}, t)$ the local energy

Continuity equation $\partial_t \vec{n} + \nabla \vec{j} = 0$ with $j = -D(\vec{n}) \nabla \vec{n}$

$\Rightarrow \partial_t \vec{n} = \nabla (D(\vec{n}) \nabla \vec{n})$

$D(\mathbf{n}) = \tau(\mathbf{n}) \frac{\langle \mathbf{v_q^2} \rangle}{\mathbf{a_{lat}}}$

Solution of the Boltzmann equation

Use variational approach

$$f_{\mathbf{q}} = f_{\mathbf{q}}^0 - \frac{\partial f_{\mathbf{q}}^0}{\partial \epsilon_{\mathbf{q}}} \sum_i \alpha_i c_{\mathbf{n}}^{(\mathrm{i})}$$

where $c_{\mathbf{q}}^{(1)} = v_{\mathbf{q}}^x$, $c_{\mathbf{q}}^{(2)} = \epsilon_{\mathbf{q}} v_{\mathbf{q}}^x$, $c_{\mathbf{q}}^{(3)} = q_x$ and $c_{\mathbf{q}}^{(4)} = (\pi/a - q_x)$

Experimental sequence

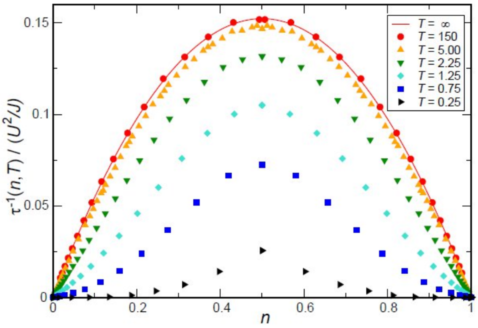


FIG. S2. Transport scattering rates as a function of density for different temperatures. Note that the transport scattering rate (in contrast to the single-particle scattering rate) becomes exponentially small in the low-density, low-temperature regime where Umklapp scattering is suppressed.