Unbounded growth of entanglement in models of many-body localization

J. H. Bardarson, F. Pollmann, and J. E. Moore

arXiv:1202.5532

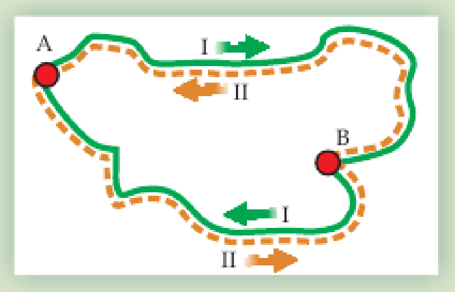
Vladimir M. Stojanović @ Journal Club

March 6, 2012

Anderson localization: the basics

“Absence of diffusion in certain random lattices”,
P. W. Anderson, Phys. Rev. **109**, 1492 (1958)

a wave-packet (or a particle) moving in a spatially-disordered, (time-independent) potential exhibits localization!



After: Lagendijk et al., Phys. Today (2009)

A. L. most easily demonstrated in optical- and matter-wave systems

What is this paper about?

Q: Can a closed quantum system of many interacting particles be localized by disorder?

Recent studies: in the absence of other degrees of freedom (e.g., phonons), e-e interaction may give rise to a "many-body localization transition" even in 1d!

Goal of this paper: show that the "many-body localized" phase differs qualitatively from the conventional (non-interacting) localized phase even in 1d!

Outcome: entanglement entropy and particle-number fluctuation show slow logarithmic evolution in time!

entanglement entropy does not saturate in the thermodynamic limit, even for very weak interactions!

Random-field ($s = 1/2$) XX Hamiltonian in $1d$

$$H_0 = J_\perp \sum_{i=1}^{N-1} \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y \right) + \sum_{i=1}^{N} h_i S_i^z$$

h_i – uniform random numbers $\in [-\eta, \eta]$

Reminder: the Jordan-Wigner (JW) transformation

$$S_1^- = c_1 \quad , \quad S_i^- = \exp[i\pi \sum_{l=1}^{i-1} c_l^\dagger c_l] c_i \;\; (i \geq 2)$$

maps H_0 onto a Hamiltonian for free fermions with n.n. hopping and random on-site potential ($S_i^z = c_i^\dagger c_i - 1/2$)

study time-evolution of an initially unentangled pure state

the von Neumann entropy: $S = -\mathrm{Tr}_{\mathrm{A}}(\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}) = -\mathrm{Tr}_{\mathrm{B}}(\hat{\rho}_{\mathrm{B}} \ln \hat{\rho}_{\mathrm{B}})$

bipartition $\mathcal{H} = \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$ by dividing the system at the center bond!

for asymptotic behavior ($t \to \infty$) use exact diagonalization:

$$H = \sum_j \lambda_j P_j \quad \Longrightarrow \quad U(t) = \sum_j e^{-i\lambda_j t} P_j$$

results obtained by averaging over $> 10^4$ field configurations $\{h_i\}$, starting from a random state $|\Psi_{t=0}\rangle = |m_1\rangle \otimes |m_2\rangle \ldots \otimes |m_L\rangle$
$m_j \in \{\uparrow, \downarrow\}$

Adding the z (Ising) coupling (fermion interactions)

$$H = \sum_{i=1}^{N-1} \left[J_\perp \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y \right) + J_z S_i^z S_{i+1}^z \right] + \sum_{i=1}^{N} h_i S_i^z$$

Q: Is there any qualitative change in the behaviour of physical quantities when a small interaction (J_z) is added?

Already known (almost...problem to approach the TD limit!)
V. Oganesyan and D. Huse, PRB **75**, **155111** (**2007**)

for $J_\perp = J_z$ (Heisenberg case) the system undergoes a dynamical transition as a function of η/J_z (in all eigenstates):

for small enough η/J_z (strong-enough interactions) localization is destroyed (i.e., spin conductivity is nonzero)!

Entanglement growth after a quench

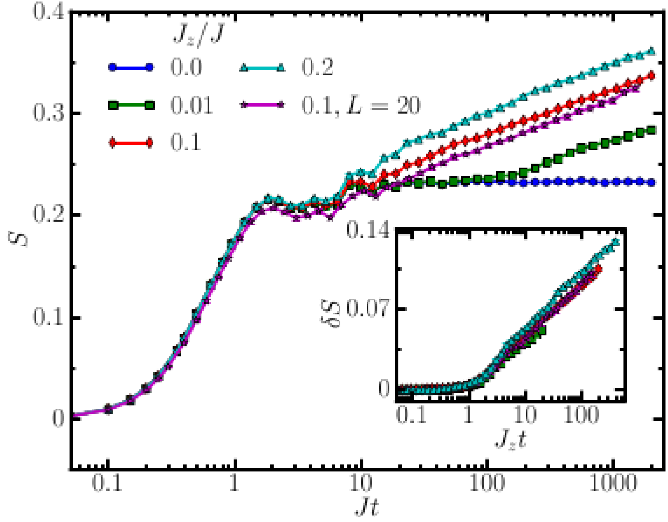
for $J_z = 0$ (no interactions) the "half-chain" entanglement entropy saturates; saturation sets in at times $\sim J_\perp^{-1}$

expectation: ...weak interaction leads to a small delay in saturation and a small increase in final entanglement...

instead, entanglement growth shows a qualitative change of behavior already for infinitesimal J_z!

It grows logarithmically even after times $>> J_\perp^{-1}$!

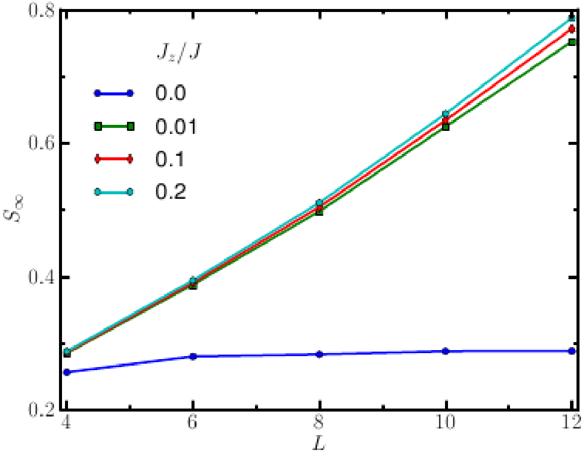
Entanglement growth after a quench



data for $\eta = 5$ and $L = 10$

Saturation value of entropy as a function of the system size

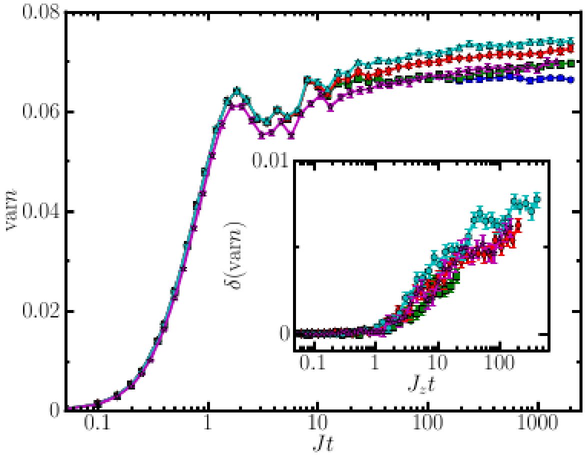
saturation values for finite L are essentially independent of J_z!



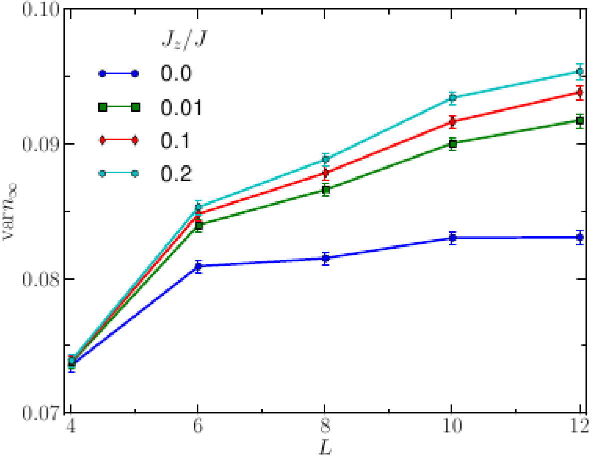
the entanglement entropy does not saturate in the TD limit!

Half-chain particle number fluctuations

i.e., the variance of the total spin on half the chain



Saturation values of the half-chain particle-number variance



unlike for entanglement, saturation values depend on the interaction strength!