Ettingshausen effect due to Majorana modes

(arXiv:1203.5793)

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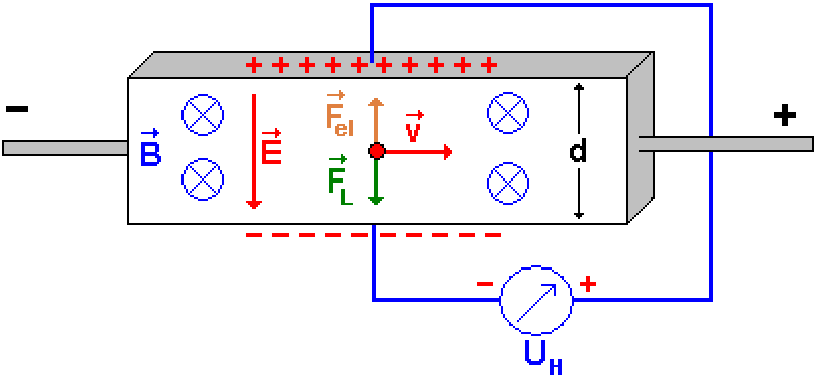
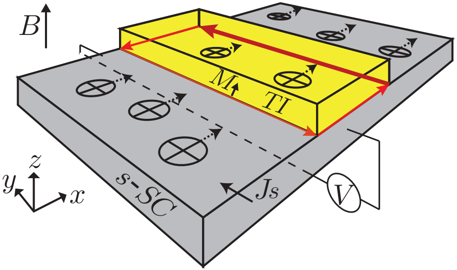


A. Ettingshausen, W. Nernst, Ann. der Phys. und Chem., **265**, 343 (1886)

Thermoelectric effect

Each vortex carries an extra entropy

$$s_0 = (k_\mathrm{B}/2) \ln 2$$



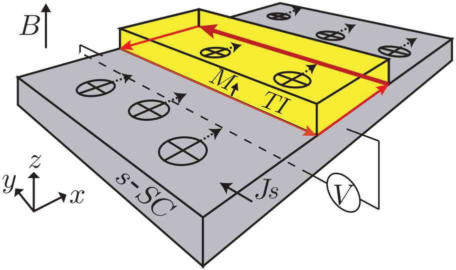
$$j_s = s_0 n_v \mathbf{u} = s_0 \frac{2eB}{h} \mathbf{u}$$

Entropy current: $$j_s = s_0 \frac{2e}{h} \mathbf{E} \times \hat{\mathbf{B}}$$ $$\hat{\mathbf{B}} \equiv \mathbf{B}/|\mathbf{B}|$$

Energy balance

The energy current flowing into/out of edge mode

$$\frac{dQ}{dt} = c_V v_\psi \delta T = \frac{\pi}{12\hbar} k_B^2 T \delta T$$



The energy flow due to the vortex motion crossing the edge

$$\frac{dQ_v}{dt} = L T j_s = T s_0 \frac{2eV}{h},$$

$$\delta T = \frac{6 \ln 2}{\pi^2} \frac{eV}{k_B}$$

Drawbacks of the proposal

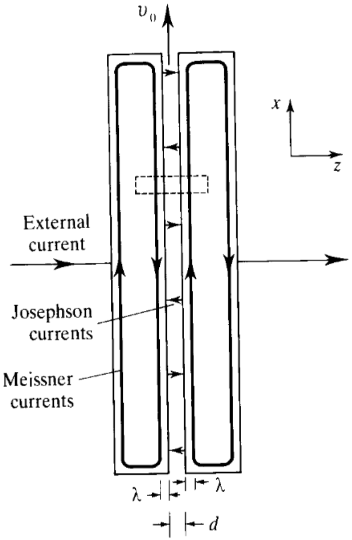
- Abrikosov vortices have a normal core
- Magnus force is ignored
- Large energy dissipation through the Joule heating
- Termalization with the environment

Sine-Gordon equation for phase

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{u^2}\frac{\partial^2}{\partial t^2}\right)\varphi = \frac{\sin\varphi}{\lambda_J^2}$$

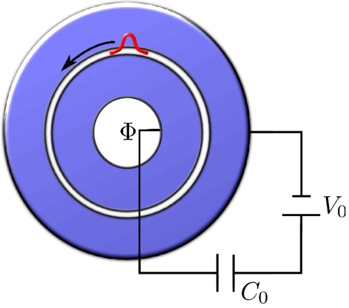
$$[\lambda_J^2 = \frac{\Phi_0}{8\pi^2 j_c(2\lambda+d)}]$$

$$\frac{\Phi_0}{2\pi}\frac{d\varphi}{dx} = \frac{d\Phi}{dx}$$



Majorana state bound to Josephson vortex

$$\boldsymbol{H = H_\phi + H_t + H_\psi}$$



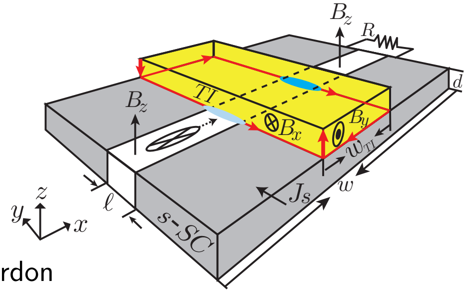
$$\boldsymbol{H_\phi = \hbar\bar{c}\int_x \left\{\beta^2 \frac{h_z^2}{2}(n-\sigma)^2 + \frac{1}{\beta^2}\left[\frac{1}{2}(\partial_x\phi)^2 + \frac{1}{\lambda_J^2}(1-\cos\phi)\right]\right\}}$$

$$\boldsymbol{H_t + H_\psi = \int dx\, \Psi^T(x) \begin{bmatrix} i v_\psi \partial_x & i m \cos(\phi/2) \\ -i m \cos(\phi/2) & -i v_\psi \partial_x \end{bmatrix} \Psi(x)}$$

E. Grosfeld and A. Stern, PNAS **108**, 1181011814 (2011).

Ettingshausen effect in Josephson junction

The time-averaged voltage drop



$$\bar{V} = \nu \Phi_0$$

The boundary condition for sine-Gordon

$$\varphi(\zeta_0) - \varphi(0) = \frac{2\pi}{\Phi_0} B_z (2\lambda_{\mathsf{L}} + \ell) \lambda_J \zeta_0$$

$$\varphi(x,t) = 4 \tan^{-1}\left[\exp\left(\pm \frac{x - vt}{\lambda_J \sqrt{1 - (v/\bar{c})^2}}\right)\right]$$

Conclusions

- **The intrinsic entropy carried by vortices due to Majorana modes leads to an Ettingshausen effect**
- **By utilizing Josephson vortices in lieu of Abrikosov ones, one can obtain experimentally measurable effect**
- **When $T \ll E_{mg}$, this thermoelectric effect provides a probe of the non-Abelian nature of Majorana fermions**
- **The effect can be potentially useful for cooling small objects, such as a QD**