arXiv:1204.1238

Majorana Bound States of Light in a One-Dimensional Array of Nonlinear Cavities

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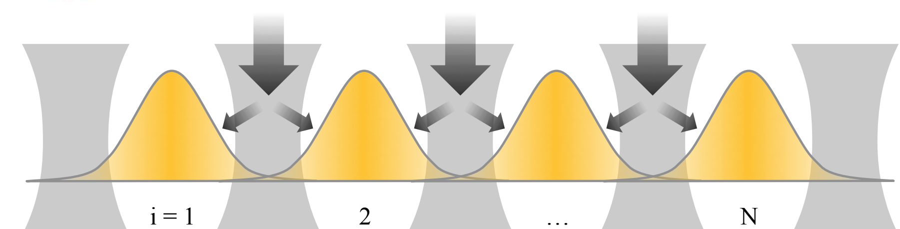
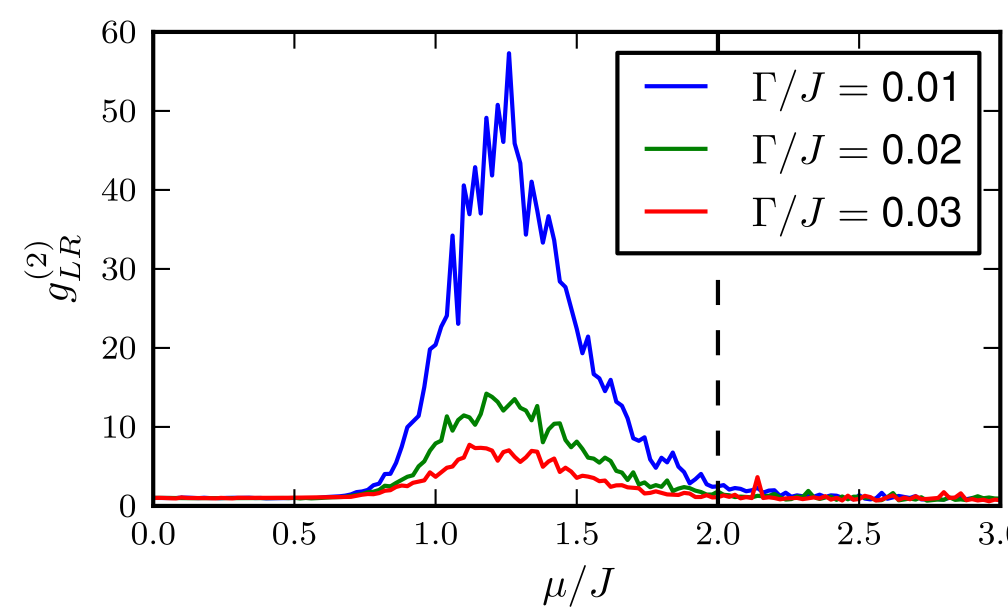
The search for Majorana fermions in p-wave paired fermionic systems has recently moved to the forefront of condensed-matter research. Here we propose an alternative route and show theoretically that Majorana modes can be realized and probed in a driven-dissipative system of strongly correlated photons consisting of a chain of tunnel-coupled cavities, where p-wave pairing effectively arises from the interplay between strong on-site interactions and two-photon parametric driving. Cross-correlation measurements carried out at the ends of the chain would reveal a strong photon bunching signature, demonstrating the nonlocal nature of these photonic Majorana modes.



- realization of Kitaev chain in driven-dissipative nonlinear cavity array
- Majorana end-modes in Kitaev chain
- detection of Majorana modes by photon correlation measurement



PROGRESS ARTICLES | INSIGHT

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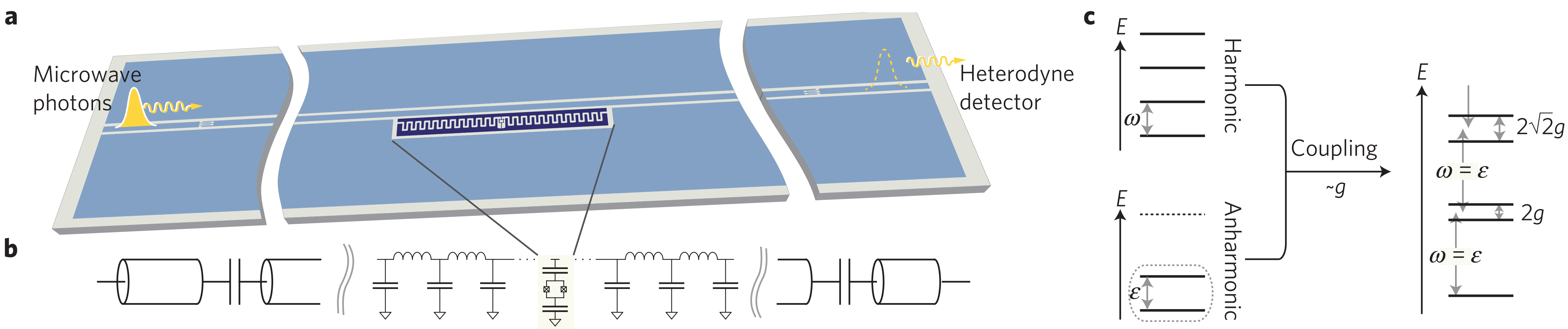
nature physics

On-chip quantum simulation with superconducting circuits

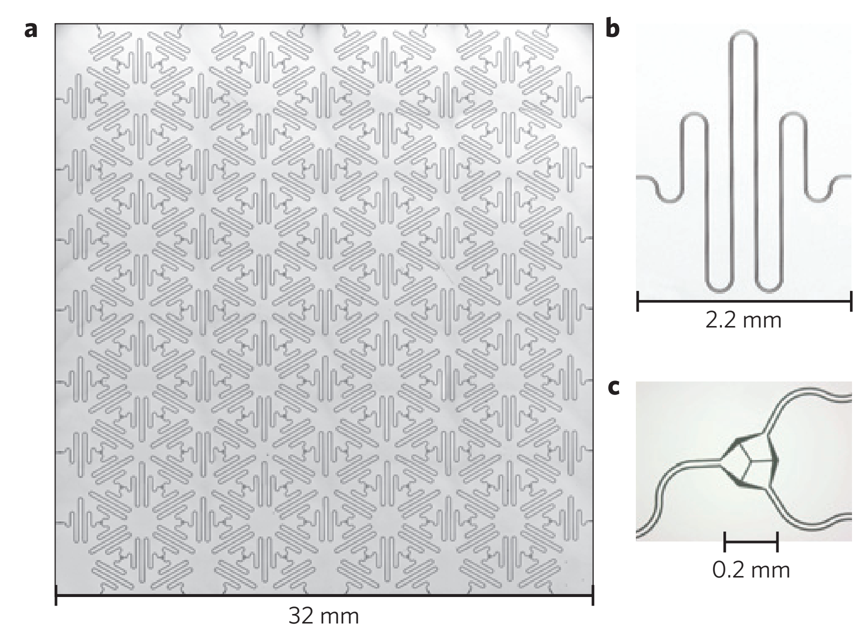
Andrew A. Houck[1]*, Hakan E. Türeci[1] and Jens Koch[2]



Jaynes-Cummings-(Hubbard) model

$$H = \sum_{i=L,R} H_i^{\mathrm{JC}} - J\left(a_L^\dagger a_R + a_R^\dagger a_L\right)$$

$$H^{\mathrm{JC}} = \omega_r a^\dagger a + \varepsilon \sigma^+ \sigma^- + g\left(a\sigma^+ + a^\dagger \sigma^-\right)$$



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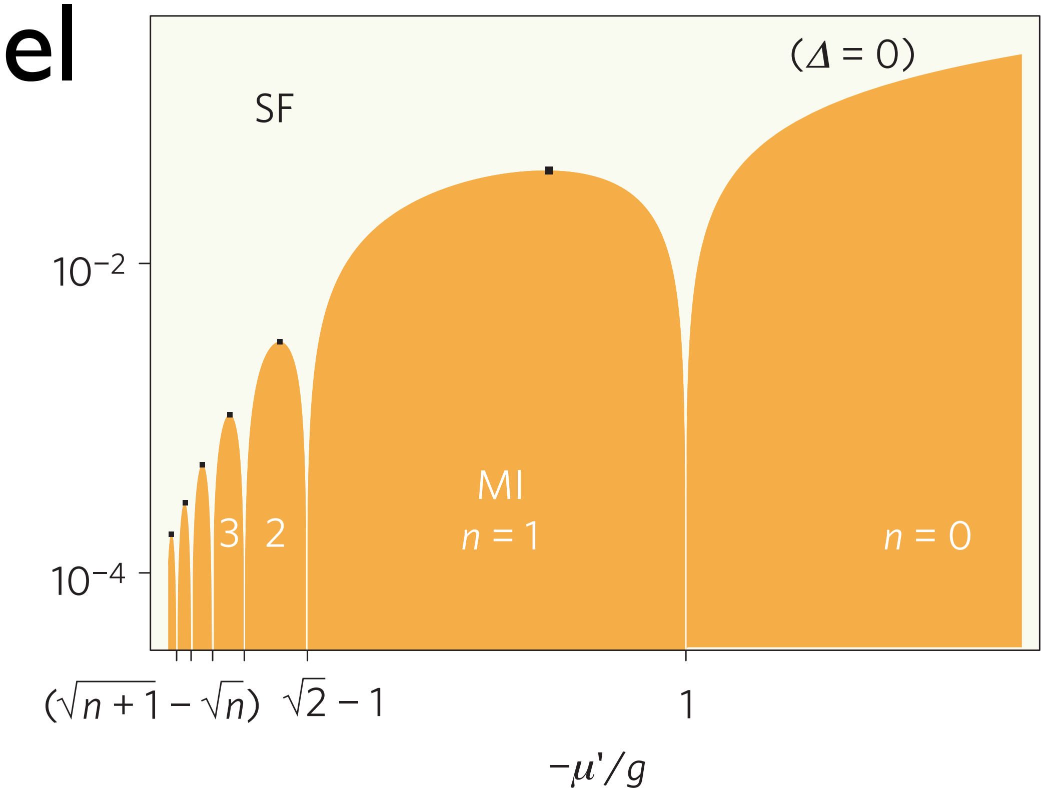


has large similarities with

→ Hartmann et al., Nature Physics **12**, 849 (2006)
→ Greentree et al., Nature Physics **12**, 856 (2006)
→ Angelakis el al., PRA **76**, 031805 (2007)
→ Koch and LeHur, PRA **80**, 023811 (2009)

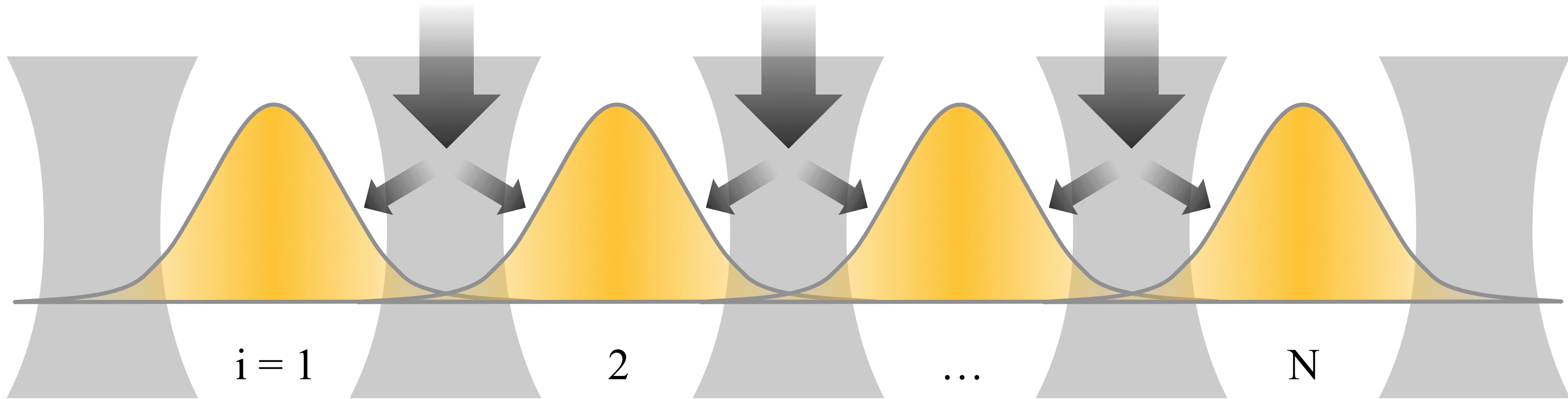
Bose-Hubbard model

$$H_0 = \omega_c \sum_{i=1}^{N} b_i^\dagger b_i + \frac{U}{2}\sum_{i=1}^{N} b_i^\dagger b_i^\dagger b_i b_i - J\sum_{i=1}^{N-1}\left(b_i^\dagger b_{i+1} + h.c.\right)$$

In this work we focus on hard-core bosons $U \gg J$

$$H_0 = \omega_c \sum_{i=1}^{N} b_i^\dagger b_i - J\sum_{i=1}^{N-1}\left(b_i^\dagger b_{i+1} + h.c.\right) \qquad b_j^2 = \left(b_j^\dagger\right)^2 = 0$$

Crucial ingredient: p-wave pairing from parametric pump



$$H_{\mathrm{drive},i} = \int \mathrm{d}^3 r\, \chi^{(2)}(\mathbf{r}) E^{(+)}_{p,i}(\mathbf{r},t) E^{(-)}_{s,i}(\mathbf{r}) E^{(-)}_{s,i}(\mathbf{r}) + h.c.$$

nonlinearity pump signal signal

Treating the pump field classically and in the hard-core limit

$$H_{\mathrm{drive}} = \Omega \sum_{i=1}^{N} (b_i^\dagger e^{-i\omega_p t} + h.c.)$$

coherent drive

$$H_{\mathrm{drive}} = \Delta \sum_{i=1}^{N-1} (b_i^\dagger b_{i+1}^\dagger e^{-2\mathrm{i}\omega_p t} + h.c.)$$

with $\Delta = -E^0_{p,i}\omega_c \int \mathrm{d}^3 r\, \chi^{(2)}(\mathbf{r}) \varphi_{p,i}(\mathbf{r}) \phi_i^*(\mathbf{r}) \phi_{i+1}^*(\mathbf{r})$

“reservoir engineering” see Diehl & Zoller

The full Hamiltonian $H = H_0 + H_{\mathrm{drive}}$

in the hard-core limit $b_i \to \sigma_i^- /2$

defining the detuning $\mu = \omega_c - \omega_p$

and moving to the rotating frame $H_1 = \omega_p \sum_i b_i^\dagger b_i$

$$H = -J_x \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x - J_y \sum_{i=1}^{N-1} \sigma_i^y \sigma_{i+1}^y + \frac{\mu}{2} \sum_{i=1}^{N} (\sigma_i^z + 1)$$

where $J_x = \frac{1}{2}(J - \Delta)$ and $J_y = \frac{1}{2}(J + \Delta)$

$|\Delta| \neq 0$ and $|\mu| < |2J|$ topological non-trivial phase

Majorana fermions are localized at the ends of the chain.

A. Yu. Kitaev, Phys. Usp. 44, 131 (2001)

For $|\Delta| = -J > 0$ and $\mu = 0$

$$H = -J \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x = \mathrm{i}J \sum_{i=1}^{N-1} c_{2i} c_{2i+1}$$

$$H = 2J \sum_{i=1}^{N-1} (\tilde{a}_i^\dagger \tilde{a}_i - \tfrac{1}{2})$$

with $\tilde{a}_i = \frac{1}{2}(c_{2i} + \mathrm{i}c_{2i+1})$

Majorana operators $\begin{cases} c_{2i-1} = (\prod_{j=1}^{i-1} \sigma_j^z)\sigma_i^x \\ c_{2i} = (\prod_{j=1}^{i-1} \sigma_j^z)\sigma_i^y \end{cases}$

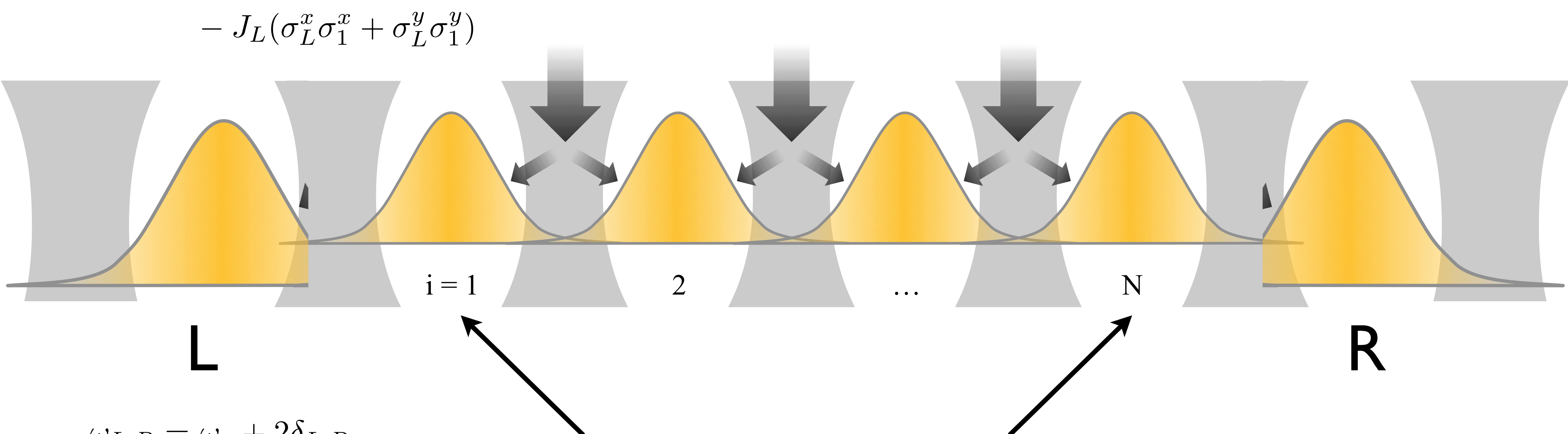
Majorana qubit

$$\boxed{\sigma_M^z = -\mathrm{i}c_1 c_{2N} = (\prod_{j=1}^{N} \sigma_j^z)\sigma_1^x \sigma_N^x}$$

It is not topologically protected as fermionized photons can lead out of the system.

A. Yu. Kitaev, Phys. Usp. 44, 131 (2001)

Majorana-mediated "photonic Cooper pair" splitting



$\omega_{L,R} = \omega_p + 2\delta_{L,R}$

localized Majorana modes

In finite system and for finite $\mu < 2J$ there is a splitting

$$H_{\text{eff}} = \delta_M\sigma_M^z + \delta_L\sigma_L^z + \delta_R\sigma_R^z - J_L\sigma_L^x\sigma_M^x - J_R\sigma_M^x\sigma_R^x$$

The (non-local) Majorana qubit mediates a nonlocal coherent exchange of photons between probe cavities.

Photon-photon correlation measurements - analytics

For $\delta_{L,R} = 0, J_{L,R} = \sqrt{2}\tilde{J}, \Gamma_{L,R} = 8\tilde{\Gamma}$

and without decoherence for the Majorana qubit

$$g_{LR}^{(2)} \equiv \frac{\langle \tilde{b}_L^\dagger \tilde{b}_R^\dagger \tilde{b}_R \tilde{b}_L \rangle}{\langle \tilde{b}_L^\dagger \tilde{b}_L \rangle \langle \tilde{b}_R^\dagger \tilde{b}_R \rangle} = 1 + \frac{\langle \sigma_L^z \sigma_R^z \rangle - \langle \sigma_L^z \rangle \langle \sigma_R^z \rangle}{(1 + \langle \sigma_L^z \rangle)(1 + \langle \sigma_R^z \rangle)} = 1 + \frac{\tilde{\Gamma}^2 \delta_M^2}{(\tilde{J}^2 + \tilde{\Gamma}^2)^2}$$

"third quantization" approach
J. Eisert and T. Prosen, arXiv:1012.5013 (2010)

Light emitted by the spatially separated probe cavities is strongly bunched in the topologically non-trivial regime.

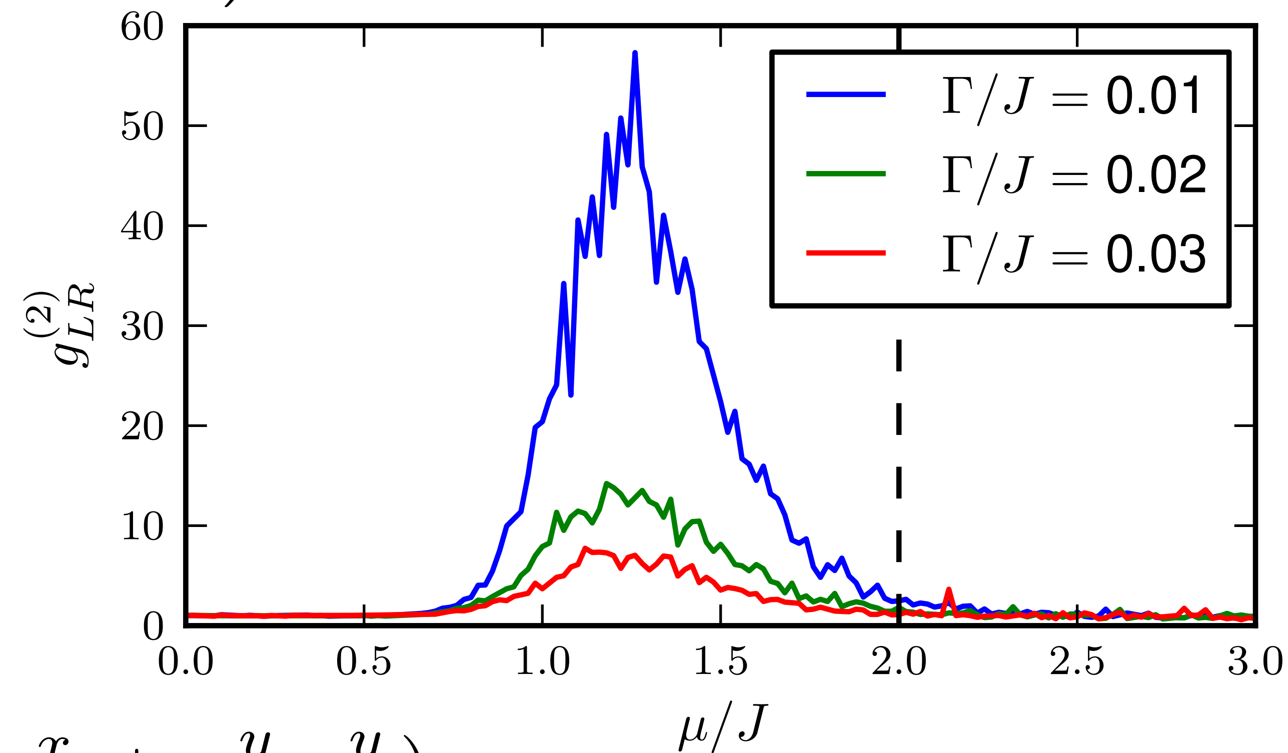
Photon-photon correlation measurements - numerics

$$\partial_t \rho = -\mathrm{i}\,[H_0 + H_{\mathrm{drive}}, \rho] + \Gamma \sum_{i=1}^{N} \left(b_i \rho\, b_i^\dagger - \tfrac{1}{2}\{b_i^\dagger b_i, \rho\} \right)$$

$$H_0 = \omega_c \sum_{i=1}^{N} b_i^\dagger b_i - J \sum_{i=1}^{N-1} (b_i^\dagger b_{i+1} + h.c.)$$

$$H_{\mathrm{drive}} = \Delta \sum_{i=1}^{N-1} (b_i^\dagger b_{i+1}^\dagger e^{-2\mathrm{i}\omega_p t} + h.c.)$$

$$H_{\mathrm{probe}} = \delta_L \sigma_L^z + \delta_R \sigma_R^z - J_L(\sigma_L^x \sigma_1^x + \sigma_L^y \sigma_1^y) - J_R(\sigma_N^x \sigma_R^x + \sigma_N^y \sigma_R^y)$$



$$\mu = \omega_c - \omega_p$$

Light emitted by the spatially separated probe cavities is strongly bunched in the topologically non-trivial regime.

$$N = 10, \Delta/J = 1, \delta_{L,R}/J = 0, J_{L,R}/J = 0.02, \Gamma = \Gamma_{L,R}$$

Conclusions

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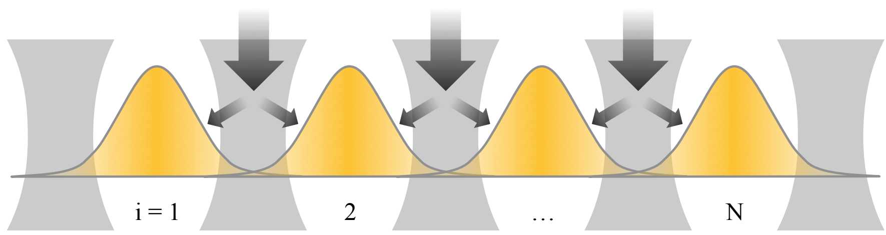

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