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Cooling by Heating: Very Hot Thermal Light Can Significantly Cool Quantum Systems

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MOTIVATION

- Classical & quantum physics:
  System in contact with a colder bath.
- Quantum physics:
  Cooling mechanical degrees of freedom using the radiation pressure of light.

➔ Interacting body should first be cold or coherent.

ALTERNATE COOLING METHOD

- Presented paradigm: Thermal hot states can be used to significantly cool down a quantum system.
- It is demonstrated that due to the driving with thermal noise, the interaction of other modes can be effectively enhanced, giving rise to a “transistorlike” effect.

It turns out that thermal noise, when appropriately used, can assist cooling.

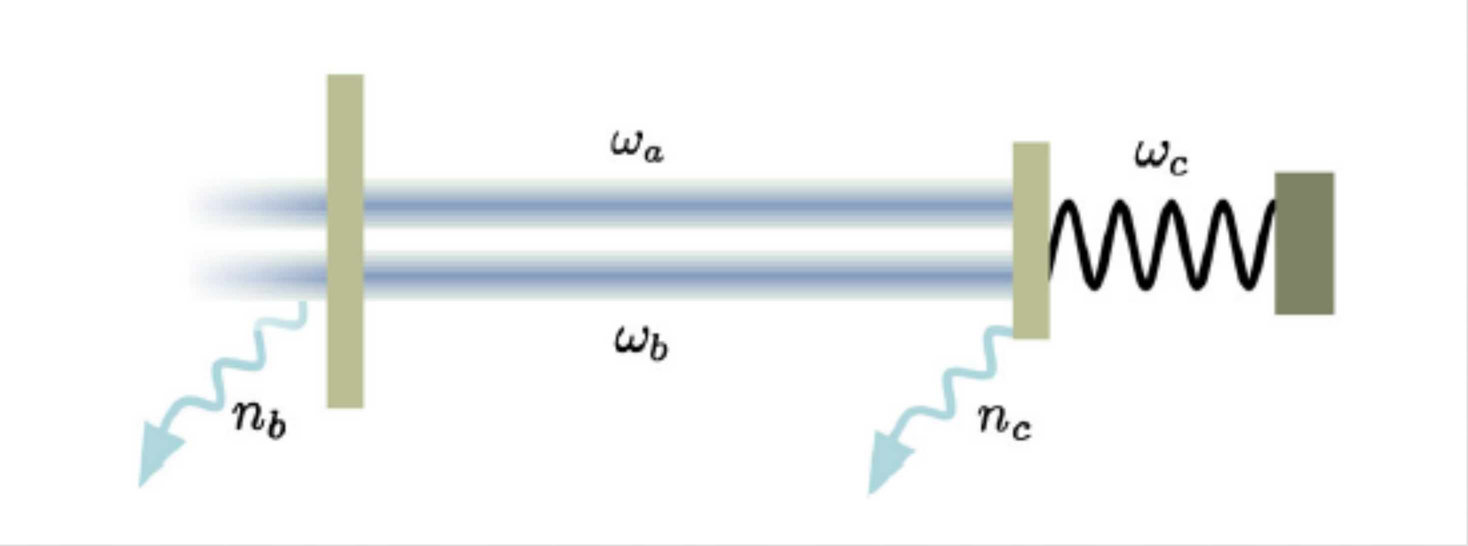
THE SYSTEM UNDER CONSIDERATION

- Hamiltonian:

$$H = H_0 + H_1$$

$$H_0 = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\omega_c c^\dagger c$$

$$H_1 = \hbar g(a+b)^\dagger (a+b)(c+c^\dagger)$$



TRANSFORMATION

- Rotating interaction picture with respect to:

$$\hbar\omega_b(a^\dagger a + b^\dagger b)$$

- Radiation pressure is invariant.
- System Hamiltonian simplifies to

$$H_0' = \hbar\Delta a^\dagger a + \hbar\omega_c c^\dagger c$$

  where $\Delta = \omega_a - \omega_b$.
- The frequencies are chosen such that $\Delta = \omega_c$, which is the optimal resonance for cooling.

QUANTUM MASTER EQUATION

$$\dot{\rho} = \mathcal{L}\rho = -\frac{i}{\hbar}[H, \rho] + (\mathcal{L}_a + \mathcal{L}_b + \mathcal{L}_c)\rho$$

with the generators being defined by

$$\mathcal{L}_a = \kappa D_a$$
$$\mathcal{L}_b = (1 + n_b)\kappa D_b + n_b \kappa D_{b^\dagger}$$
$$\mathcal{L}_c = (1 + n_c)\gamma D_c + n_c \kappa D_{c^\dagger}$$

and $D_x(\rho) = 2x\rho x^\dagger - \{x^\dagger x, \rho\}$

WEAK COUPLING APPROXIMATION

Mode *b* is considered as an external "bath".

The Liouvillian can be decomposed as

$$\mathcal{L} = \mathcal{L}_{\mathrm{sys}} + \mathcal{L}_{\mathrm{int}} + \mathcal{L}_{\mathrm{bath}}$$

where

$$\mathcal{L}_{\mathrm{sys}} = -\frac{i}{\hbar}[H_0', \cdot] + \mathcal{L}_a + \mathcal{L}_c$$

$$\mathcal{L}_{\mathrm{int}} = -\frac{i}{\hbar}[H_1, \cdot]$$

$$\mathcal{L}_{\mathrm{bath}} = \mathcal{L}_b$$

MASTER EQUATION FOR THE REDUCED SYSTEM $\rho_{a,c} = \mathrm{tr}_b[\rho]$

$$\dot{\rho}_{a,c}(t) = \mathcal{L}_{\mathrm{sys}}\rho_{a,c}(t) + \mathrm{tr}_b \mathcal{L}_{\mathrm{int}} \int_0^\infty ds\ e^{\mathcal{L}_{\mathrm{r}} s} \mathcal{L}_{\mathrm{int}} \rho_{a,c}(t-s) \otimes \rho_b$$

with $\mathcal{L}_{\mathrm{r}} = \mathcal{L}_{\mathrm{sys}} + \mathcal{L}_{\mathrm{bath}}$ and $\mathcal{L}_{\mathrm{b}}\rho_b = 0$.

$$\dot{\rho}_{a,c}(t) = \mathcal{L}_{\mathrm{sys}}\rho_{a,c}(t) - \frac{1}{\hbar^2}\mathrm{tr}_b \left[H_1, \int_0^\infty ds\ e^{\mathcal{L}_{\mathrm{r}} s} \left[H_1, \rho_{a,c}(t-s) \otimes \rho_b \right] \right]$$

SEQUENTIAL APPROXIMATION OF THE INTERACTION HAMILTONIAN AND THE DAMPING MECHANISM (A)

The master equation up to second order in the coupling *g*.

$$\dot{\rho}_{a,c}(t) = \mathcal{L}_{\mathrm{sys}}\rho_{a,c}(t) - \frac{1}{\hbar^2}\mathrm{tr}_b\left[H_1, \int_0^\infty ds \left[e^{\mathcal{L}_{\mathrm{r}}^\dagger s}(H_1), \rho_{a,c}(t-s)\otimes\rho_b\right]\right]$$

Corresponding to a “dissipative interaction picture” with respect to $\mathcal{L}_{\mathrm{r}}$.

SEQUENTIAL APPROXIMATION OF THE INTERACTION HAMILTONIAN AND THE DAMPING MECHANISM (B)

Start from $H_1 = \hbar g(a+b)^\dagger(a+b)(c+c^\dagger)$ and neglect the term proportional to $a^\dagger a$ because we assume mode a to be weakly perturbed from its ground state.

Bath b is allowed to have an arbitrary temperature.

➔ $$H_1' = \hbar g(a^\dagger b + b^\dagger a + \delta)(c + c^\dagger)$$

With $\delta = b^\dagger b - n_b$.

SEQUENTIAL APPROXIMATION OF THE INTERACTION HAMILTONIAN AND THE DAMPING MECHANISM (C)

- As $\omega_a - \omega_b = \omega_c$, the rotating wave approximation is

$$H_1'' = \hbar g(a^\dagger bc + ab^\dagger c^\dagger) + \hbar g\delta(c + c^\dagger)$$

- For the partial trace two-time correlation functions of mode b are needed,

$$\langle b e^{\mathcal{L}_{\mathrm{r}}^\dagger s} b^\dagger \rangle = e^{-\kappa s} n_b$$

$$\langle \delta e^{\mathcal{L}_{\mathrm{r}}^\dagger s} \delta \rangle = e^{-2\kappa s}(n_b^2 + n_b).$$

SEQUENTIAL APPROXIMATION OF THE INTERACTION HAMILTONIAN AND THE DAMPING MECHANISM (D)

Within the time scale given by the exponentials, one can neglect the effect of the mechanical reservoir.

$$e^{\mathcal{L}_{\mathrm{r}}^{\dagger} s} a = e^{-(\kappa + i\Delta)s} a = e^{-(\kappa + i\omega_c)s} a$$

$$e^{\mathcal{L}_{\mathrm{r}}^{\dagger} s} c = e^{-(\gamma + i\omega_c)s} c \simeq e^{-i\omega_c s} c$$

And the integration can be performed.

SEQUENTIAL APPROXIMATION OF THE INTERACTION HAMILTONIAN AND THE DAMPING MECHANISM (E)

- In $\mathcal{L}_{\mathrm{heat}}$ only counterrotating terms are kept.

$$\mathcal{L}_{\mathrm{cool}} = \frac{g^2}{2\kappa}\big((1+n_b)D_{ac^\dagger} + n_b D_{a^\dagger c}\big)$$

$$\mathcal{L}_{\mathrm{heat}} = \frac{2\kappa g^2(n_b^2+n_b)}{4\kappa^2+\omega_c^2}(D_{c^\dagger}+D_c)$$

- The effect of $\mathcal{L}_{\mathrm{heat}}$ is a renormalization of the mean occupation number of the mechanical bath.

$$n_c \mapsto \tilde{n}_c = n_c + \frac{2\kappa g^2(n_b^2+n_b)}{\gamma(4\kappa^2+\omega_c^2)}$$

SEQUENTIAL APPROXIMATION OF THE INTERACTION HAMILTONIAN AND THE DAMPING MECHANISM (E)

- With the renormalized Liouvillian, the master equation can be written

$$\dot{\rho}_{a,c} = \left(\tilde{\mathcal{L}}_{\text{sys}} + \mathcal{L}_{\text{cool}}\right)\rho_{a,c}$$

- The equations for the number operators:

$$\dot{\hat{n}}_a = -2\kappa\hat{n}_a - \frac{g^2}{\kappa}\left((n_b+1)\hat{n}_a - n_b\hat{n}_c - \hat{n}_a\hat{n}_c\right)$$

$$\dot{\hat{n}}_c = -2\gamma\hat{n}_c - \frac{g^2}{\kappa}\left(n_b\hat{n}_c - (n_b+1)\hat{n}_a - \hat{n}_a\hat{n}_c\right) + 2\gamma\tilde{n}_c$$

SEQUENTIAL APPROXIMATION OF THE INTERACTION HAMILTONIAN AND THE DAMPING MECHANISM (F)

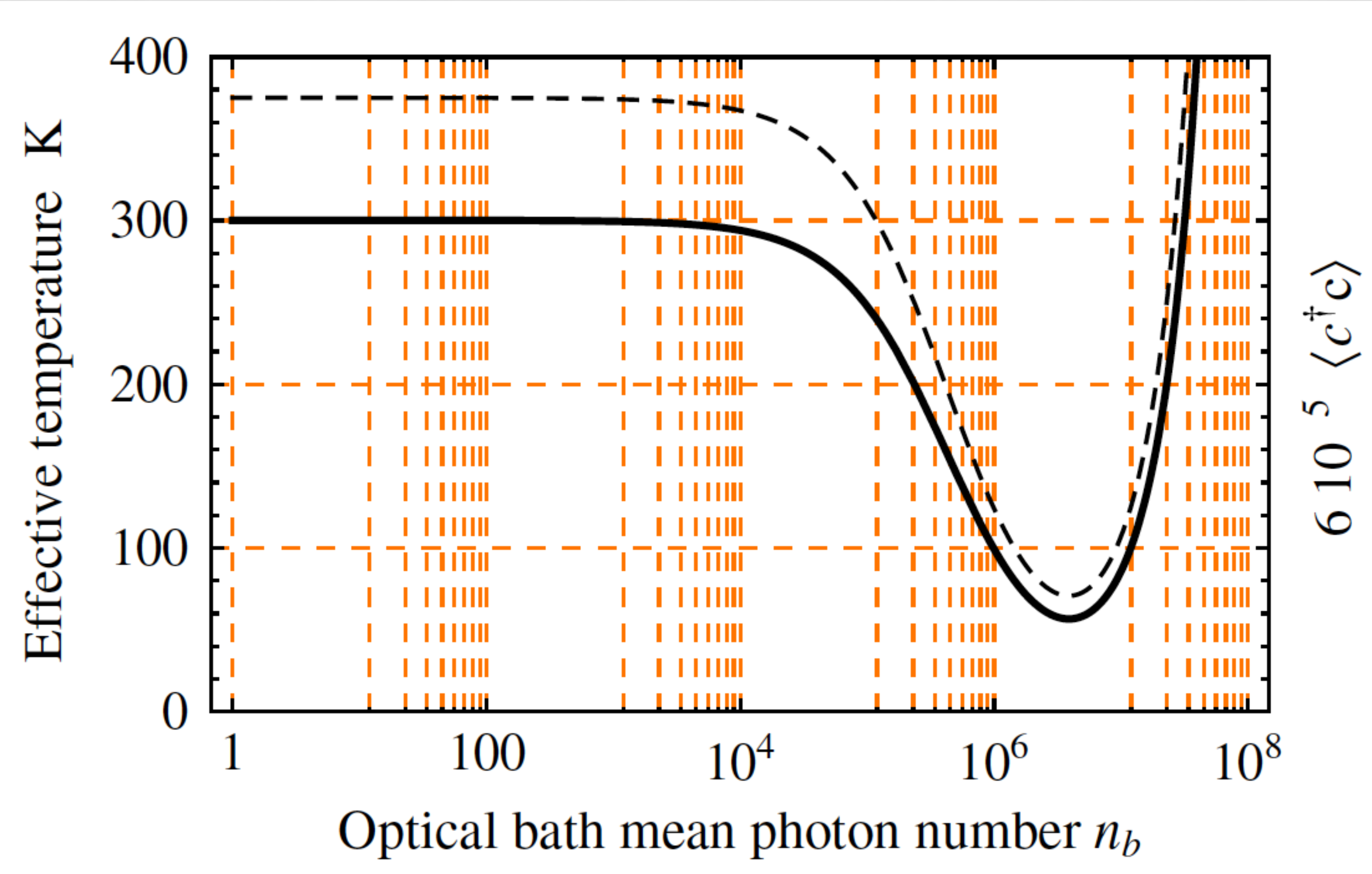
Assuming $\langle \hat{n}_a \hat{n}_c \rangle \simeq \langle \hat{n}_a \rangle \langle \hat{n}_c \rangle$, analytical expressions for the steady state expectation values are found

$$\langle \hat{n}_c \rangle = \frac{\tilde{n}_c - \eta}{2} + \left(\frac{(\tilde{n}_c + \eta)^2}{4} - \frac{\kappa n_b \tilde{n}_c}{\gamma} \right)^{1/2}$$

$$\langle \hat{n}_a \rangle = \frac{(\tilde{n}_c - \langle \hat{n}_c \rangle)\gamma}{\kappa}$$

with $\eta = 1 + n_b(1 + \kappa/\gamma) + 2\kappa^2/g^2$.

RESULT



400
300
200
100
0
Effective temperature K
1
100
10^4
10^6
10^8
Optical bath mean photon number n_b
$6\ 10^{5}\ \langle c^{\dagger}c\rangle$

CONCLUSION

- The notion of cooling by heating is established.
- Cooling processes can be assisted by means of incoherent hot thermal light.
- It would be interesting to fully flesh out the potential for the effect to assist in generating nonclassical states “the bit” or entanglement.