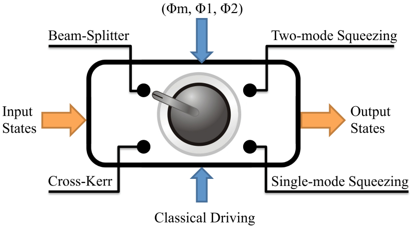
Journal Club, May 8, 2012

arXiv:1204.5450v1 [quant-ph]

Parametric four-wave mixing toolbox for superconducting resonators

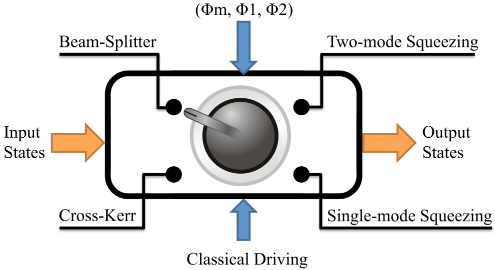
A. V. Sharypov, Xiuhao Deng, and Lin Tian

University of Califonia, Merced



Proposal

Superconducting circuit that acts as a toolbox to generate a basic set of quantum operations on two microwave photon modes.



Ingredients:

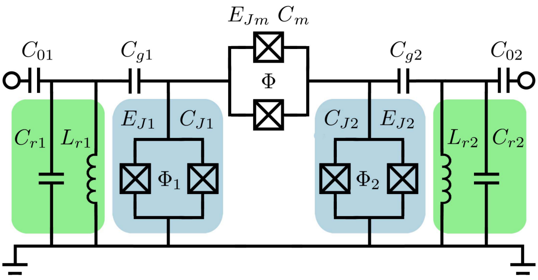
- two coupled superconducting qubits → highly tunable 4 -level system
- two superconducting resonators $(\hat{a}_1, \hat{a}_2)$
- qubit-resonator couplings + separate qubit drivings
- designed **dispersive four-wave mixing** process

→ Effective quantum operations on the modes $\hat{a}_1$ and $\hat{a}_2$.

Circuit

$$\boxed{H_{tot} = H_q + H_r + H_p}$$

- four-level system: $$H_q = \sum_j E_j |j\rangle\langle j|, \qquad (j = a, b, c, d),$$
- resonators and couplings: $$H_r = \sum_i \hbar\omega_{a_i} \hat{a}_i^\dagger \hat{a}_i + \hbar g_i \sigma_{xi} (\hat{a}_i^\dagger + \hat{a}_i)$$
- classical drivings: $$H_p = \sum \hbar\Omega_i(t)\sigma_{xi}, \quad \Omega_i(t) \sim \cos\omega_i t$$



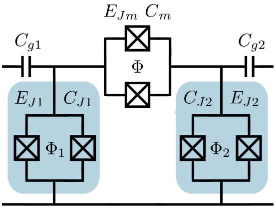
Four-level system

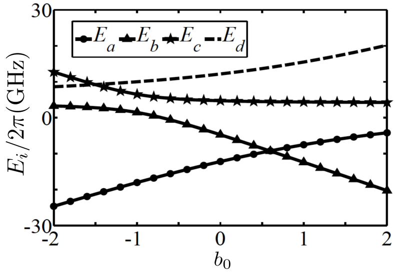
e.g. 2 charge qubits coupled via Josephson junction

$$H_q = \frac{E_{J1}}{2}\sigma_{z1} + \frac{E_{J1}}{2}\sigma_{z2} + H_{int},$$

$$H_{int} = E_{mx}\left(\sigma_{x1}\sigma_{x2} + b_0\sigma_{y1}\sigma_{y2} + b_0\sigma_{z1}\sigma_{z2}\right),$$

$$b_0 = \frac{E_{Jm}}{4E_{mx}}, \qquad E_{mx} \simeq \frac{C_m e^2}{C_{\Sigma 1}C_{\Sigma 2} - C_m^2}.$$





$E_{J1} = 8.9$ GHz, $E_{J2} = 13.9$ GHz, $E_{mx} = 4$ GHz

$$\sigma_{x1} = \cos(\theta_+ - \theta_-)(\sigma_{ab} + \sigma_{dc}) + \sin(\theta_+ - \theta_-)(\sigma_{db} - \sigma_{ac}) + h.c.$$

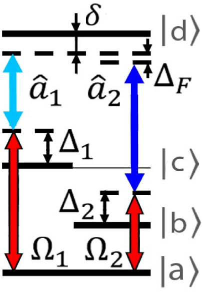
$$\sigma_{x2} = -\sin(\theta_+ + \theta_-)(\sigma_{ab} - \sigma_{dc}) + \cos(\theta_+ + \theta_-)(\sigma_{ac} + \sigma_{db}) + h.c.$$

where $\sigma_{ij} = |i\rangle\langle j|$.

Beam-splitter operation: $H_{bm} = \hbar\chi_{bm} e^{i\phi} \hat{a}_1^\dagger \hat{a}_2 + h.c.$

e.g. to swap the states of $\hat{a}_1$ and $\hat{a}_2$, Hadamard gate for photon qubits.

$$\begin{pmatrix} \hat{a}_1(t) \\ \hat{a}_2(t) \end{pmatrix} = \begin{pmatrix} \cos\varphi & -e^{i\phi}\sin\varphi \\ -e^{-i\phi}\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \hat{a}_1(0) \\ \hat{a}_2(0) \end{pmatrix}, \qquad \varphi = \chi_{bm} t.$$



Selecting the couplings

- $\hat{a}_1 \leftrightarrow \sigma_{dc}$,
- $\hat{a}_2 \leftrightarrow \sigma_{db}$,
- $\Omega_1 \leftrightarrow \sigma_{ca}$,
- $\Omega_2 \leftrightarrow \sigma_{ba}$,

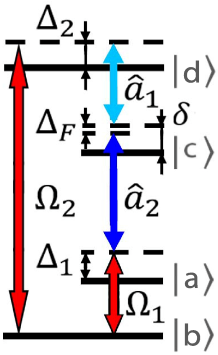
it leads to correction of the GS energy: $\sigma_{aa} H_{bm}$,

$$H_{bm}/\hbar = \sum_i \delta\epsilon_i^{bm} \hat{a}_i^\dagger \hat{a}_i + \left(\chi_{bm} \hat{a}_1^\dagger \hat{a}_2 e^{i\Delta_F t} + h.c.\right)$$

$$\chi_{bm} = \frac{\Omega_1 \Omega_2 \tilde{g}_1 \tilde{g}_2}{\Delta_1 \Delta_2 \delta}, \qquad \delta\epsilon_i^{bm} = \frac{\Omega_i^2 \tilde{g}_i^2}{\Delta_i^2 \delta}.$$

Two-mode squeezing: $H_{sq} = i\hbar\chi_{sq}\hat{a}_1^\dagger\hat{a}_2^\dagger + h.c.$

$$\begin{pmatrix}\hat{a}_1(t)\\ \hat{a}_2^\dagger(t)\end{pmatrix} = \begin{pmatrix}\cosh\varphi & \sinh\varphi\\ \sinh\varphi & \cosh\varphi\end{pmatrix}\begin{pmatrix}\hat{a}_1(0)\\ \hat{a}_2^\dagger(0)\end{pmatrix}, \qquad \varphi = \chi_{sq}t.$$



Selected couplings:

- $\hat{a}_1 \leftrightarrow \sigma_{dc}$,
- $\hat{a}_2 \leftrightarrow \sigma_{ac}$,
- $\Omega_1 \leftrightarrow \sigma_{ab}$,
- $\Omega_2 \leftrightarrow \sigma_{db}$.

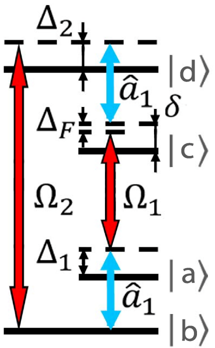
Effective Hamiltonian: $\sigma_{aa}H_{sq}$,

$$H_{sq}/\hbar = \sum_i \delta\epsilon_i^{sq}\hat{a}_i^\dagger\hat{a}_i + \left(\chi_{sq}\hat{a}_1^\dagger\hat{a}_2^\dagger e^{i\Delta_F t} + h.c.\right)$$

$$\chi_{sq} = \frac{\Omega_1\Omega_2\tilde{g}_1\tilde{g}_2}{\Delta_1\Delta_2\delta}, \qquad \delta\epsilon_i^{sq} = \frac{\Omega_i^2\tilde{g}_i^2}{\Delta_i^2\delta}.$$

Single-mode squeezing: $H_{sq1} = i\hbar\chi_{sq1}(\hat{a}_i^\dagger)^2 + h.c.$

$$\hat{a}_i(t) = \hat{a}_i(0)\cosh\varphi + \hat{a}_i^\dagger(0)\sinh\varphi, \qquad \varphi = 2\chi_{sq1}t.$$



Selected couplings:

- $\hat{a}_1 \leftrightarrow \sigma_{ab}$ and σ_{dc}
- $\hat{a}_2$ effectively decoupled
- $\Omega_1 \leftrightarrow \sigma_{ca}$,
- $\Omega_2 \leftrightarrow \sigma_{db}$.

$$H_{sq1}/\hbar = \delta\epsilon_1^{sq}\hat{a}_1^\dagger\hat{a}_1 + \left(\chi_{sq1}(\hat{a}_1^\dagger)^2 e^{-i\Delta_F t} + h.c.\right)$$

$$\chi_{sq1} = \frac{\Omega_1\Omega_2\tilde{g}_1^2}{\Delta_1\Delta_2\delta}, \quad \delta\epsilon_1^{sq1} = \frac{\tilde{g}_1^2}{\delta}\left(\frac{\delta}{\Delta_1} + \frac{\Omega_1^2}{\Delta_1^2} + \frac{\Omega_2^2}{\Delta_2^2}\right).$$

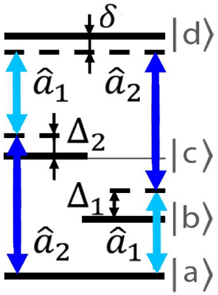
Arbitrary linear transformations: H_{bm}, H_{sq}, H_{sq1} with $H_{ph} = \hbar\Delta_{ph}\hat{a}_i^\dagger\hat{a}_i$:

$$\hat{a}_i \quad \longrightarrow \quad \sum_j A_{ij}\hat{a}_j + B_{ij}\hat{a}_j^\dagger + C_i.$$

Cross-Kerr nonlinearity: $H_{ck} = \hbar\chi_{ck}\hat{a}_1^\dagger\hat{a}_1\hat{a}_2^\dagger\hat{a}_2$

Controlled gate on photon qubits, e.g. CPHASE

$$\left(|0_1\rangle + |1_1\rangle\right)\left(|0_2\rangle + |1_2\rangle\right) \quad \longrightarrow \quad |0_1 0_2\rangle + |0_1 1_2\rangle + |1_1 0_2\rangle - |1_1 1_2\rangle$$



Selected couplings:

- $\hat{a}_1 \leftrightarrow \sigma_{ba} + \sigma_{dc}$
- $\hat{a}_2 \leftrightarrow \sigma_{ca} + \sigma_{db}$
- no classical drivings: $\Omega_{1,2} = 0$

Corrected GS enenergy: $\sigma_{aa} H_{ck}$,

$$H_{ck}/\hbar = \sum_i \delta\epsilon_i^{ck}\hat{a}_i^\dagger\hat{a}_i + \chi_{ck}\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_2\hat{a}_1$$

$$\chi_{ck} = \frac{\tilde{g}_1^2\tilde{g}_2^2}{\delta}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right)^2, \quad \delta\epsilon_i^{ck} = \frac{\tilde{g}_i^2}{\Delta_i}.$$

Error sources

- unwanted transitions in the resonator-qubit couplings and classical driving:

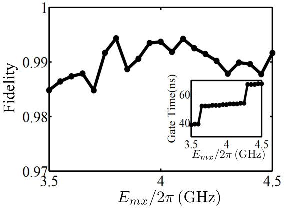
$$(\hat{a}_1, \hat{a}_2, \Omega_1, \Omega_2) \leftrightarrow (\sigma_{ab}, \sigma_{ac}, \sigma_{bd}, \sigma_{cd})$$

- cross-talk between circuit elements:

$$\sigma_{x1}(\hat{a}_2^\dagger + \hat{a}_2) \quad \text{or} \quad (\hat{a}_1^\dagger + \hat{a}_1)(\hat{a}_2^\dagger + \hat{a}_2)$$

- decoherence of the resonators
- toolbox preserved in the ground state: weakly affected by the qubit decoherence

CPHASE gate using H_{tot}



Conclusion

- **Toolbox**: one single circuit can generate various quantum operations, for both discrete-state and continuous-variable protocols.
- It makes use of a dispersive FWM approach to engineer effective couplings and quantum operations between the resonator modes.