All-electric qubit control in heavy hole quantum dots via non-Abelian geometric phases

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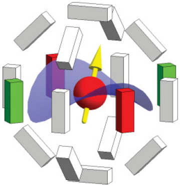
Journal Club

Daniel Becker
22 May 2012

Proposal

- use (pseudo-) spin of heavy holes (HH) in GaAs quantum dots as qubits
- realize arbitrary single-qubit gates with pure electric quadrupole fields
- preserve time-reversal symmetry (TRS)
- implement qubit-gates via adiabatic transformation of Hamiltonian (holonomic quantum computing)



Possible Advantages

heavy holes (HH) in quantum dots as qubits

- predicted long coherence times
- coupling to nuclear spins Isig-like
- advanced level of optical control
- naturally realized in many semiconductors

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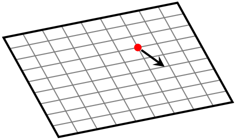
all-electrical control

- electrical fields easier to control experimentally
- preserves time-reversal symmetry
- absence of certain (phonon mediated) decoherence mechanisms (higher temperatures?)

Holonomy, Geometric Phase, and Gauge Potentials

flat manifold $\mathcal{M}$:



holonomy

degree of failure in preserving geometrical properties in parallel transport along closed paths γ on manifold $\mathcal{M}$

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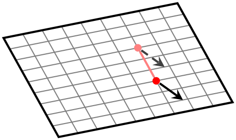

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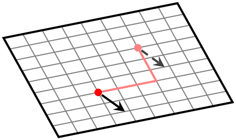

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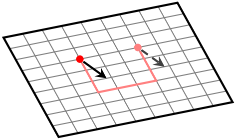

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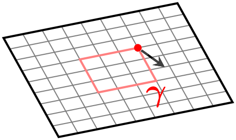

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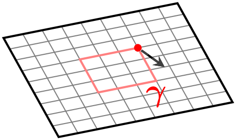

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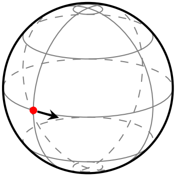

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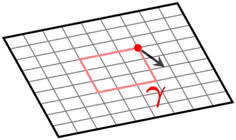

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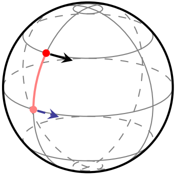

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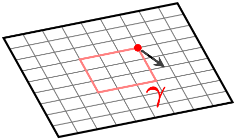

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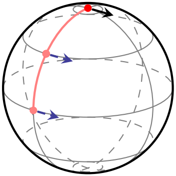

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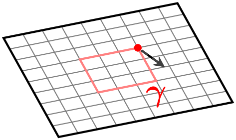

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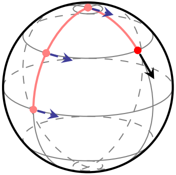

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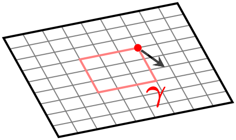

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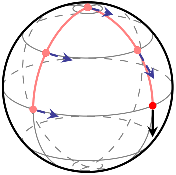

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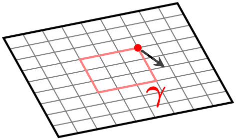

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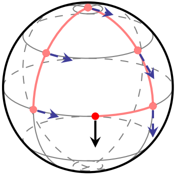

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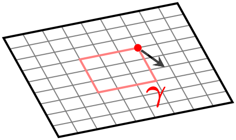

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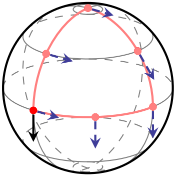

rotation $\hat{U}_\gamma$

close relation between holonomy, curvature, and topology of manifold

holonomy

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- holonomy operation (geometric phase) determined by gauge potential (connection) A: $\hat{U}_\gamma = e^{\int_\gamma A}$

Holonomic Quantum Computation

P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94-99 (1999)

translation to quantum mechanics

identify $\mathcal{M}$ with Hamiltonian manifold $\mathcal{H} = \{\hat{H}(\gamma) | \gamma \in \mathcal{V}\}$

- $\hat{H}(\gamma)$ has n-fold degenerate subspace with corresponding projector $\hat{P}$
- closed path γ in space $\longrightarrow$ adiabatic change of external parameter set $\gamma(t)$ with $\gamma(2\pi) = \gamma(0)$
- adiabatic connection $A\left(\frac{\mathbf{d}}{\mathbf{d}t}\right) = -\left[\frac{\mathbf{d}\hat{P}(t)}{\mathbf{d}t}, \hat{P}(t)\right]$

main idea

implement universal set of quantum gates with geometrical phases $\hat{U}_\gamma$ for properly chosen closed loops in parameter space

Example for Non-Abelian Gauge Potential

F. Wilczek and A. Zee, PRL **52**, 2111 (1984)

3-fold degenerate $4d$ Hamiltonian

$\hat{H}(t) = \hat{R}(t)\hat{H}_0\hat{R}^{-1}(t)$ with $\hat{H}_0 = \text{diag}(0,0,0,1)$

- $\hat{R} = \hat{R}_3[\phi_3]\hat{R}_2[\phi_2]\hat{R}_1[\phi_1]$ with $\hat{R}_i[\phi_i]$ rotate components $i \leftrightarrow 4$
- gauge potential $A_i = \hat{P}\hat{R}^{-1}(\partial_{\phi_i}\hat{R})\hat{P}$ with $\hat{P} = \text{diag}(1,1,1,0)$

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$$A_1 = 0 \qquad A_2 = \begin{pmatrix} 0 & \sin(\phi_1) & 0 & 0 \\ -\sin(\phi_1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 0 & \sin(\phi_1)\cos(\phi_2) & 0 \\ 0 & 0 & \sin(\phi_2) & 0 \\ -\sin(\phi_1)\cos(\phi_2) & -\sin(\phi_2) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

simple dynamical systems can have (non-abelian) gauge structure

Manifold and Gauge Potential for the HH

quadrupole hamiltonian $\hat{H}_Q = \hat{J}_i \mathcal{Q}^{ij} \hat{J}_j$ for holes in GaAs

- due to TRS: 2 two-fold degenerate subspaces
- quadrupole tensor $\mathcal{Q}$ element of $5d$ matrix space
- angular momentum operators $\hat{\boldsymbol{J}}$ for spin $3/2$

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can be expressed in basis of Hamiltonians Γ_i:

$\hat{H}_Q = \sum_{i=0}^{4} x_i \Gamma_i$ with

$$\Gamma_0 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \Gamma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix}, \text{ and } \Gamma_4 = \begin{pmatrix} 0 & -\boldsymbol{i1} \\ \boldsymbol{i1} & 0 \end{pmatrix}$$

SO(5) Clifford algebra: $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$

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$$A_{i<j} = \hat{P}_0 \left(\tfrac{1}{2}[\Gamma_i, \Gamma_j]\right) \hat{P}_0 \quad \text{with} \quad \hat{P}_0 = \text{diag}(1, 0, 0, 1)$$

Single-Qubit Operations with Quadrupole Fields

general single-qubit gate

$$\mathcal{U}(\hat{n}, \phi) = \exp\left(\boldsymbol{i}\phi\frac{\hat{n}\boldsymbol{\sigma}}{2}\right) = \exp(\boldsymbol{i}\sum_i \sigma_i\phi_i/2)$$

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Pauli matrices in HH subspace:

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for arbitrary $\hat{n}$ and ϕ

choose $\hat{a} = (a_0, a_1, a_2, a_3)$ with $|\hat{a}| = 1$ with $\frac{(a_1,a_2,a_3)}{|(a_1,a_2,a_3)|} = \hat{n}$ and

$$\phi = 2\pi(1 - |(a_1,a_2,a_3)|) = 2\pi\left(1 - \sqrt{1 - a_0^2}\right)$$

Example: $\pi/2$ Rotation of HH-Qubit around y-Axis

wanted: $a_1, \ldots, a_0$ so that $\hat{n} = -\hat{e}_y$ and $\phi = \pi/2$

$$\mathcal{U}(-\hat{e}_y, \pi/2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

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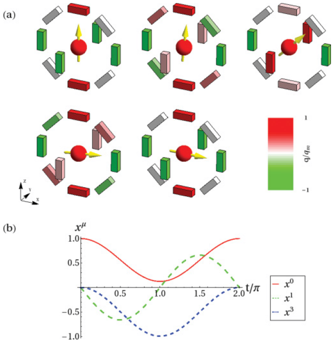

$$\hat{H}(t) = e^{t[(\sqrt{7}/4A_0)-(3/4)A_2]}\Gamma_3 e^{-t[(\sqrt{7}/4A_0)-(3/4)A_2]}$$

HH-LH Splitting due to Confinement

realistic Hamiltonian

$\hat{H} = \hat{H}_Q + \frac{\Delta E_0}{2}\tau_z$ with $\Delta E_0 > \Delta E(\hat{H}_Q)$

HH/HL splitting with qubit-charge distance $r = 50$nm
and quadrupole potential $e\Phi = 50$meV: $\Delta E(\hat{H}_Q) \approx 0.57$meV

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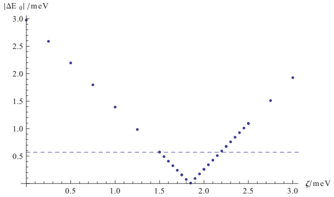
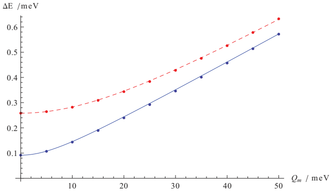
possible solution: induce linear mechanical strain in z-direction (GaAs)



confinement splitting versus strain



total HH/LH splitting versus quadrupole potential

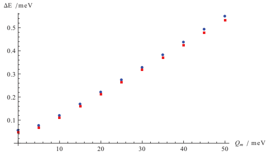
Other Deviations

method robust against:

- residual dipole fields
- deviations from quadrupole potential wih $l = 2$
- deviations from quadratic confinement

TABLE I. Characteristic terms of the axial multipole expansion.

$l = 0$		Overall shift in energy that does not change ΔE
$l = 1$	$r \cos\phi$	Shift of the center of the bound state assuming that quadrupole and confining potentials $(\Phi_1 + \Phi_4)$ are quadratic in r; ΔE unchanged
$l = 2$	$r^2, r^2 \cos 2\phi$	Included in the model as $\Phi_1 + \Phi_4$
$l = 3$	$r^3 P_3 = r^3(\frac{3}{8}\cos\phi + \frac{5}{8}\cos 3\phi)$	Lowest order that appears in dipole expansion and can induce quadratic Stark effect
$l = 4$	$r^4 \cos 4\phi$	Deviation from quadrupole symmetry by four equally charged gates
	$r^4 \cos 2\phi$	Allowed by quadrupole symmetry leading to the same effective Hamiltonian $H(\mathcal{Q})$ with $J = \frac{3}{2}$ but with the induced value ΔE only a few percent in comparison with the $l = 2$ term; does not influence holonomy operations
	r^4	Correction to the confinement potential, which removes stability against the $l = 1$ perturbation
$l = 6$	$r^6 \cos 6\phi$	Lowest-order perturbation that appears in quadrupole expansion



effect of r^4 corrections to confinement

Possible Issues

- extra long coherence times in HH systems often rely on tuning with magnetic fields (break TRS)
- phonon-mediated spin-decoherence mainly important for higher temperatures
- implementation of quadrupole gates around every qubit necessary
- how good can strain suppress the HH/LH splitting in reality?