PRL **107**, 150504 (2011)

Energy Landscape of 3D Spin Hamiltonians with Topological Order

Sergey Bravyi (IBM) and Jeongwan Haah (Caltech)

Why can we store a classical bit, but not a qubit?

bit	$\longleftrightarrow$	**qubit**
protect against logical Z	$\longleftrightarrow$	protect against logical X and Z

unstable:

1D Ising model:	$\longleftrightarrow$	2D (3D) toric code
$O(L^0)$ energy gap		

stable: (below T_c)

2D Ising model	$\longleftrightarrow$	4D toric code
$O(L^1)$ energy gap		

Can we find something similar in $<$4D?

Stabilizer Hamiltonians

Lattice $\mathbb{Z}_L^D$ with $O(1)$ qubits per site

$H = -\sum_a G_a$
G_a: mutually commuting local products of single-qubit Pauli operators

$G_a \psi = +\psi$: ψ is *stabilized* by G_a
$G_a \psi = -\psi$: defect
stabilized space $\mathcal{K} \subseteq \left(\mathbb{C}^2\right)^{\otimes L^D}$: groundstate space of H (if frustration-free)
$\mathcal{G} = \langle G_a \rangle$: stabilizer group: abelian, $-I \notin \mathcal{G}$

logical operators: elements of $C(\mathcal{G}) \setminus \mathcal{G}$: perform non-trivial operations on the groundstate space

No-go results in 2D

Bravyi & Terhal 2008, Kay & Colbeck 2008, Haah & Preskill 2010:
Every 2D stabilizer Hamiltonian with local interactions has an $O(1)$ energy gap between different groundstates.

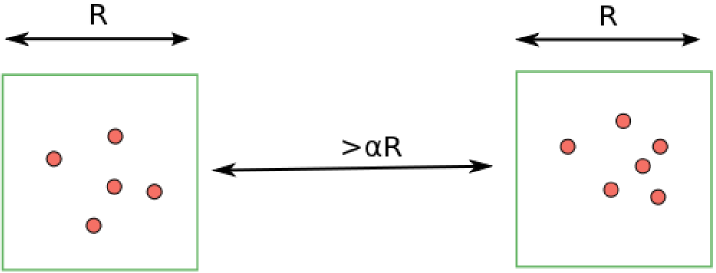
Problem: string-like logical operators:
Defects can move without energy costs ($\sim$ domain walls in 1D Ising model).

Haah's breakthrough

What about 3D?

Recent breakthrough (Haah, PRA **83**, 042330 (2011)):
Existence of 3D stabilizer Hamiltonians with local interactions without string-like logical operators

Defects cannot move further than a certain constant distance away without creating other defects (**no-strings rule**).



Result: Assumptions

Assumptions on the Hamiltonian $-\sum_a G_a$:

- *Frustration-free*: for a groundstate ψ_0 we have $G_a\psi_0 = \psi_0$, $\forall a$ (guaranteed if G_a's indep. and $-I \notin \mathcal{G}$)
- *Topological quantum order*:
  A Pauli operator with local support that creates no defects is a stabilizer.
  $\Leftrightarrow$ Logical operators have $O(L)$ distance.
- *No-strings rule*
  (violated by every 2D local stabilizer Hamiltonian, satisfied by e.g. Haah's 3D Hamiltonian)

Result: Precitions

- Any sequence of local errors creating an isolated defect from the vacuum with no other defects within distance R must cross an energy barrier at least $c \cdot \log R$ for some constant $c > 0$.
- The energy barrier for any logical operator is at least $c \cdot \log L$.
- This bound is tight up to a constant factor (c.f. Haah's 3D Hamiltonian)
- Proof: Based on renormalization group technique, studies syndromes which are 'dense' or 'sparse' on different length-scales, uses induction over different length-scale levels (no-strings rule is scale-invariant!).

The scaling is not as favorable as the one of the 4D toric code, but the first known in $< 4D$ which is not $O(L^0)$.

Lifetime of the stored quantum information

- In Haah's 3D Hamiltonian, the gap between different groundstates grows as $\Delta = c \cdot \log L$
- Naive expectation (Haah & Bravyi, PRL **107**, 150504 (2011)): $\tau \sim e^{\beta\Delta} = L^{c\beta}$
- Best known lower bound: $\tau = e^{c'\beta^2}$ for some finite L (Haah & Bravyi, arXiv:1112.3252 (2011))
- No further improvement expected (J. Haah, private communication, 2012)