To close or not to close: the fate of the superconducting gap across the topological quantum phase transition in Majorana-carrying semiconductor nanowires

Tudor D. Stanescu , Sumanta Tewari , Jay D. Sau , and S. Das Sarma
arXiv:1206.0013

Class D spectral peak in Majorana quantum wires

Dmitry Bagrets and Alexander Altland
arXiv:1206.0434

Enhanced zero-bias Majorana peak in disordered multi-subband quantum wires

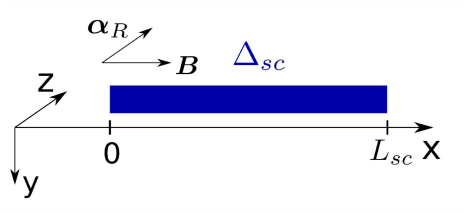
Falko Pientka, Graham Kells, Alessandro Romito, Piet W. Brouwer, and Felix von Oppen
arXiv:1206.0723

Zero-bias peaks in spin-orbit coupled superconducting wires with and without Majorana states

Jie Liu, Andrew C. Potter, K.T. Law, and Patrick A. Lee
arXiv:1206.1276

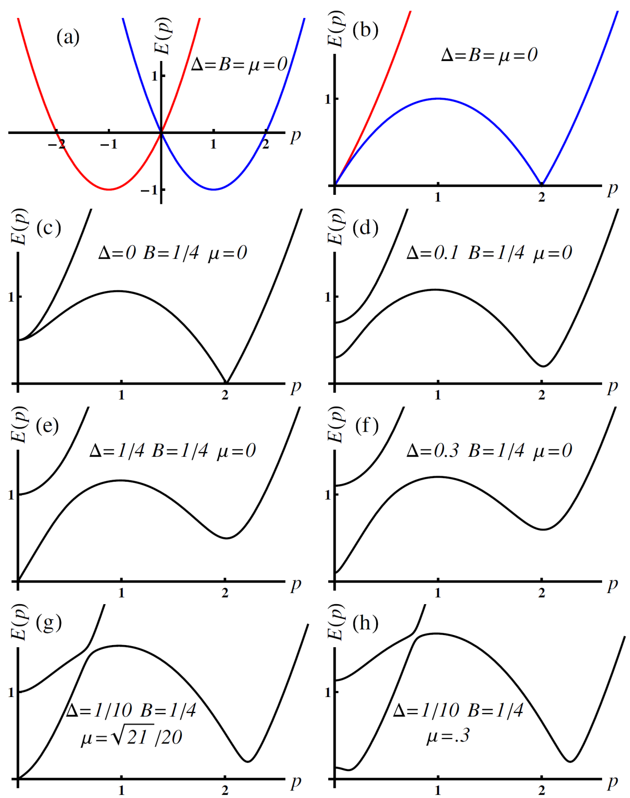
Journal Club *12.06.12* *Jelena Klinovaja*

Majorana Fermions in Semiconducting Nanowires



$$H = \int \Psi^\dagger(y)\mathcal{H}\Psi(y)dy; \qquad \Psi^\dagger = (\psi_\uparrow^\dagger, \psi_\downarrow^\dagger, \psi_\downarrow, -\psi_\uparrow)$$

$$\mathcal{H} = [p^2/2m - \mu(y)]\tau_z + up\sigma_z\tau_z + B(y)\sigma_x + \Delta(y)\tau_x.$$



$$E_0 = E(p=0) = |B - \sqrt{\Delta^2 + \mu^2}|$$

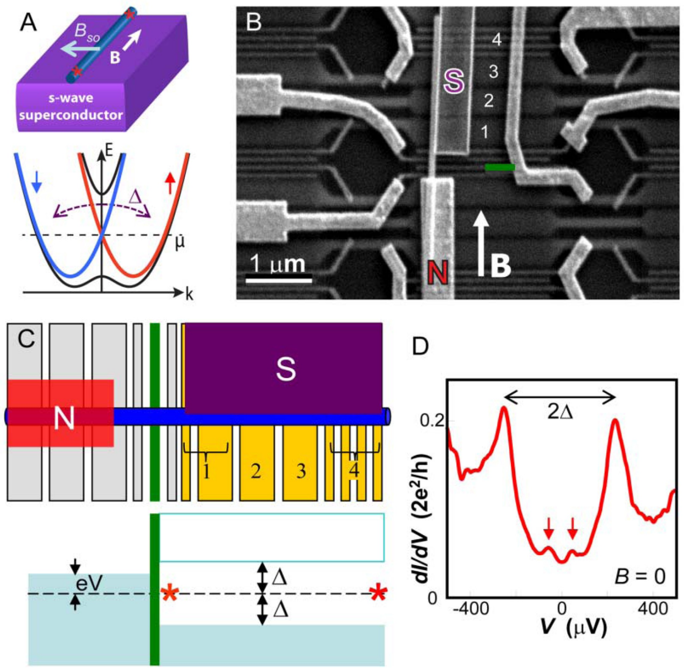
topological gap

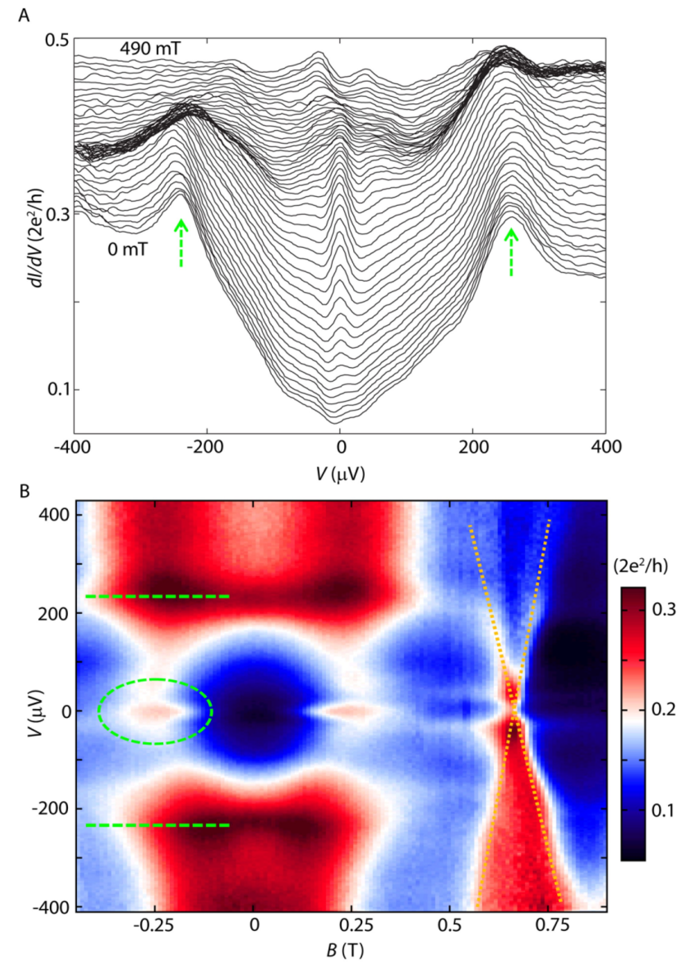
$$B^2 > \Delta^2 + \mu^2$$

topological criterion

R. M. Lutchyn, J. D. Sau, S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010)
Y. Oreg, G. Refael, F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).

Delft experiment





$l_{so} \approx 200$ nm

$\alpha \approx 0.2$ eV•Å

$\Delta \approx 250$ μeV

no closing of the topological gap!!!

V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, L. P. Kouwenhoven, Science **336**, 1003 (2012).

LDOS *vs.* DOS

S. Das Sarma

A rectangular semiconducting nanowire: $L_x \gg L_y \sim L_z$ $(u_\uparrow, u_\downarrow, v_\uparrow, v_\downarrow)$

chemical potential

Zeeman term $\Gamma = g^* \mu_B B/2$

$$H_{nm}(k) = [\epsilon_{nm}(k) - \mu\delta_{nm}]\tau_z + \Gamma\delta_{nm}\sigma_x\tau_z + \alpha k\delta_{nm}\sigma_y\tau_z - i\alpha_y q_{nm}\sigma_x + \Delta_{nm}\sigma_y\tau_y,$$

spin orbit interaction

$\alpha = 0.2\,\mathrm{eV\AA}$

superconductivity

σ_i spin

τ_i particle-hole

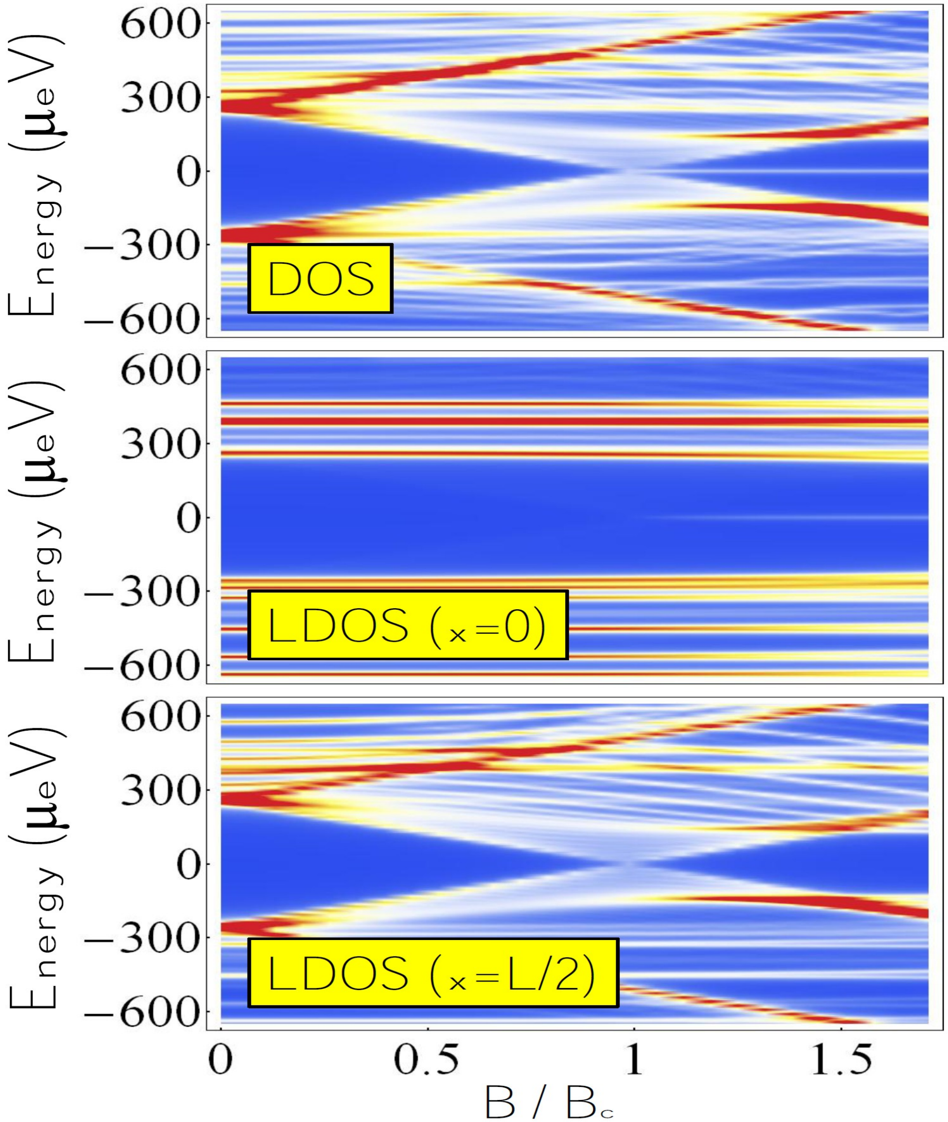
modes:

$n = (n_y, n_z)$ $m = (m_y, m_z)$

$\sin(n_y \pi y/L_y)\sin(n_z \pi z/L_z)$

LDOS *vs.* DOS

S. Das Sarma



$$B = B_c \approx 0.2\,\mathrm{T}$$

$$\mu = \Delta/2$$

$$\Delta \sim 250\ \mu\mathrm{eV}$$

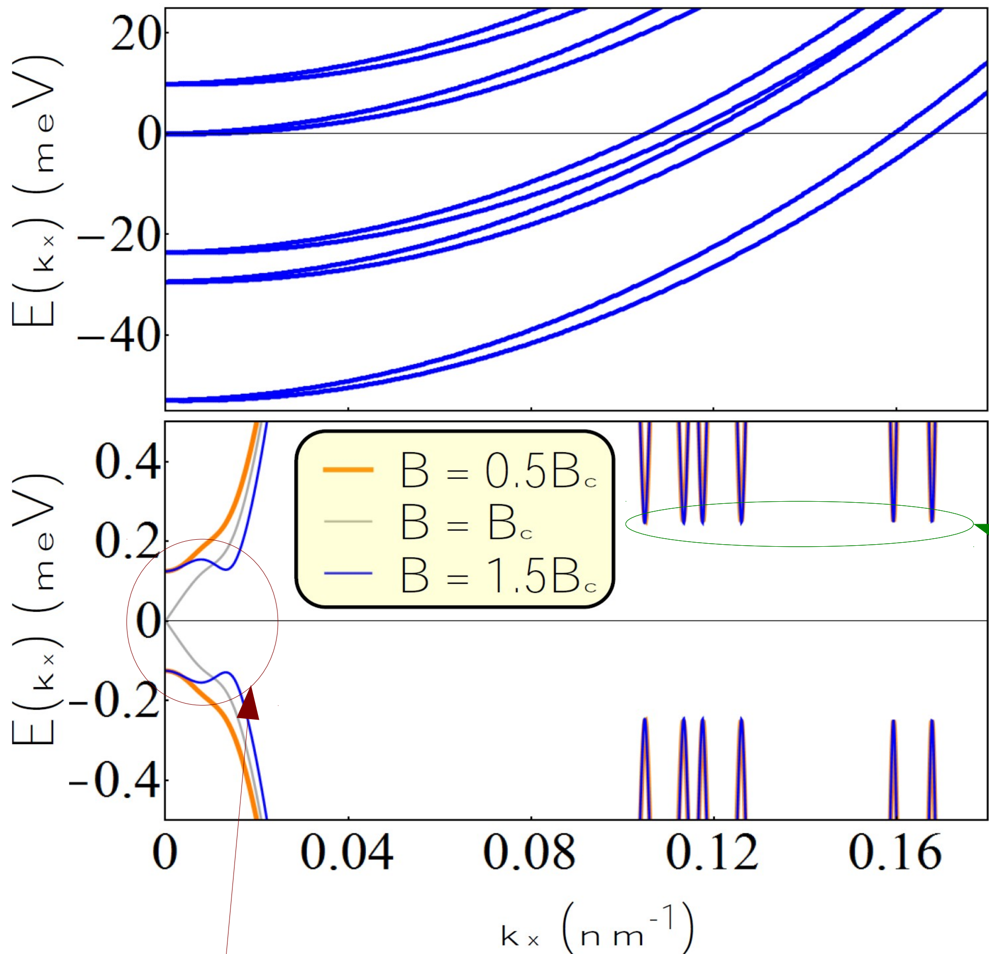
Assumption: limit of point contact tunneling

"The main difference between our calculated LDOS and the measured differential tunneling charge conductance current is some matrix element effects which are neither precisely known nor of any qualitative significance."

LDOS *vs.* DOS

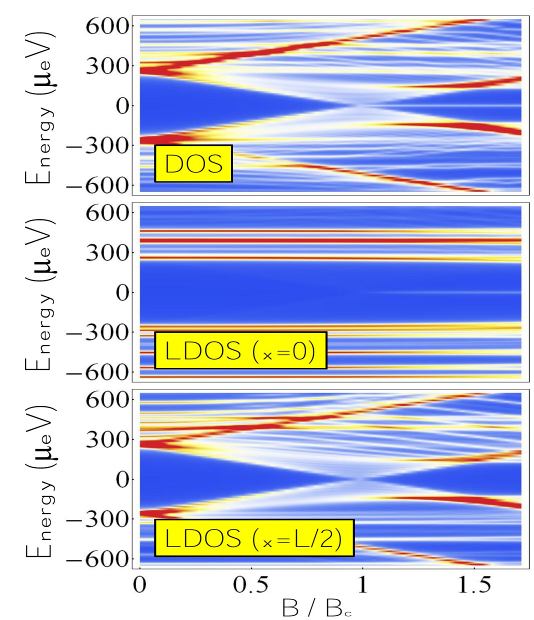
S. Das Sarma

four occupied bands $\mu = \Delta/2$



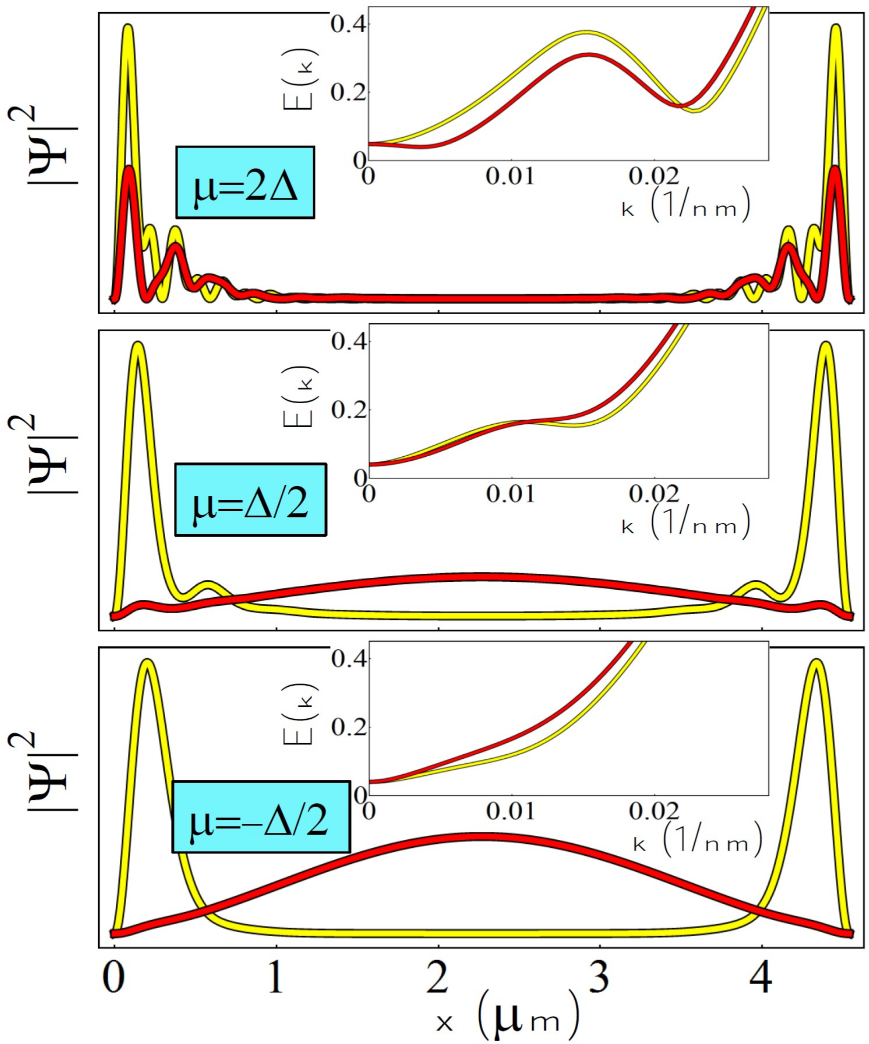
low-energy bands
weakly depend on B

top band - “Majorana band”
strongly depend on B
topological gap vanishes and reopens



LDOS *vs.* DOS

S. Das Sarma



$B = 0.9B_c$ — non-topological

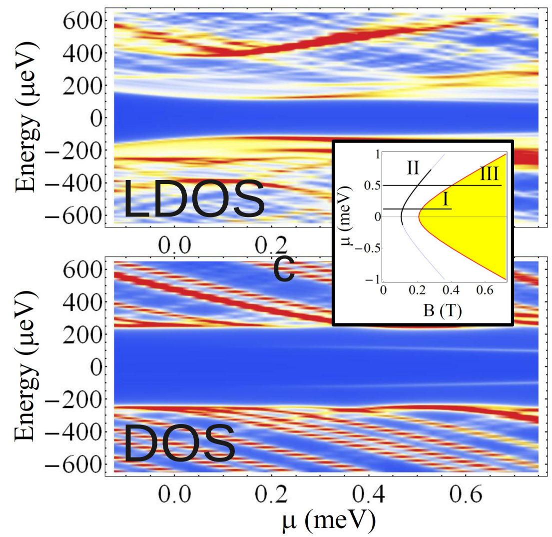
$B = 1.1B_c$ — topological

"In the non-topological SC phase, we find that the lowest energy states are delocalized when the chemical potential is below a certain value"

$$\mu_c(\Gamma) \sim \mathcal{O}(\Delta)$$

LDOS *vs.* DOS

S. Das Sarma

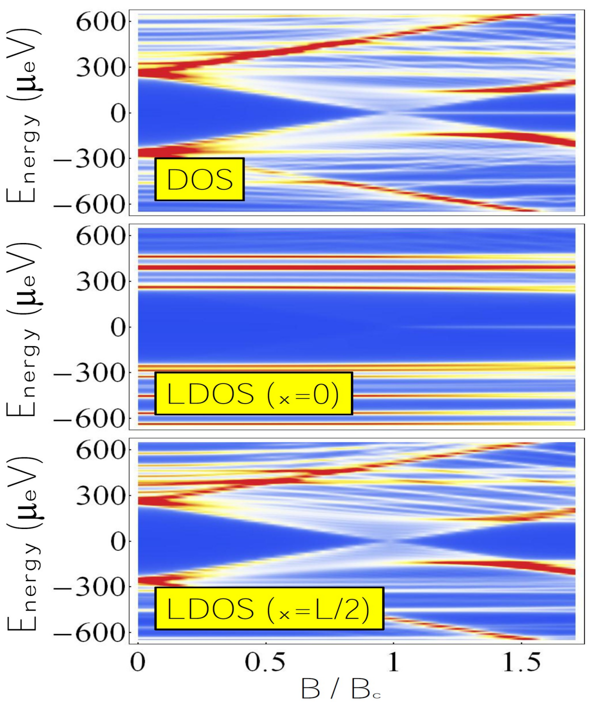


delocalized

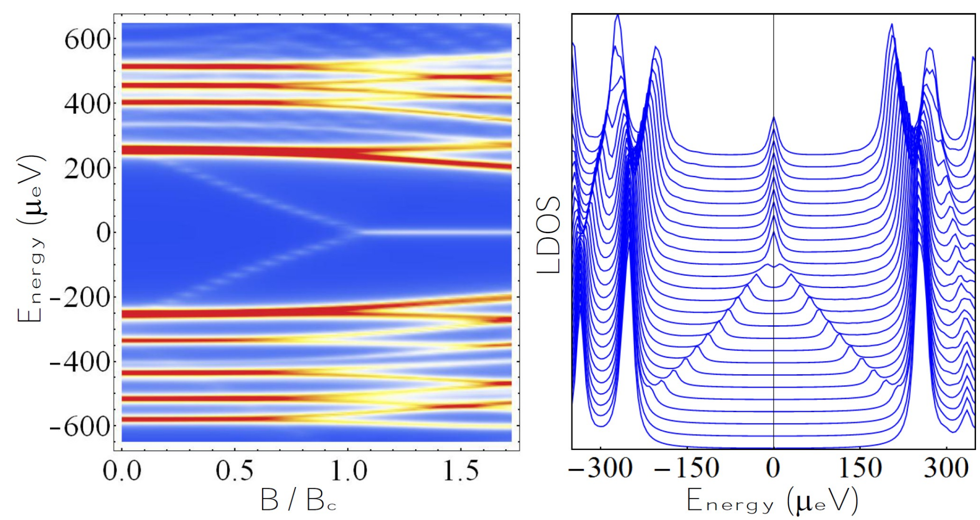
$\mu < \mu_c$

localized

$\mu > \mu_c$



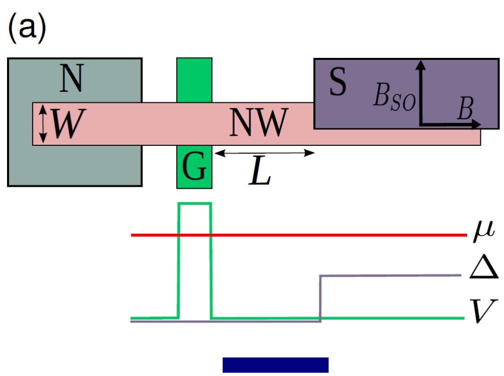
no closing of the topological gap is observed

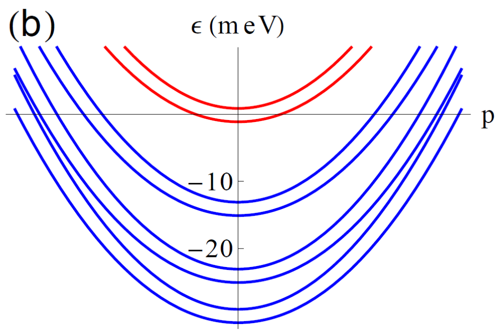


closing of the topological gap is observed

Enhanced zero-bias Majorana peak - disorder

Piet Brouwer
Felix von Oppen





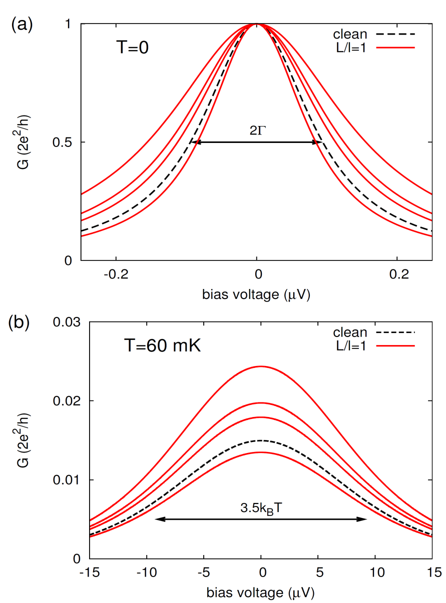
$$\mathcal{H} = \left(\frac{\mathbf{p}^2}{2m} + \alpha p_x \sigma_y - \alpha_y p_y \sigma_x + U(x) + V_{\rm dis}(\mathbf{r}) - \mu \right) \tau_z - B\sigma_x + \Delta(x)\tau_x,$$

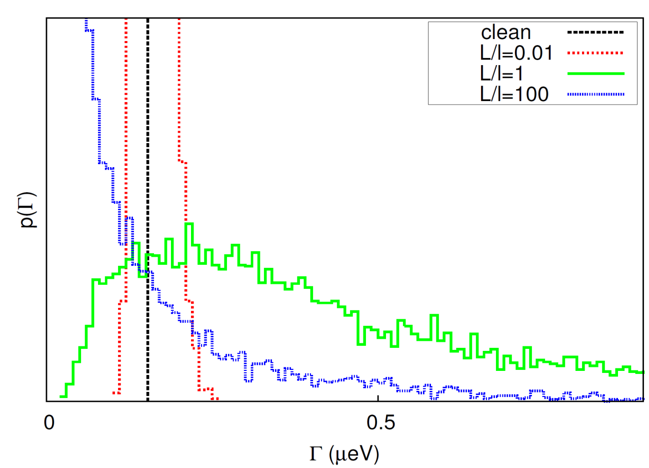
$$\langle V_{\rm dis}(\mathbf{r}) V_{\rm dis}(\mathbf{r}') \rangle = \frac{v_F^2}{l_{2\rm d}} \delta(\mathbf{r} - \mathbf{r}')$$

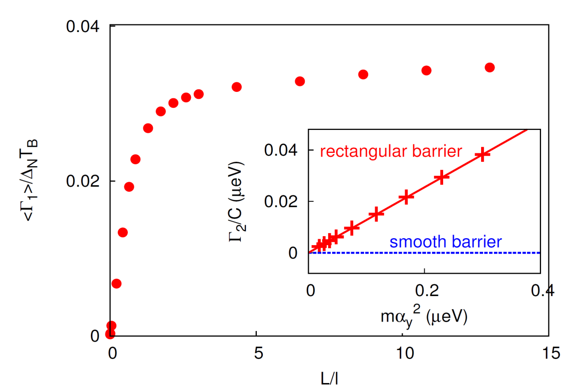
disorder between the tunnel barrier and superconductor

$$G(V) = \frac{e^2}{h} \mathrm{tr}\left[1 + r_{\rm he}(eV) r_{\rm he}(eV)^\dagger - r_{\rm ee}(eV) r_{\rm ee}(eV)^\dagger\right]$$

Zero-bias conductance peak

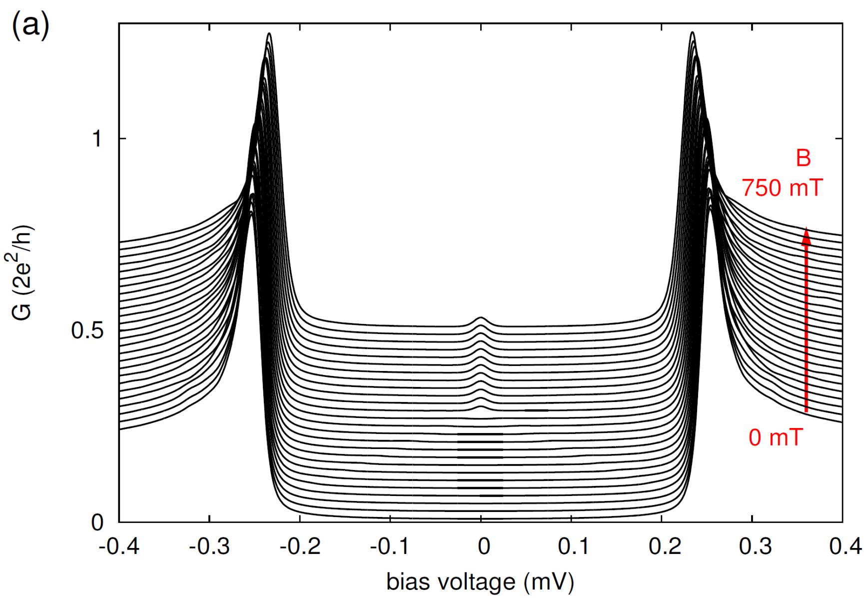




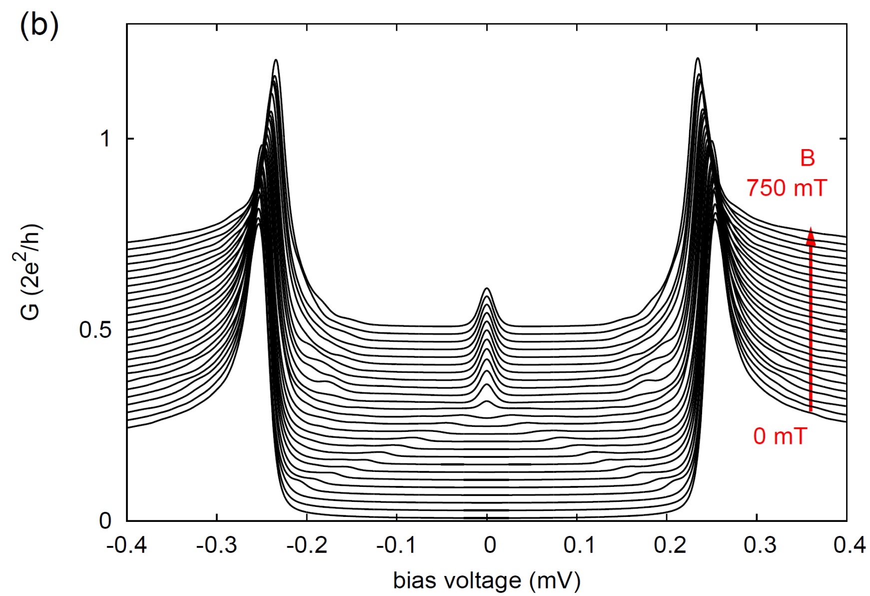


Enhanced zero-bias Majorana peak - disorder

Piet Brouwer
Felix von Oppen



clean wire



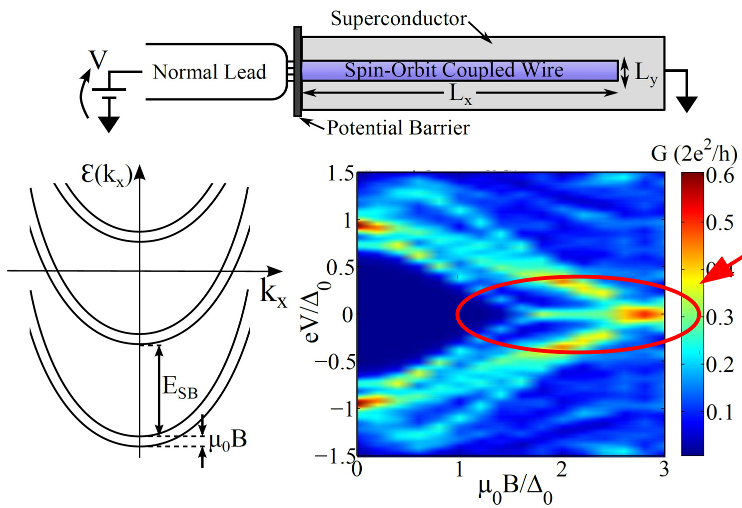
weak disorder

$$l = 10L$$

$$L = 10nm$$

Andrew Potter
Patric Lee

Zero-bias peaks with and without Majorana end-states



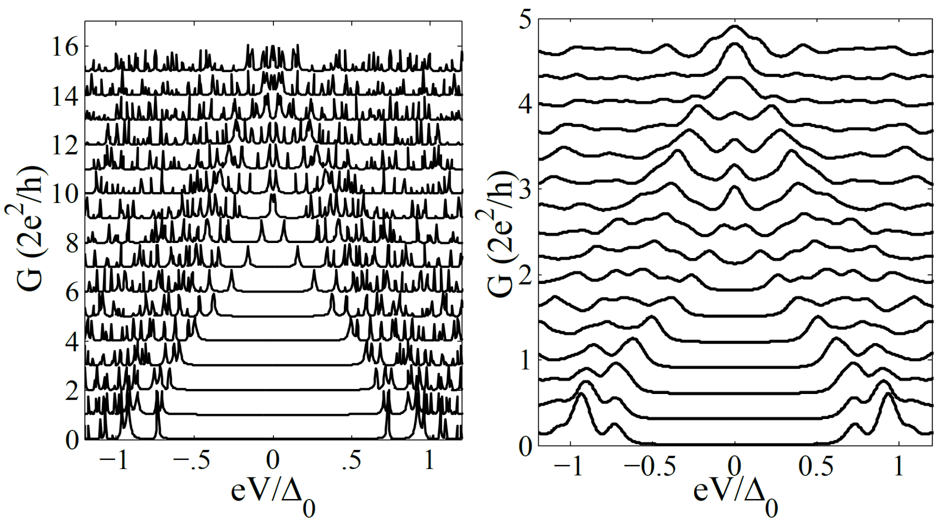
ZBP in the non-topological regime due to disorder

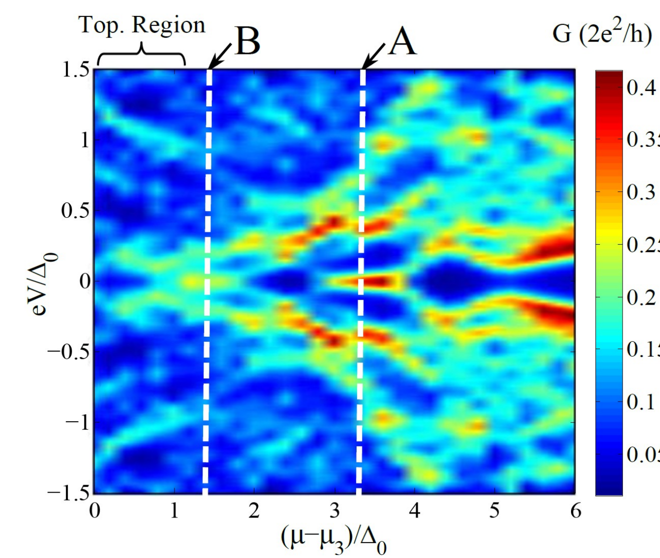
$$\ell \approx 3\mu m$$

$$\overline{V(\mathbf{r})V(\mathbf{r}')} = W^2 \delta_{\mathbf{r},\mathbf{r}'}$$

ZBP's exist in the non-topological state

ZBP's persist even when disorder is sufficiently strong

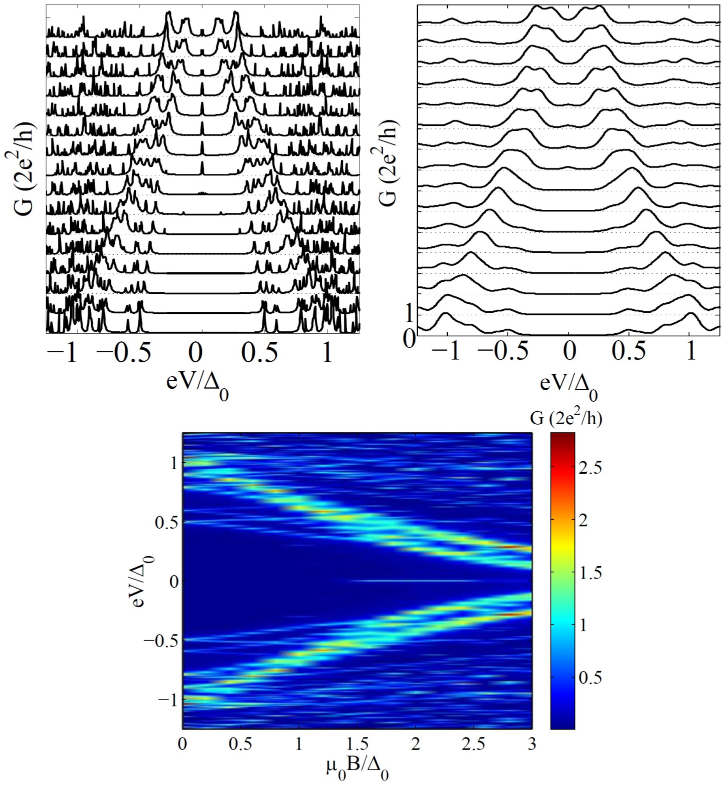




$$\ell \approx 10\mu m$$

Zero-bias peaks with and without Majorana end-states

Andrew Potter
Patric Lee

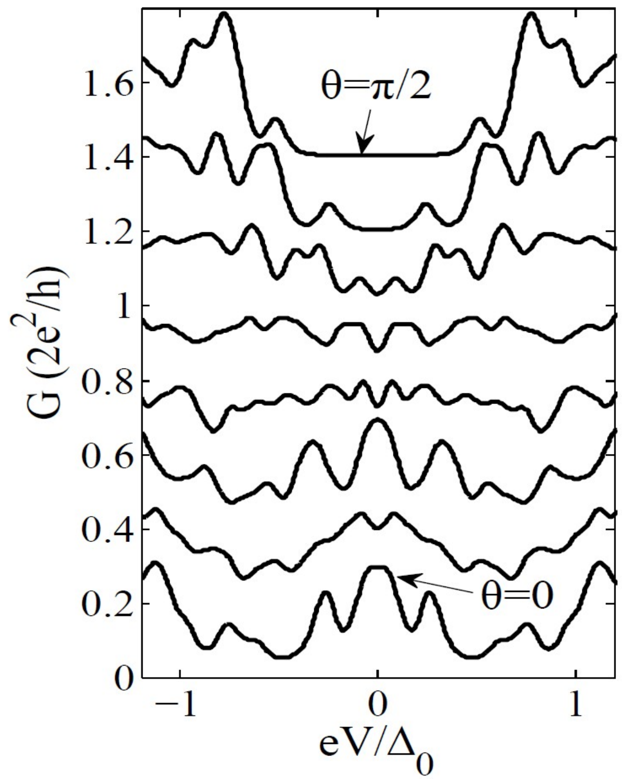


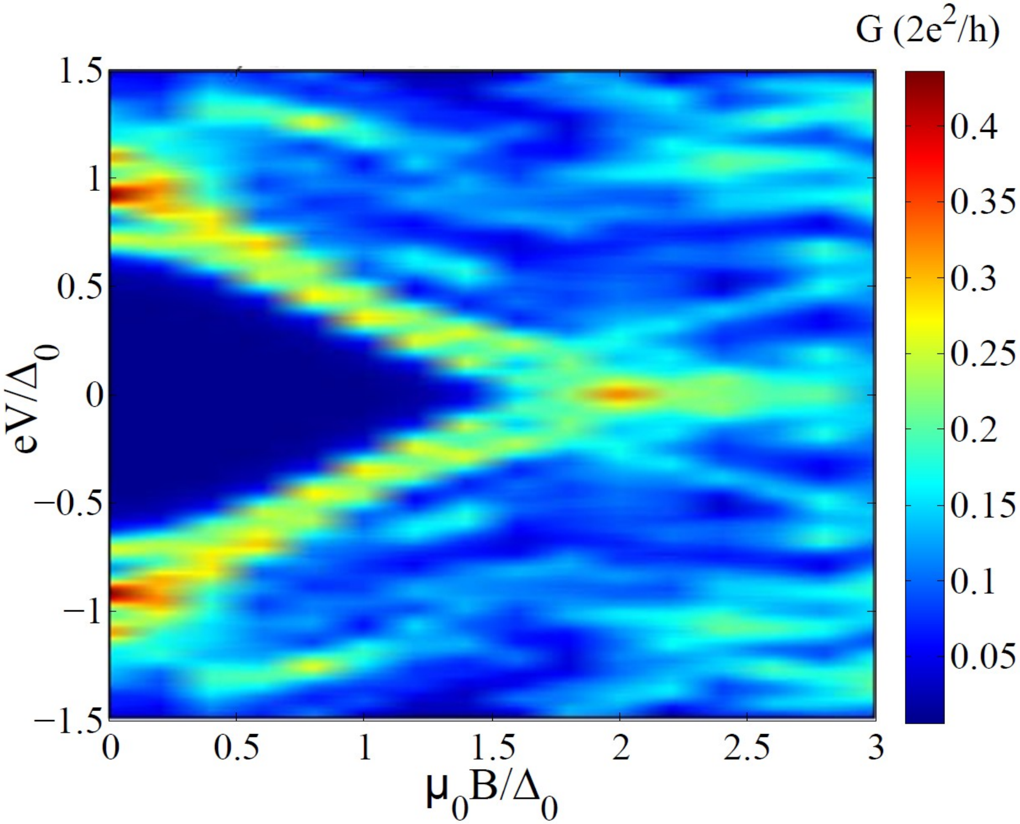
Long clean wire

"We note that, even for a clean wire, the observation of this quantized peak requires much 2-3 times longer wires than those used in the experiment"

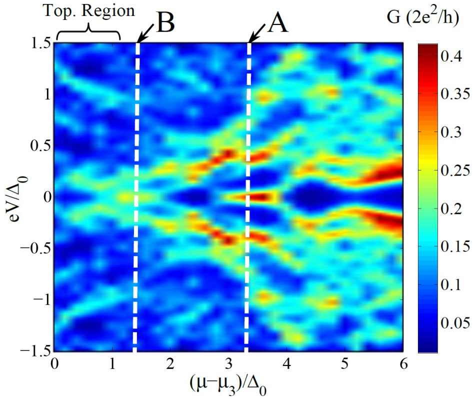
Zero-bias peaks with and without Majorana end-states

Andrew Potter
Patric Lee





Angle dependence of non-topological ZBP



Conclusion

Jie Liu, Andrew C. Potter, K.T. Law, and Patrick A. Lee; arXiv:1206.1276

(i) Majorana end-states are destroyed and do not give rise to quantized ZBPs.
(ii) ZBPs of a nontopological origin often appear due to disorder
(iii) Non-topological ZBPs are typically stable
(iv) Non-topological ZBPs appear and disappear under nearly identical conditions to those of true Majorana peaks.

Dmitry Bagrets and Alexander Altland arXiv:1206.0434

(i) rigidly positioned at zero energy,
(ii) strongly affected by temperature
(iii) not affected by magnetic field,
(iv) relies on parametric conditions similar to those required by the Majorana peak,
(v) carries unit integrated spectral weight (a single quasi 'state'),
(vi) independent of other midgap structures (Kondo resonances or Andreev bound states),
(vii) shows sensitivity to the parity of channel number,
unlike the Majorana particle,
(viii) relies on the presence of a moderately weak amount of disorder

“These results strike a note of caution for interpreting recent experimental evidence of Majorana states in tunneling data”