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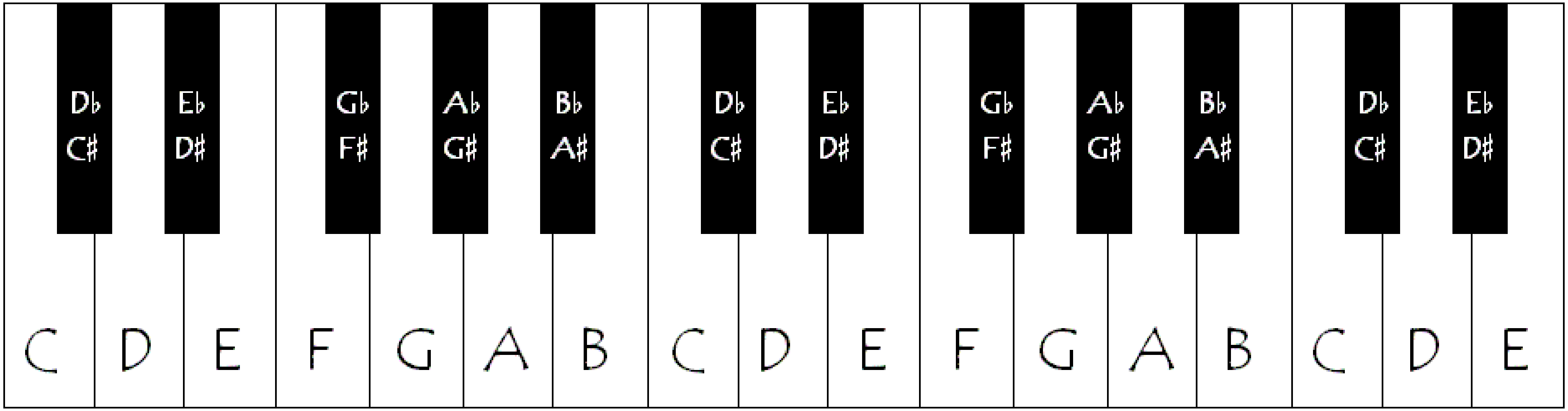
Entropy-based Tuning of Musical Instruments

Haye Hinrichsen

Fakultät für Physik und Astronomie, Universität Würzburg, Germany

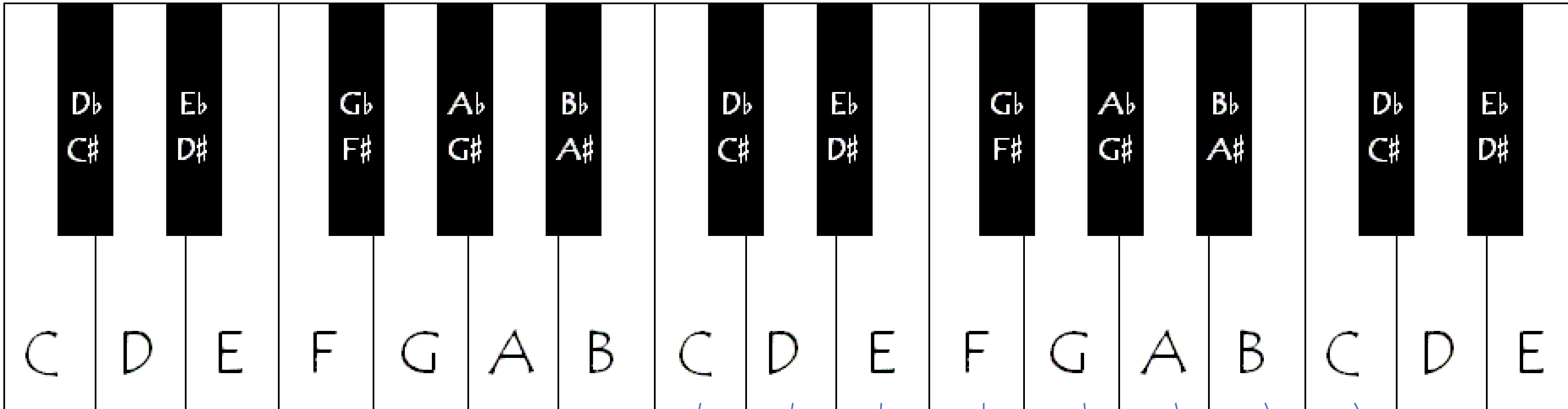
Journal Club, 19 June 2012

Tuning Systems



Crucial for the sound of chords and melodies:
<u>Frequency ratios!</u>

Tuning Systems

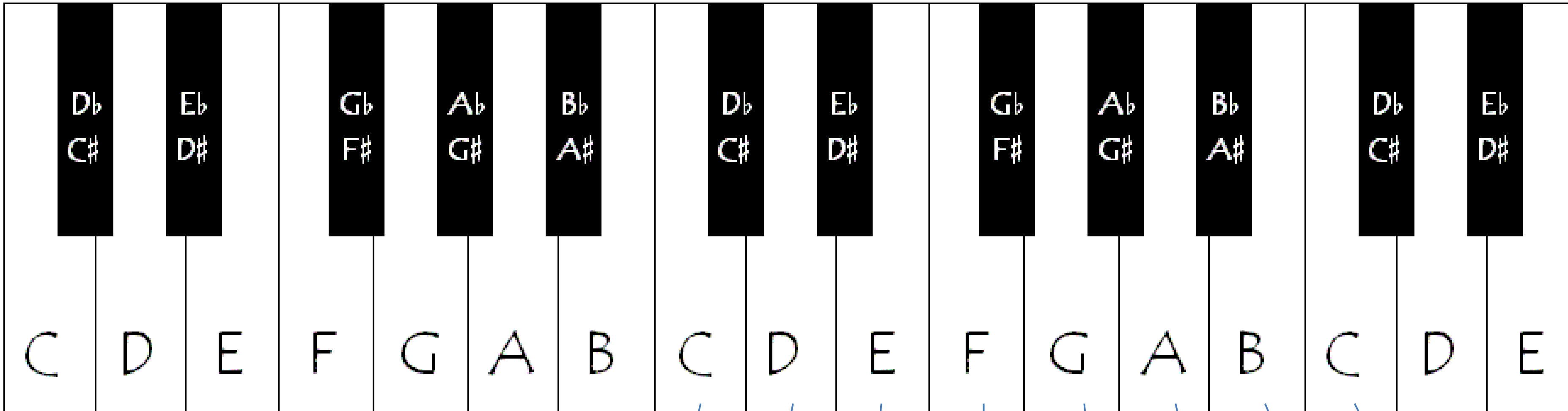


Crucial for the sound of chords and melodies: **<u>Frequency ratios!</u>**

"Just Intonation": 1 9/8 5/4 4/3 3/2 5/3 15/8 2

Here: common example for C-Major scale

Tuning Systems



Crucial for the sound of chords and melodies: **Frequency ratios!**

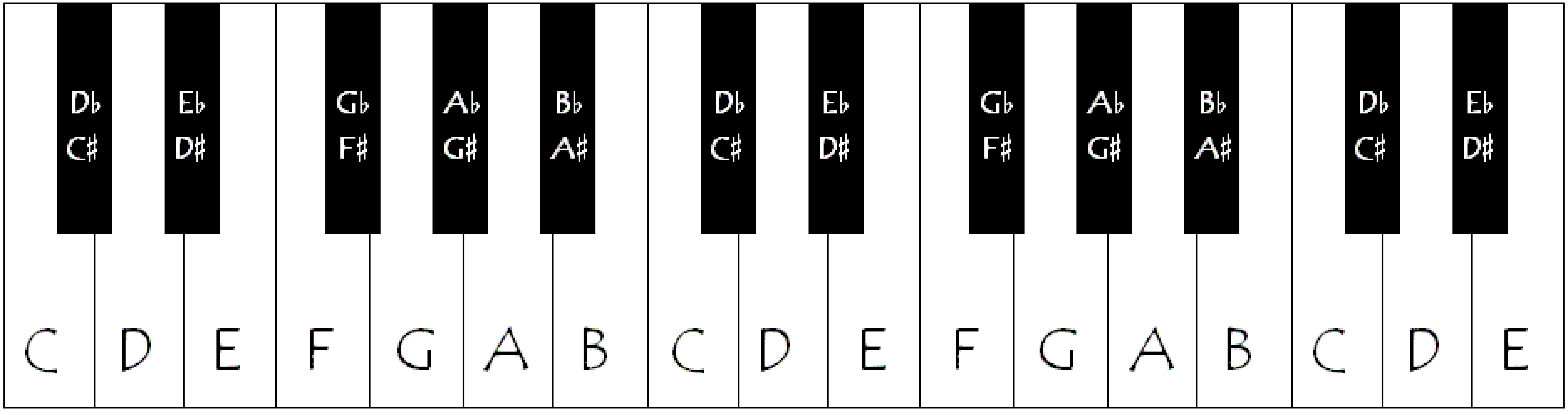
"Just Intonation": 1 9/8 5/4 4/3 3/2 5/3 15/8 2

Here: common example for C-Major scale

"Equal Temperament": $1 \quad 2^{2/12} \quad 2^{4/12} \quad 2^{5/12} \quad 2^{7/12} \quad 2^{9/12} \quad 2^{11/12} \quad 2$

Same ratio ($2^{1/12}$) between adjacent notes

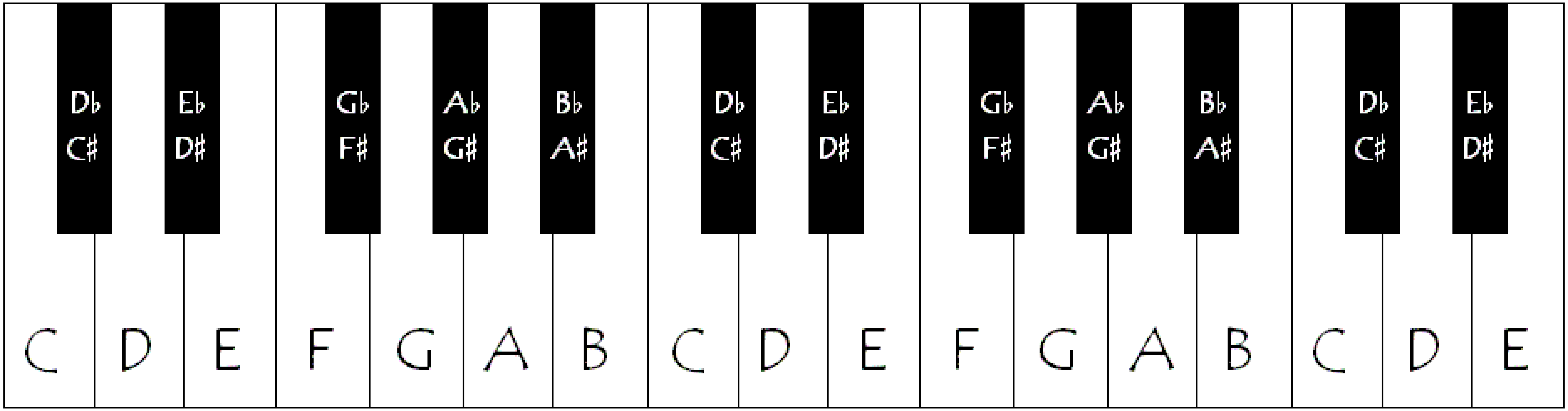
Equal Temperament



Since ~ 19th century, Western music is based on **Equal Temperament**

Adjacent notes differ by factor $\mathbf{2^{1/12}}$ in frequency → Translational Invariance

Equal Temperament



Since ~ 19th century, Western music is based on **Equal Temperament**

Adjacent notes differ by factor $\mathbf{2^{1/12}}$ in frequency ⟶ Translational Invariance

Professional Piano Tuning: Aural

Picture from Wikipedia, by Henry Heatly

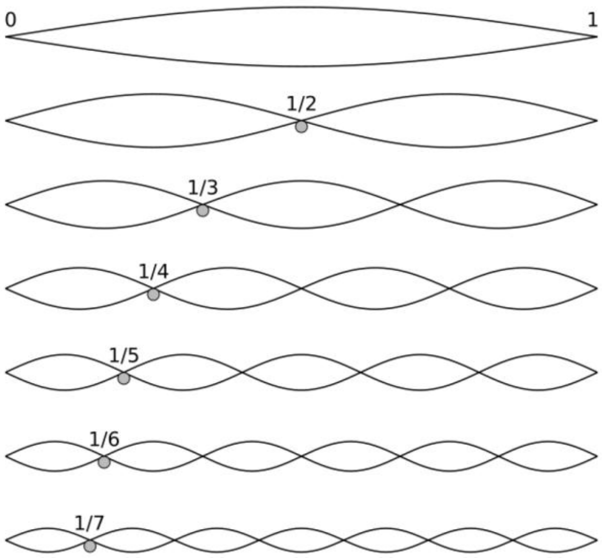
Why can't we tune it ourselves?

Overtones & Stiffness of Strings

Besides its **fundamental mode (frequency f_1)**, a string features several **overtones** of **frequencies f_n**

Ideal string: $\ddot{y} \propto -y''$ $\quad f \propto |k|$

$$\longrightarrow \quad f_n = n f_1$$



$$n = 1, 2, \ldots$$

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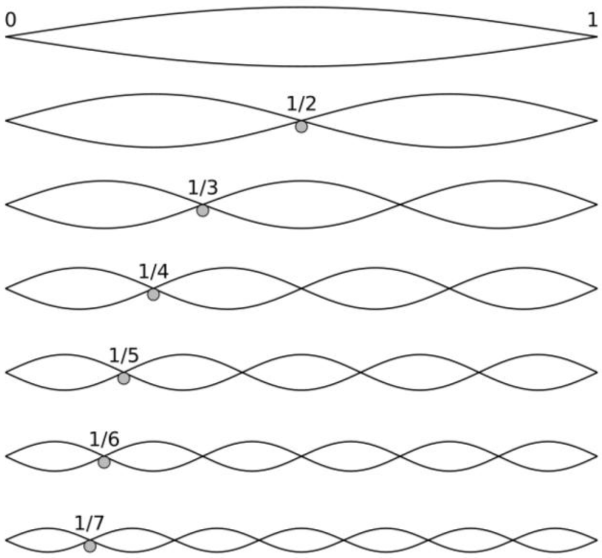

Ideal string: $\ddot{y} \propto -y''$ $\quad f \propto |k|$

$$\longrightarrow \quad f_n = n f_1$$

Stiff bar: $\ddot{y} \propto -y''''$ $\quad f \propto k^2$

Realistic string: $\ddot{y} \propto -y'' - \epsilon y''''$ $\quad f^2 \propto k^2 + \epsilon k^4$

$$\longrightarrow \quad f_n \propto n f_1 \sqrt{1 + B n^2}$$

***B**: Inharmonicity coefficient*

$n = 1, 2, \ldots$

Overtones & Stiffness of Strings

Further complications:

- Inharmonicity coefficient is different for each string (depends on length, diameter, tension, material properties, ...)
- For each string, the amplitudes of the overtones are different (depending on position of hammer, ...)

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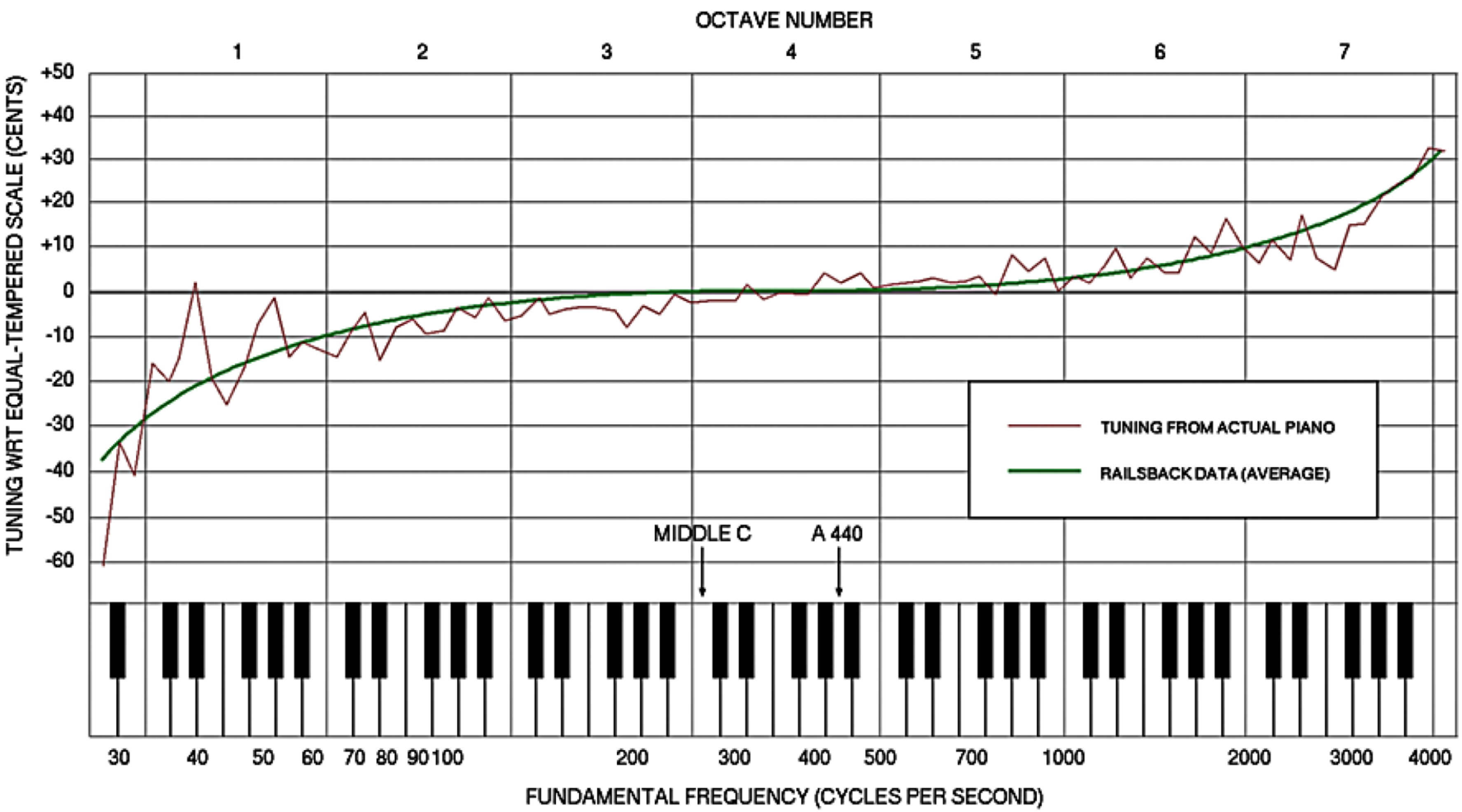
Tuning Curve of High-Quality Aural Tuning



Green: Average

Red: Individual piano

Tuning via Entropy

Idea of the paper:

Human brain perceives sounds as “pleasant” (“in tune”) when there is some kind of order

Entropy is a measure of disorder

→ **Find tuning curve via entropy minimization**

Entropy-Based Tuning: Preparation

Step 1: Play and record each of the keys

Entropy-Based Tuning: Preparation

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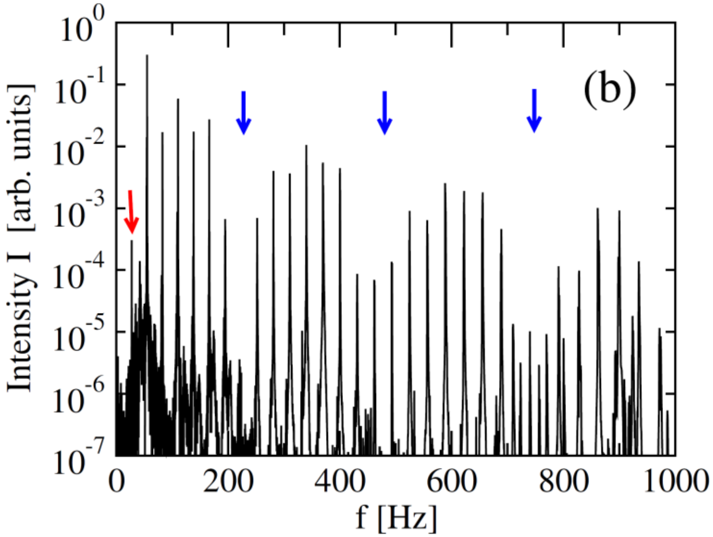
Step 2: Calculate power spectrum

$I(f) = |\text{Fourier transform}|^2$

Power spectrum for the lowest key (out of 88)

Red arrow: Fundamental mode

Blue: Suppressed overtones (position of hammer, ...)



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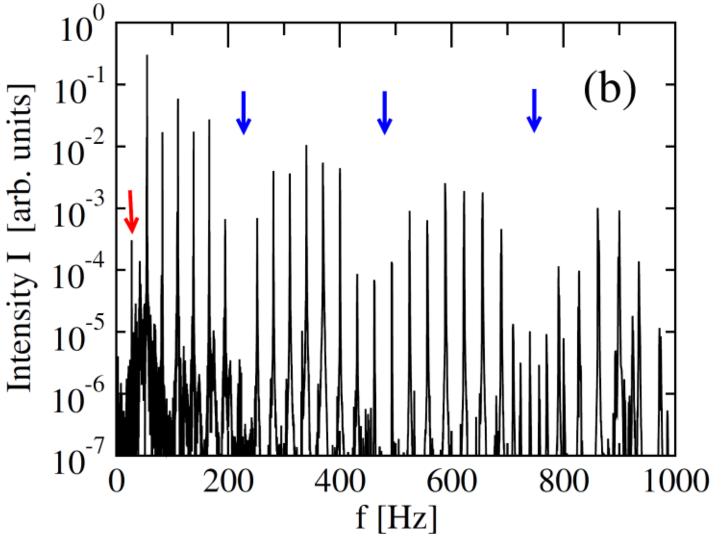

Step 1: Play and record each of the keys

Step 2: Calculate power spectrum

I(f) = |Fourier transform|^2

Step 3: Calculate **A-weighted sound pressure level L_A(f)** (in dBA)

Can be considered a rough measure of frequency-dependent energy deposition in the inner ear (cochlea)

$$L_A(f) = \Big(2.0 + 20\log_{10} R_A(f)\Big)\ L(f)$$

Filter function: Outer → Inner ear

$$L(f) = 10\log_{10}\Big(\frac{I(f)}{I_0}\Big) \qquad R_A(f) = \frac{12200^2 f^4}{(f^2+20.6^2)(f^2+12200^2)\sqrt{(f^2+107.7^2)\,(f^2+737.9^2)}}$$

Entropy-Based Tuning: Algorithm (Start)

Start configuration:

- Quantize frequency, ranging from 10 Hz to 10 kHz, in steps of cents:

$$f_m = 2^{m/1200} \cdot 10 \text{ Hz} \qquad 0 \le m \le 12000$$

- For each of the 88 keys *k*, map the A-leveled sound pressure level $L_A(f)$ onto f_m to obtain $L_m^{(k)}$
- Shift $L_m^{(k)}$ such that the fundamental modes of the keys correspond exactly to that of an equal temperament (with A4 = 440 Hz)
- Compute the sum p_m over all keys: $p_m = \sum_{k=1}^{88} L_m^{(k)}$
- Normalize: $\sum_m p_m = 1$

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Start configuration is a quantized (cents) probability distribution based on the power spectrum generated in the inner ear when the piano is exactly tuned to equal temperament

Entropy-Based Tuning: Algorithm (Dynamics)

Entropy: $$H = -\sum_m p_m \ln p_m$$

Monte-Carlo dynamics:

- Randomly shift one of the keys by ± 1 cent
- Compute again the sum p_m over all keys: $p_m = \sum_{k=1}^{88} L_m^{(k)}$
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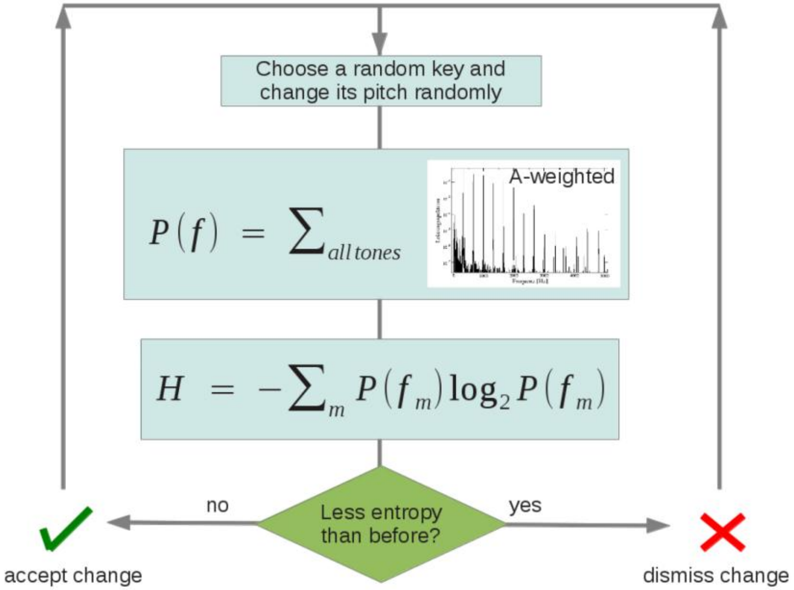
- Compute the entropy
- If entropy decreased, keep the change, otherwise undo it



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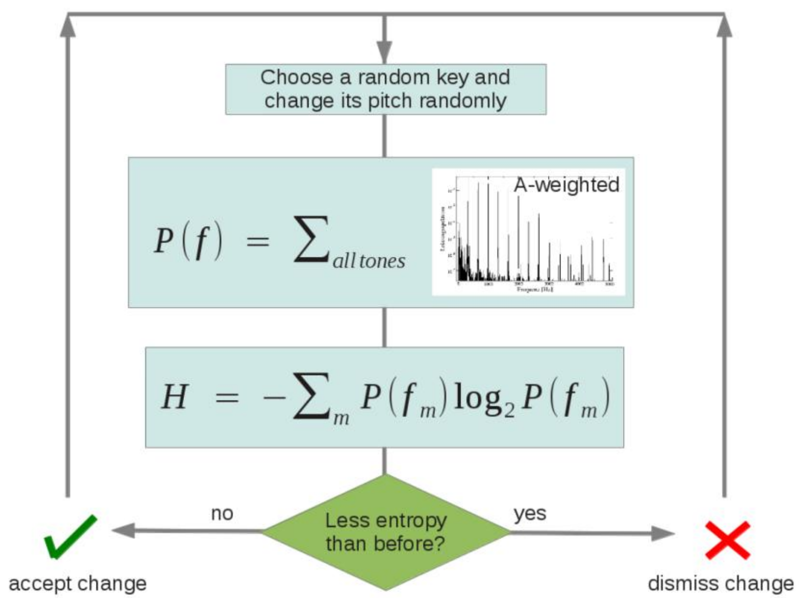
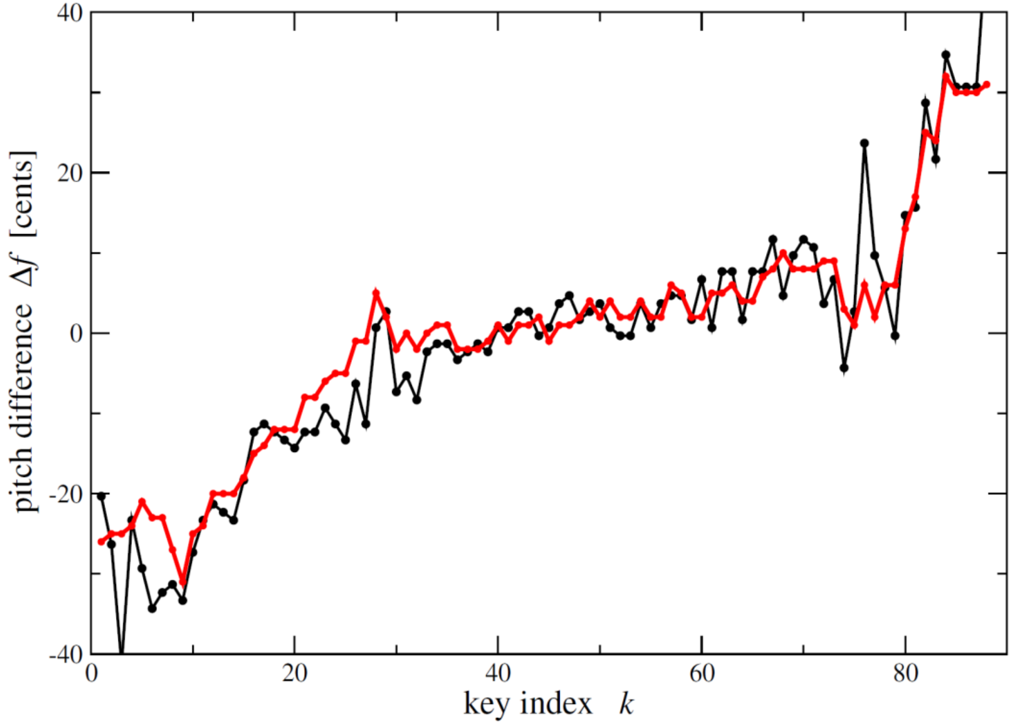
→ **Find minimum and return the tuning curve**

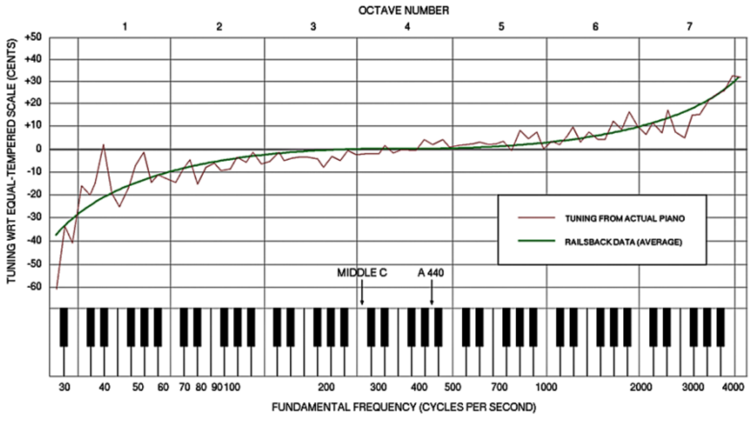


Results



Red: Theoretical result

Black: Aural tuning



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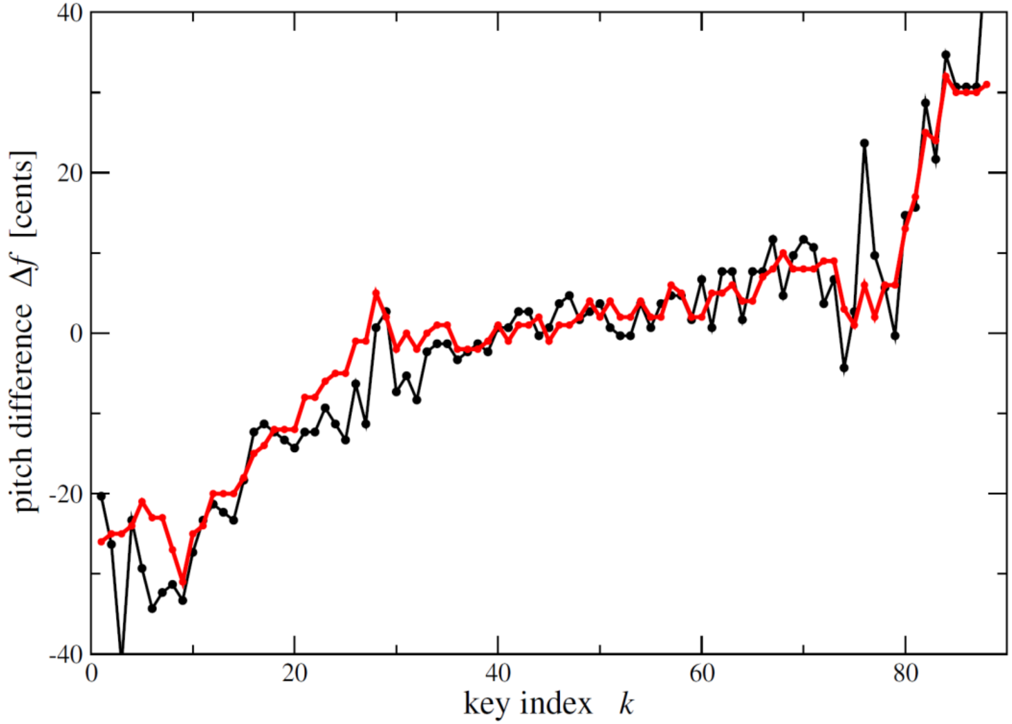

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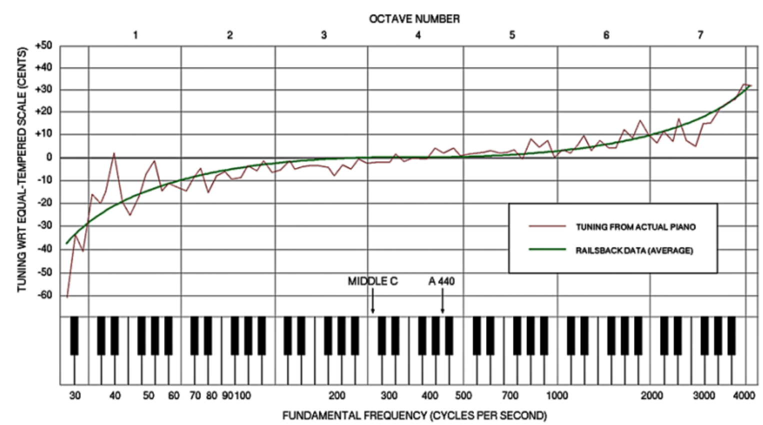
Method reproduces the stretch curve

Fluctuations are correlated (!), especially in the treble and the bass



Media Interest: Articles, Blogs, ...

English

IOP PhysicsWorld.com
MIT Technology Review
The Wall Street Journal
Daily Mail – Mail Online
Discover Magazine
Pano News Archiv
Microsoft Future Tech
Physics4me
The Week behind
Quantummaniac
33rd Square
Piano Tuner Technicians Forum
Tune a Piano Yourself Blog
Editorial RBEF
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Deutschlandradio Kultur
Pressestelle Uni Würzburg
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Nürnberger Zeitung (NZ)
Wiley Interscience pro-physik
Codex Flores: Viel Aufregung...
Medizin&Technik: Wir wollen Spaß
Neurosociology & Neuromarketing
Interview Klassikradio
Interview BR2
Mainpost
...

http://theorie.physik.uni-wuerzburg.de/~hinrichsen/

Conclusions

Author: Several open questions and remaining tasks

- Method tested on only one piano so far
- Apparently there are many local minima, and the present algorithm gives similar but not reproducible results
- Step-size of one cent is smaller than the resolution of the ear
- When additional filter function for “inner ear $\rightarrow$ brain” (“loudness”) are included, one obtains unreasonable stretches in the bass
- ... (see article)

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MIT Technology Review, ... : *“Algorithm Spells the End for Professional Musical Instrument Tuners”*

Science **336**, 1283 (2012)

Room-Temperature Quantum Bit Memory Exceeding One Second

P. C. Maurer,[1] G. Kucsko,[1] C. Latta,[1] L. Jiang,[2] N. Y. Yao,[1]
S. D. Bennett,[1] F. Pastawski,[3] D. Hunger,[3] N. Chisholm,[4]
M. Markham,[5] D. J. Twitchen,[5] J. I. Cirac,[3] and M. D. Lukin[1]

[1]*Department of Physics, Harvard University, Cambridge, USA*
[2]*Institute for Quant. Inf. and Matter, California Institute of Technology, Pasadena, USA*
[3]*Max-Planck-Institut für Quantenoptik, Garching, Germany*
[4]*School of Engineering and Applied Sciences, Harvard University, Cambridge, USA*
[5]*Element Six, Ascot, UK*

Journal Club, 19 June 2012

Main Results

System

Single ^{13}C nuclear spin near a nitrogen-vacancy (NV) center in an isotopically pure diamond (99.99% spinless ^{12}C)

Experimental Results (Room temperature)

^{13}C nuclear spin (spin ½) can preserve its polarization for several minutes

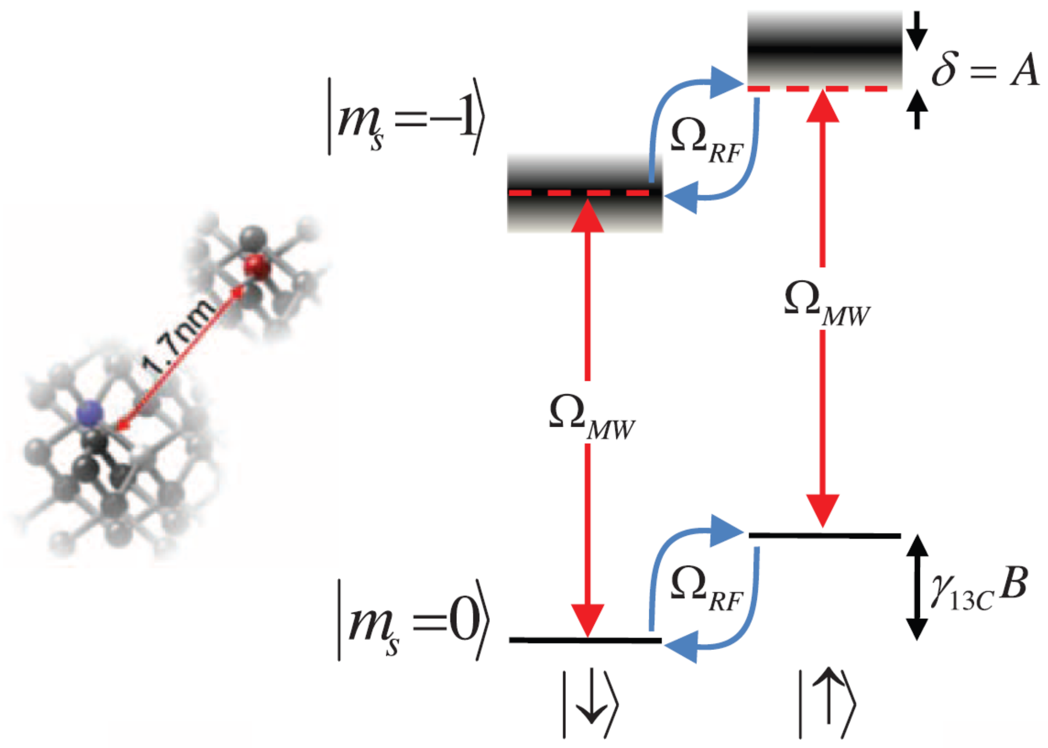
Coherence times longer than one second are achieved by decoupling the nuclear spin from its environment

Basic System

Electronic spin of NV Center: Spin 1, m_s = 1, 0, -1

Nearby ^{13}C nuclear spin: Spin 1/2, I_z = 1/2, -1/2

A magnetic field B is applied along the NV symmetry axis (z axis)



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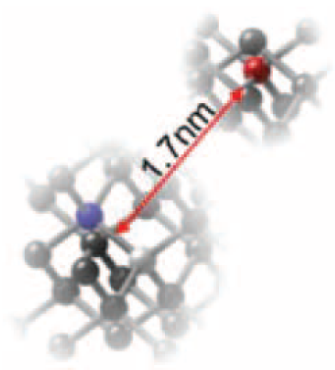

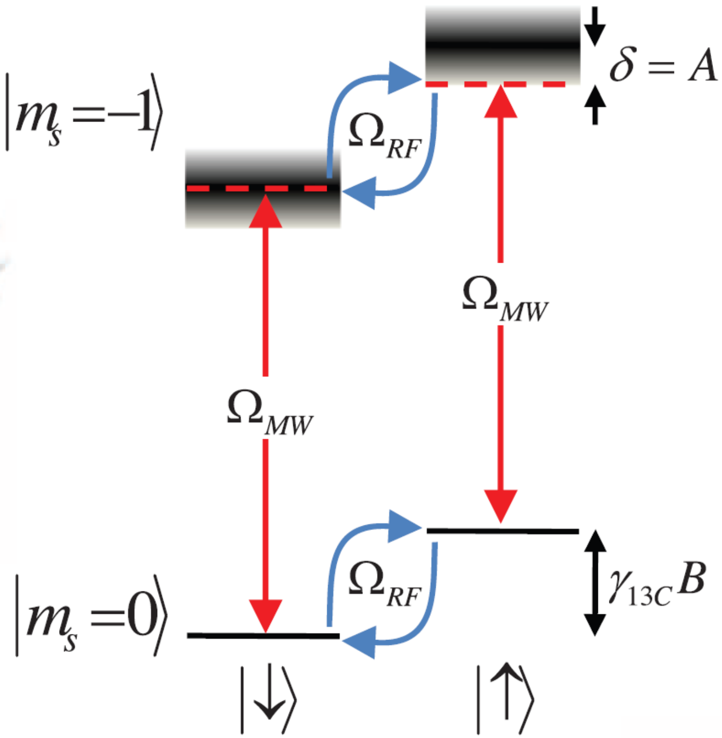

Measurement of Hyperfine Interaction

Simple Hamiltonian:

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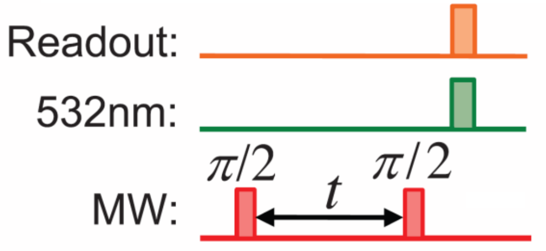
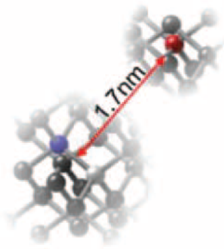
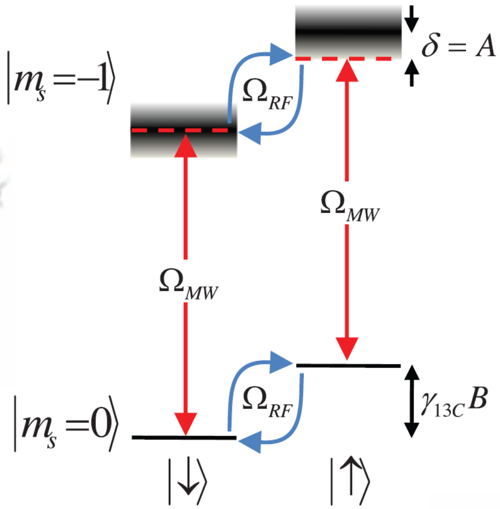
Ramsey-type experiment:







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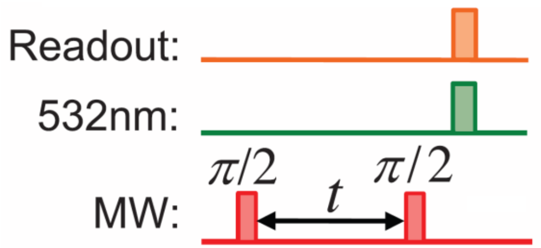

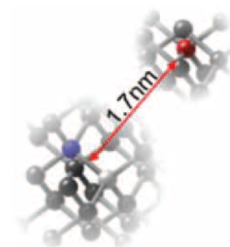

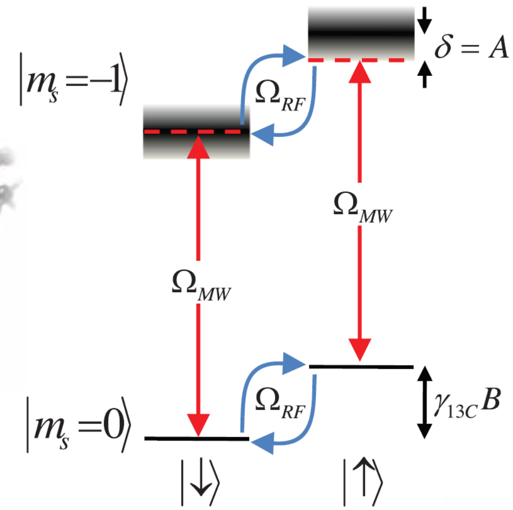

In the presence of a ^{13}C nuclear spin, one expects an additional collaps in the signal at time $t = \tau = \pi / A_{||}$, for which one finds $\langle m_s \rangle \simeq -1/2$ when the system is initially in the state $|0\rangle \left(|\uparrow\rangle + |\downarrow\rangle\right)$

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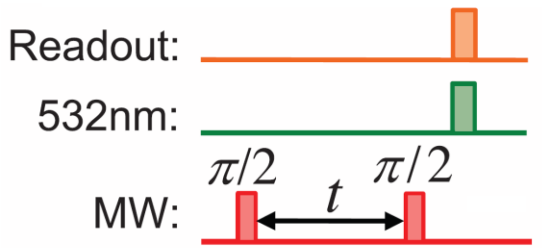

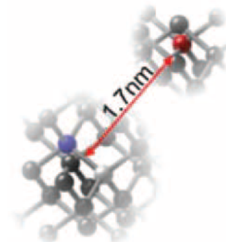

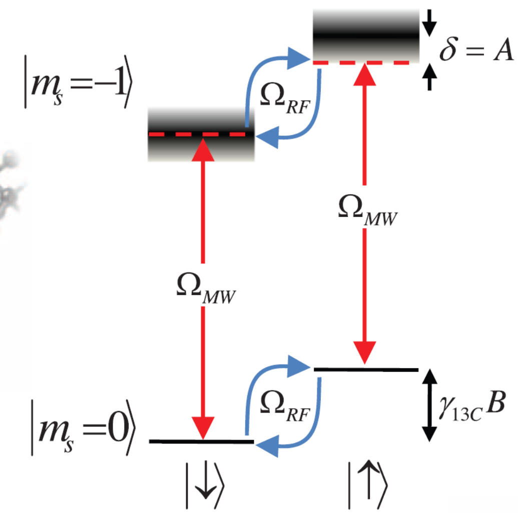

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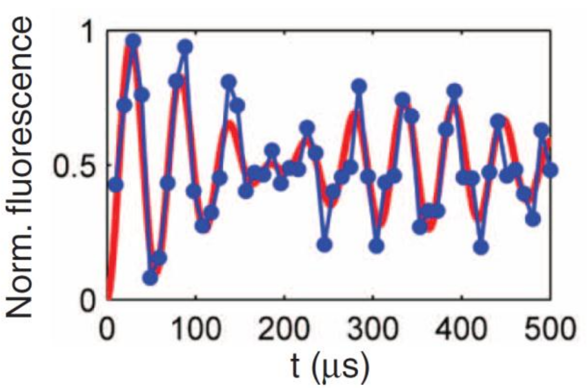


In this sample, around 1 out of 10 NV centers had a ^{13}C nuclear spin close by (1-2 nm). Here: ~ 1.7 nm

$$T_{2e}^* = 470 \pm 100 \ \mu s$$

$$A_{||} = (2\pi)\,(2.66 \pm 0.08)\ \text{kHz}$$

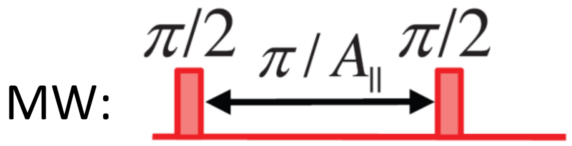
Measured also accurately via an NMR experiment

$C_n NOT_e$ gate

Simple Hamiltonian:

$$H = -E_Z m_s + E_n I_z + \hbar A_{||} m_s I_z$$

Pulse sequence:



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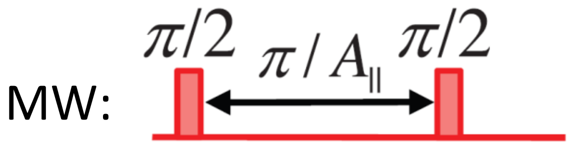

One finds

$$e^{-i\alpha} = 1$$

$$|0\rangle\,|\uparrow\rangle \rightarrow |0\rangle\,|\uparrow\rangle$$
$$|0\rangle\,|\downarrow\rangle \rightarrow |-1\rangle\,|\downarrow\rangle$$

$$\alpha = \frac{\pi E_Z}{\hbar A_{||}} + \frac{\pi}{2}$$

$$e^{-i\alpha} = -1$$

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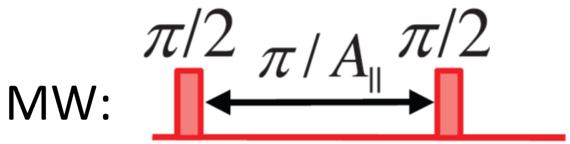

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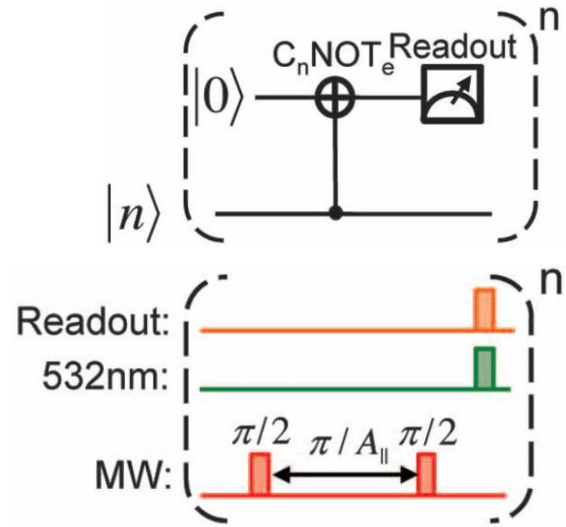
Corresponds to a $C_n NOT_e$ logical gate

Nuclear spin state can be read out via electron spin

Initialization via projective measurement



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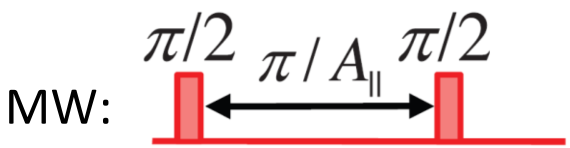

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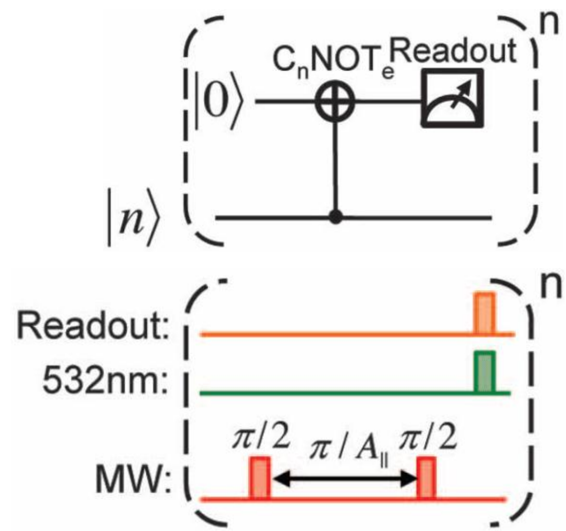
Experiment:
244.42 ± 0.02 G

$$e^{-i\alpha} = -1 \qquad \begin{aligned} |0\rangle\,|\uparrow\rangle &\rightarrow |-1\rangle\,|\uparrow\rangle \\ |0\rangle\,|\downarrow\rangle &\rightarrow |0\rangle\,|\downarrow\rangle \end{aligned}$$



Corresponds to a C_nNOT_e logical gate

Nuclear spin state can be read out via electron spin

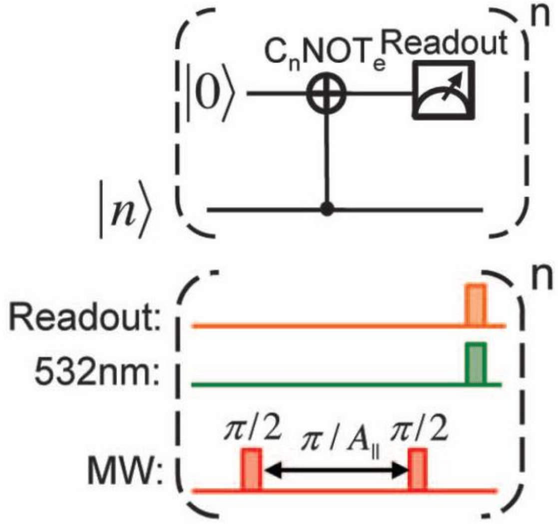
Initialization via projective measurement

C_nNOT_e gate

Simple Hamiltonian:

$$H = -E_Z m_s + E_n I_z + \hbar A_{||} m_s I_z$$

Repetitive readout



C_nNOT_e gate

Simple Hamiltonian:

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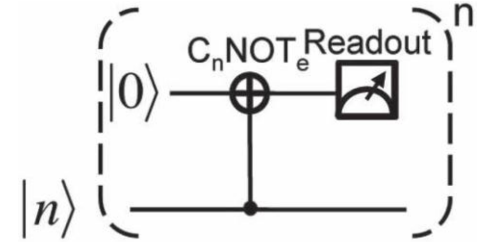
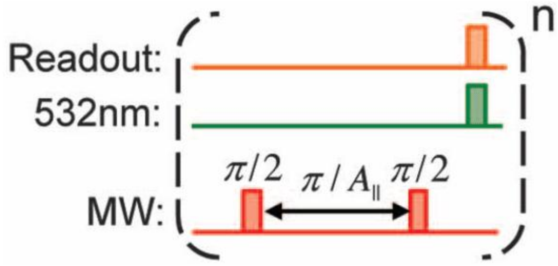
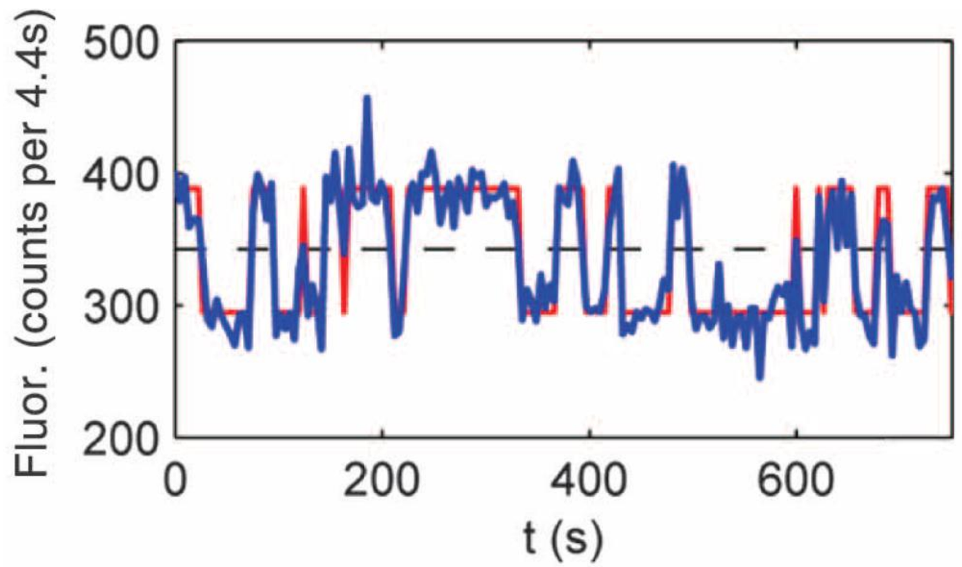




Repetitive readout



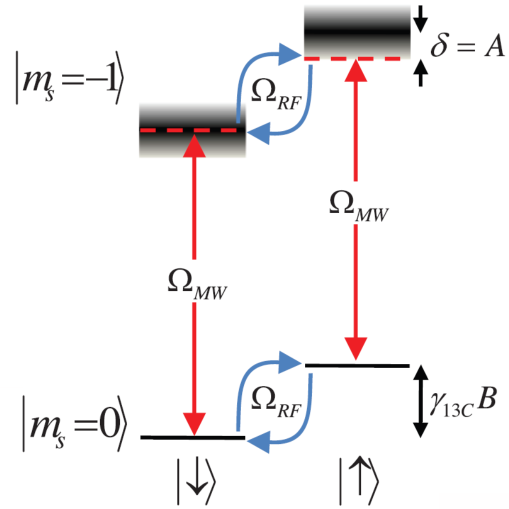
Nuclear spin preserves orientation for about half a minute

In the dark, no decay was observed at time scale of 200 s

Scheme allows initialization of nuclear spin state with > 97% fidelity, and readout with 92% fidelity

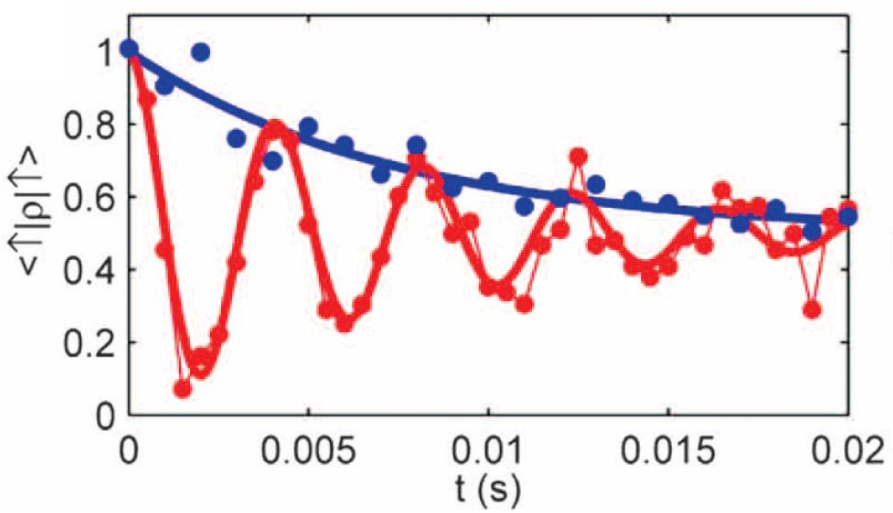
Decoherence

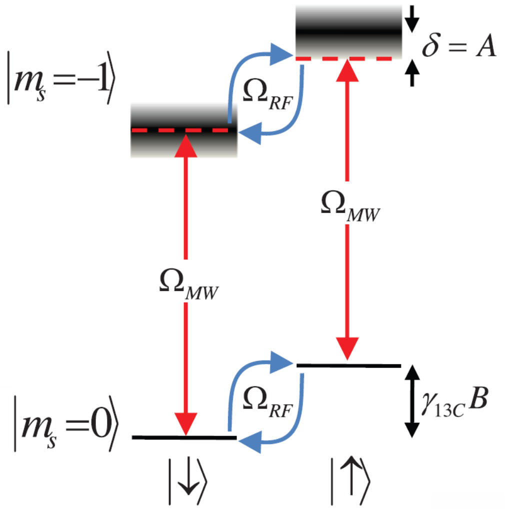
Ramsey experiment on the nuclear spin via rf pulses



Decoherence

Ramsey experiment on the nuclear spin via rf pulses





Red: Coherent oscillations of nuclear spin (Ramsey)

Blue: Relaxation of electronic spin state

$T_{2n}^{*} = 8.2 \pm 1.3$ ms

$T_{1e} = 7.5 \pm 0.8$ ms

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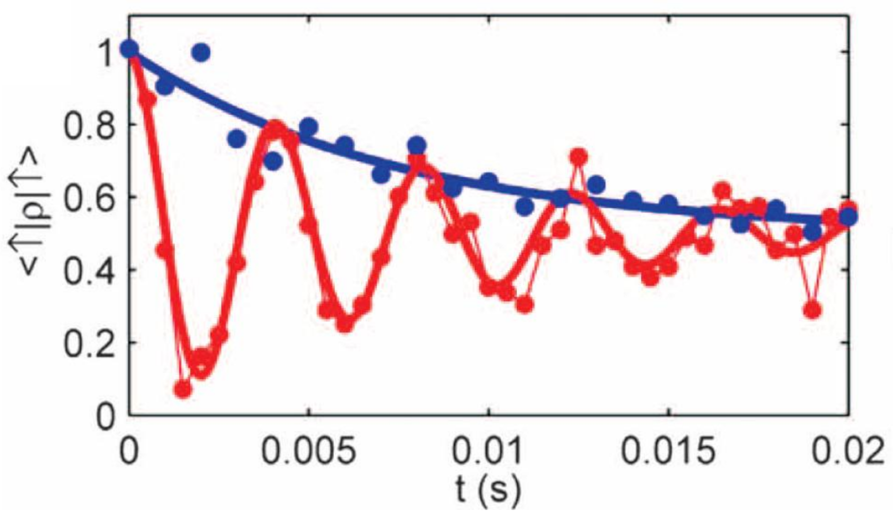

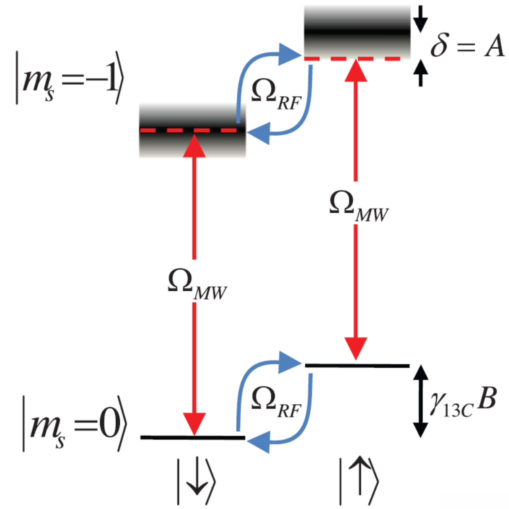

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Short dephasing time is determined by coupling to nearby electronic states with m_s = 1, -1. Can these be decoupled from the nuclear spin?

Decoherence

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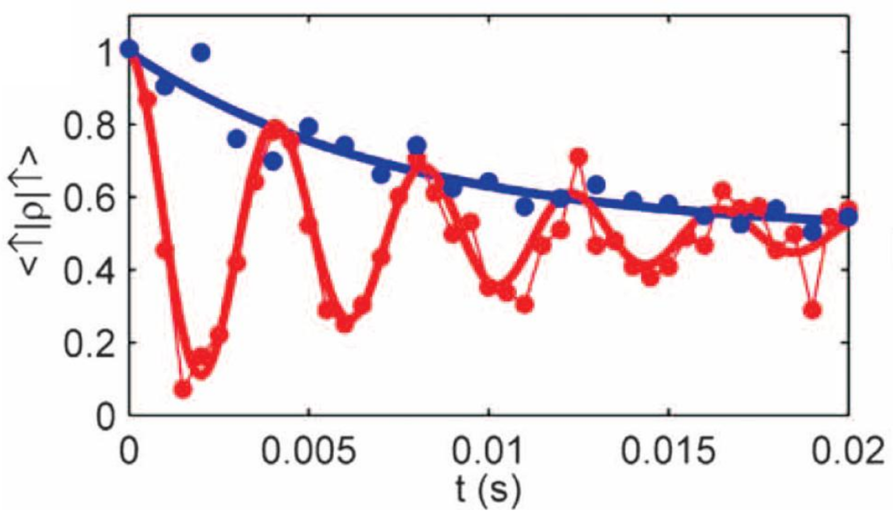

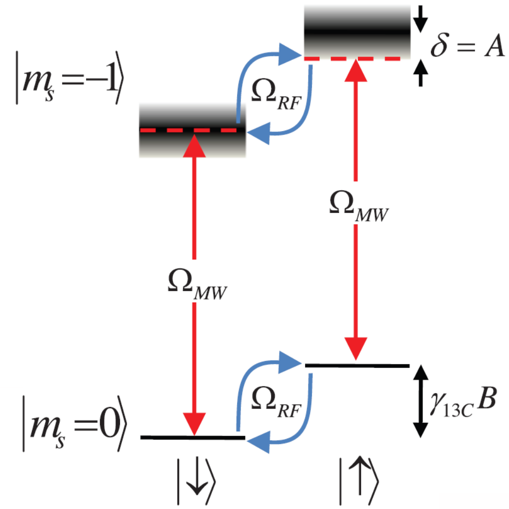

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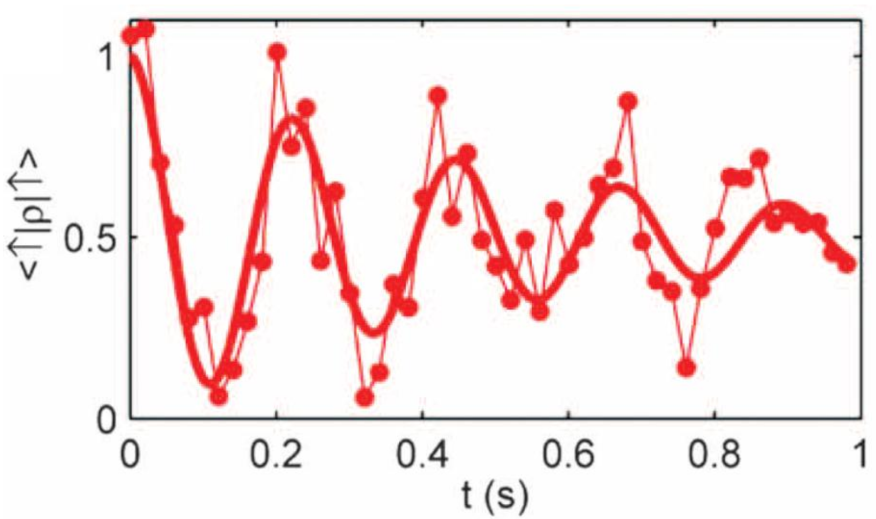
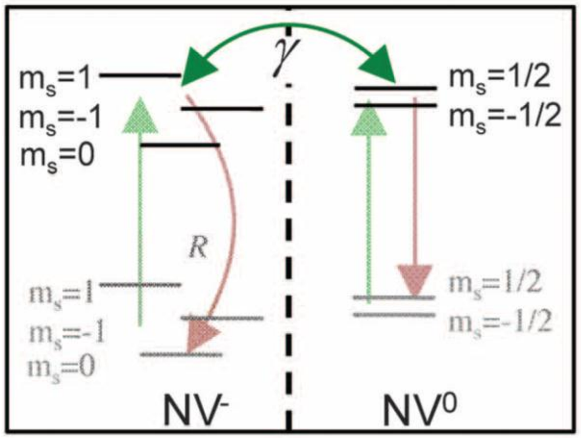
Short dephasing time is determined by coupling to nearby electronic states with m_s = 1, -1. <u>Can these be decoupled from the nuclear spin?</u>

Yes, by exciting the NV center with a focused green laser beam!

Decoherence with Laser

Ramsey experiment on the nuclear spin via rf pulses





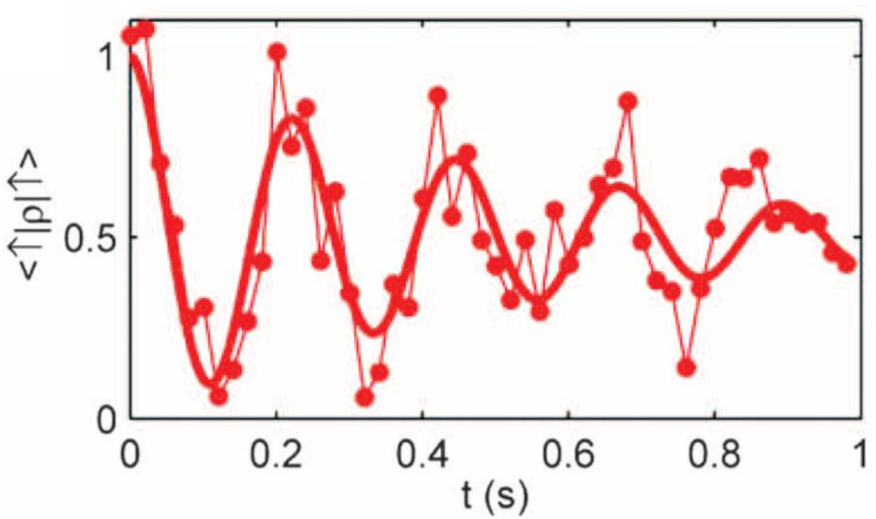
The authors also explain their results with simulations of a many-state system (Supplement)

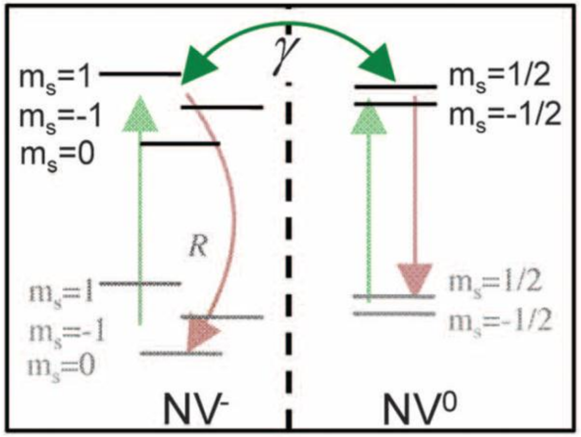
$$T_{2n}^{*} = 0.53 \pm 0.14 \text{ s}$$

“Dissipative decoupling” via laser illumination prolongs dephasing time by two orders of magnitude

Decoherence with Laser

Ramsey experiment on the nuclear spin via rf pulses





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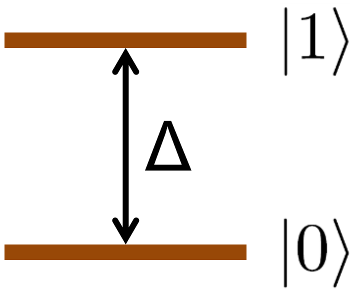
$$T_{2n}^{*} = 0.53 \pm 0.14 \text{ s}$$

"Dissipative decoupling" via laser illumination prolongs dephasing time by two orders of magnitude

Further improvement is possible:
Dynamical decoupling from other ^{13}C nuclear spins

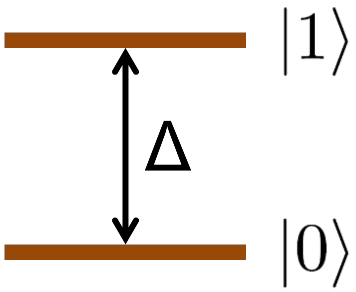
Dynamical Decoupling

Two-level system (Qubit)



Dynamical Decoupling

Two-level system (Qubit)



Prepare system in eigenstate of σ_x

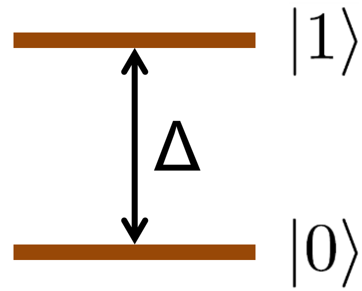
$$|\psi\rangle_{t=0} = |0\rangle + |1\rangle$$

Time evolution

$$|\psi\rangle_t = |0\rangle + e^{-i\frac{\Delta}{\hbar}t}|1\rangle$$

Dynamical Decoupling

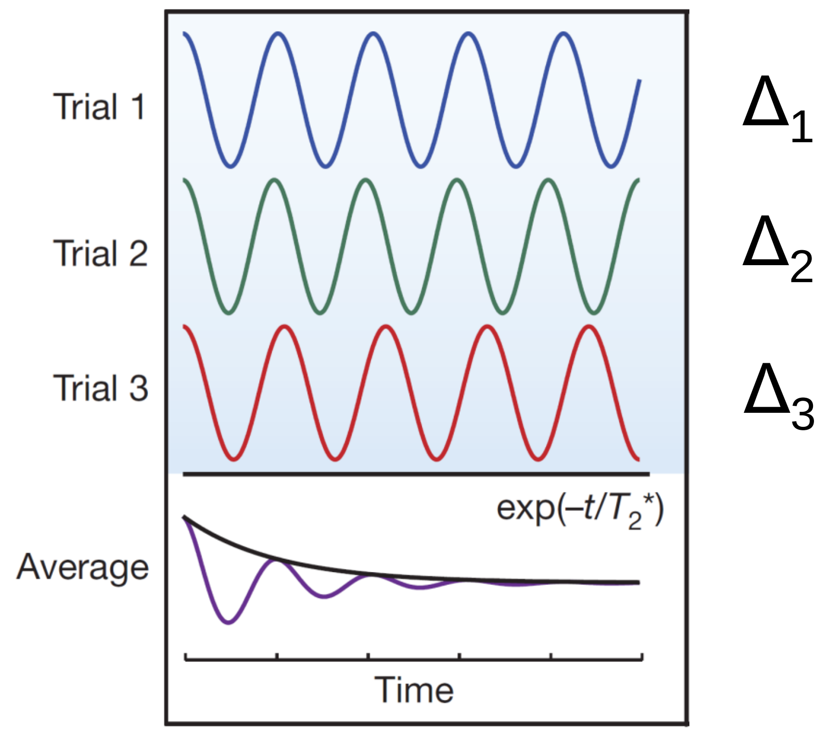
Two-level system (Qubit)



Average over different Δ

Prepare system in eigenstate of σ_x

$$|\psi\rangle_{t=0} = |0\rangle + |1\rangle$$



Time evolution

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Figure from Ladd *et al*., Nature 2010

Dynamical Decoupling

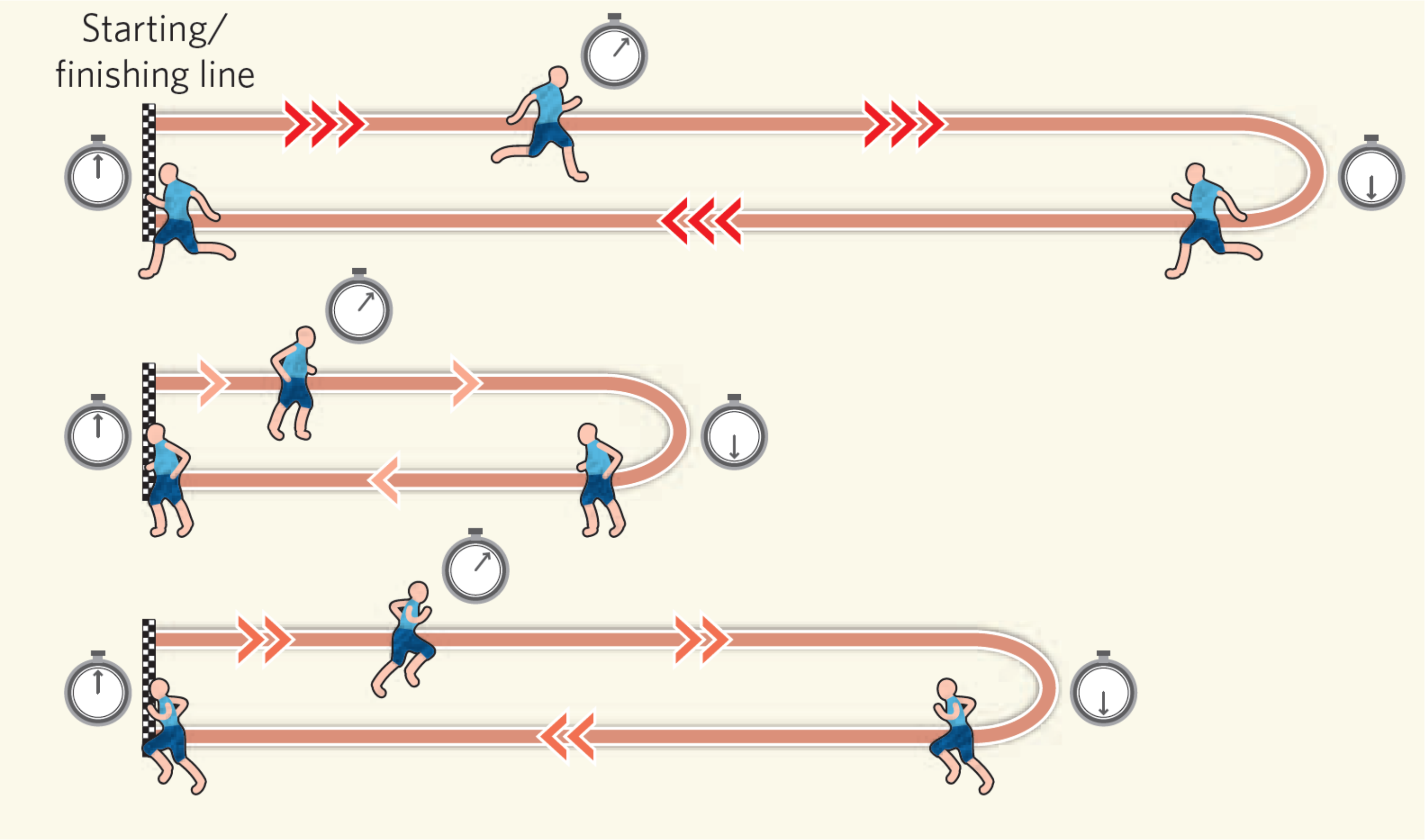
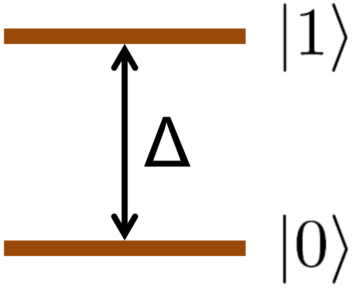


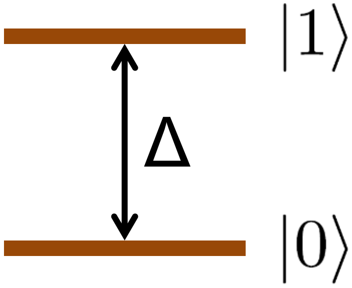
Figure from Buckley/Awschalom, Nature (News & Views) 2009

Dynamical Decoupling

Apply a single π pulse (echo pulse) at time t = τ



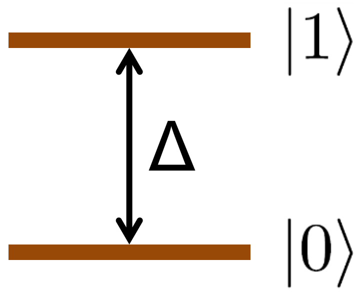
Dynamical Decoupling



Apply a single π pulse (echo pulse) at time t = τ

Eigenstate of σ_x $$|\psi\rangle_{t=0} = |0\rangle + |1\rangle$$

Dynamical Decoupling

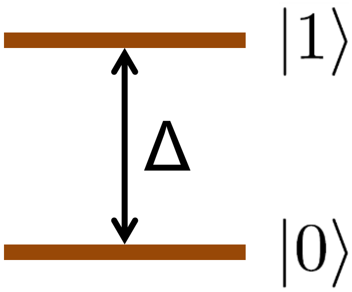


Apply a single π pulse (echo pulse) at time t = τ

Eigenstate of σ_x $$|\psi\rangle_{t=0} = |0\rangle + |1\rangle$$

$$|\psi\rangle_{t<\tau} = |0\rangle + e^{-i\frac{\Delta}{\hbar}t}|1\rangle$$

Dynamical Decoupling



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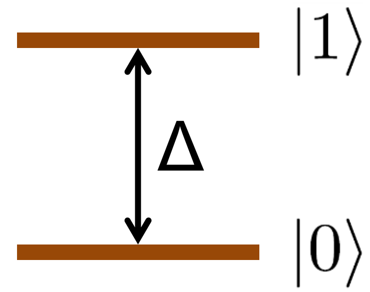
Dynamical Decoupling



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π pulse

$$|\psi\rangle_{t=\tau} = |1\rangle + e^{-i\frac{\Delta}{\hbar}\tau}\,|0\rangle$$

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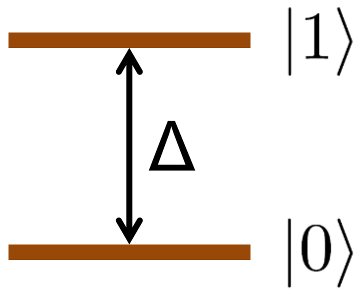

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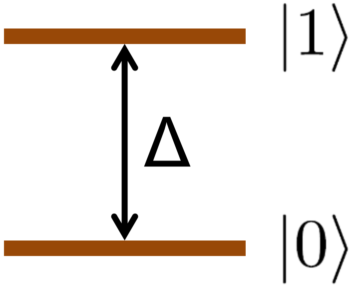
π pulse

$$|\psi\rangle_{t=\tau} = |1\rangle + e^{-i\frac{\Delta}{\hbar}\tau}|0\rangle$$

$$|\psi\rangle_{t>\tau} = e^{-i\frac{\Delta}{\hbar}(t-\tau)}|1\rangle + e^{-i\frac{\Delta}{\hbar}\tau}|0\rangle$$

$$= |0\rangle + e^{-i\frac{\Delta}{\hbar}(t-2\tau)}|1\rangle$$

Dynamical Decoupling



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$$= |0\rangle + e^{-i\frac{\Delta}{\hbar}(t-2\tau)}\,|1\rangle$$

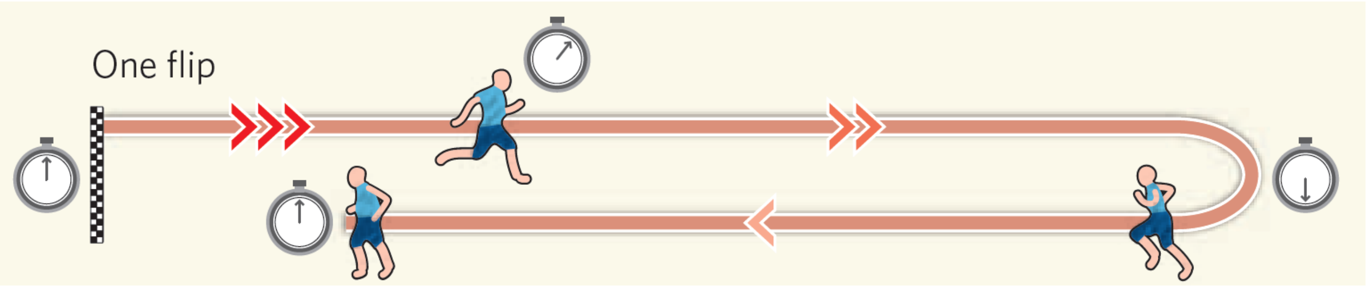
t = 2 τ

$$|\psi\rangle_{t=2\tau} = |0\rangle + |1\rangle$$

Independent of Δ

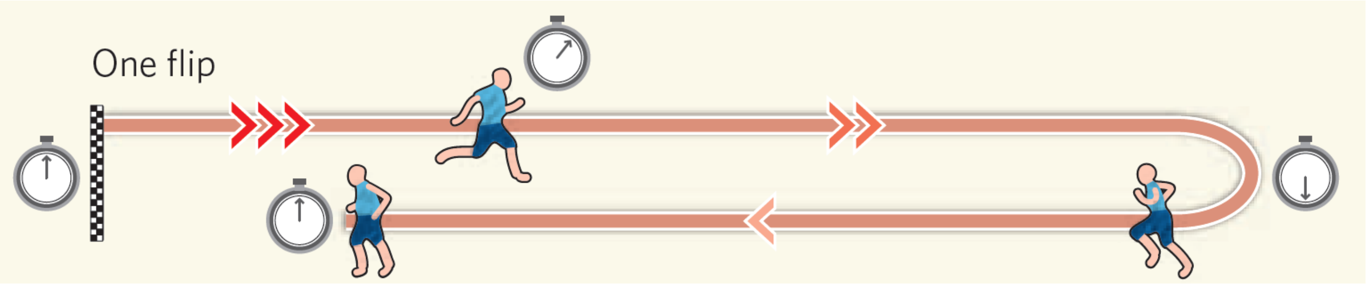
Peak in the signal

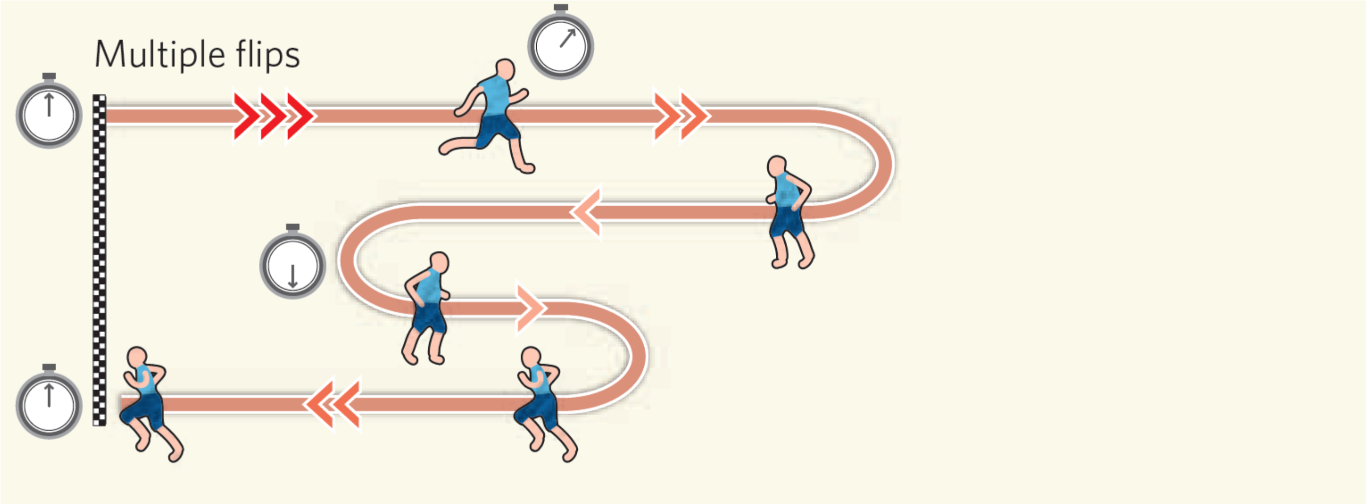
Dynamical Decoupling



Figures from Buckley/Awschalom
Nature (News & Views) 2009

Dynamical Decoupling





<u>More advanced pulse sequences:</u>

Carr-Purcell-Meiboom-Gill (CPMG)

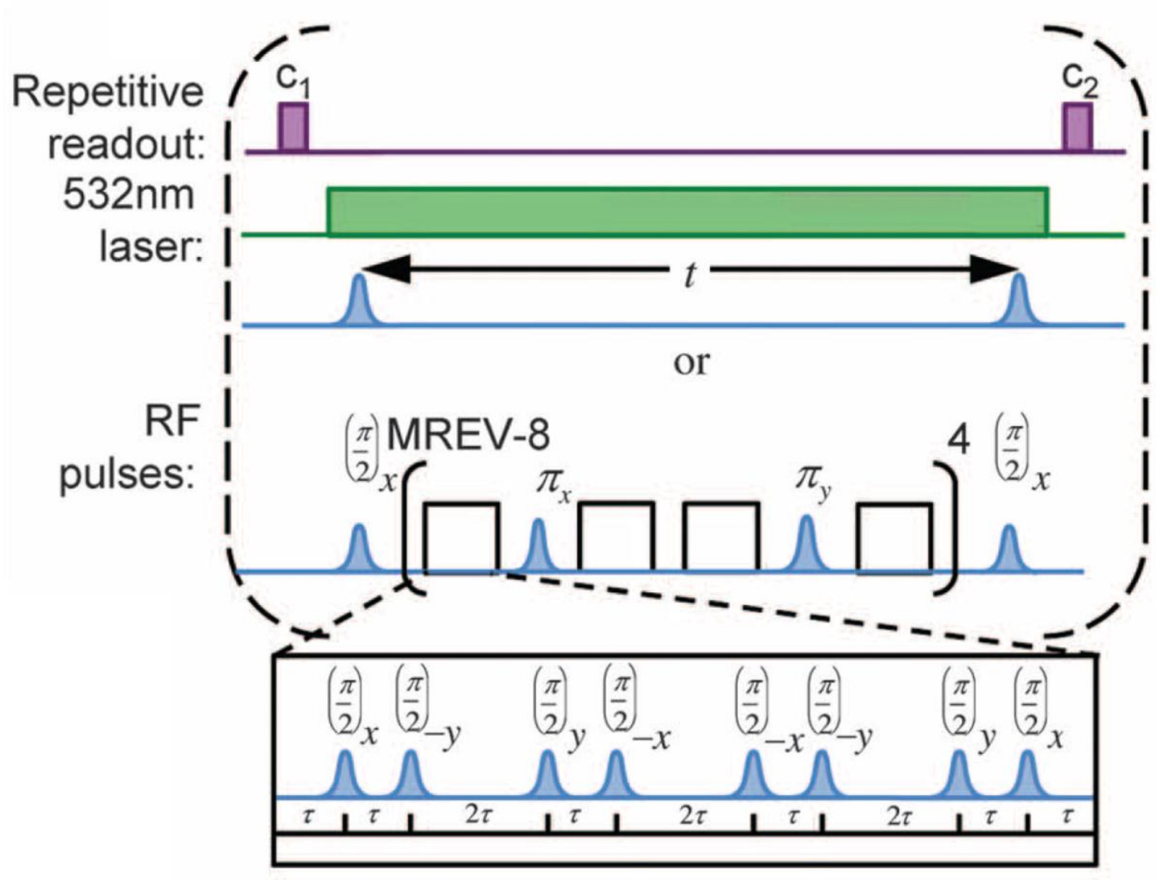
Concatenated dynamical decoupling (CDD)

…

Figures from Buckley/Awschalom
Nature (News & Views) 2009

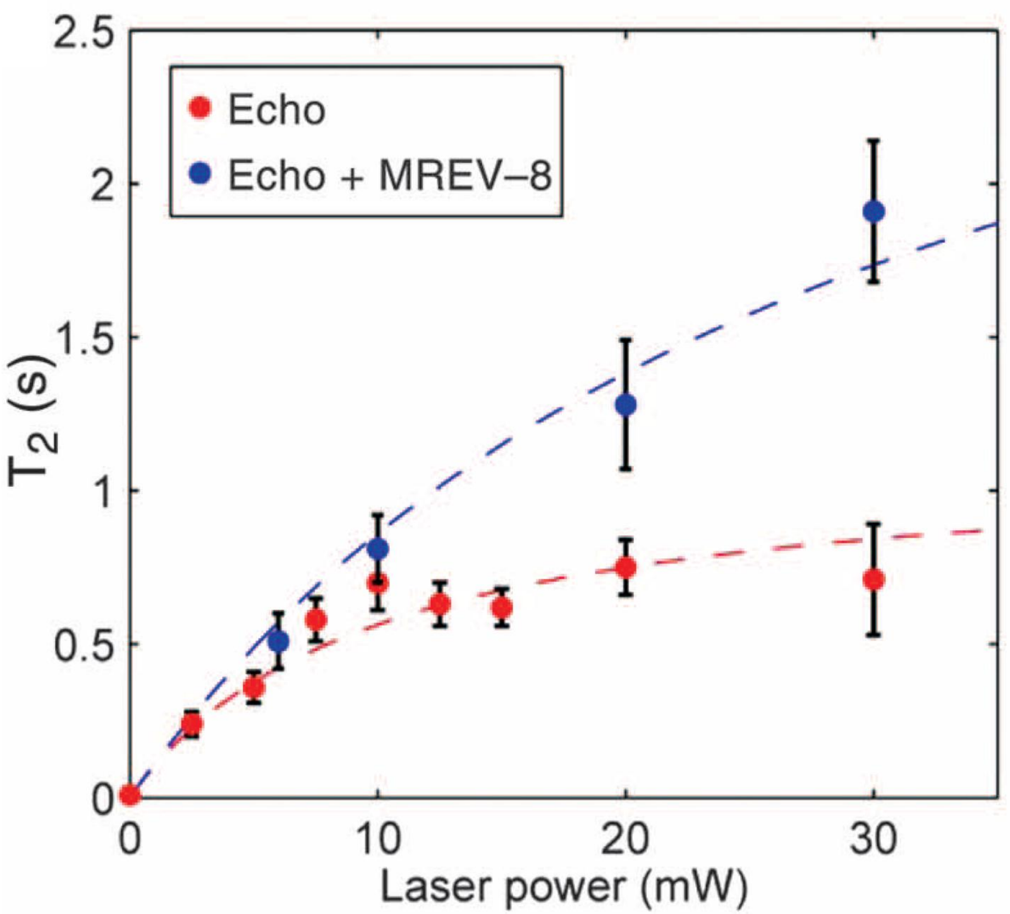
Dynamical Decoupling

In the paper, a **modified Mansfield-Rhim-Elleman-Vaughan (MREV)** decoupling sequence is used



Decoherence with Laser and Decoupling Sequence

For laser illumination PLUS advanced dynamical decoupling sequence, coherence times longer than one second have been measured



Conclusions

- Single ^{13}C nuclear spins near NV centers are candidates for solid state qubits
- The paper demonstrates that they feature very long relaxation times T_1 (many seconds to minutes) and coherence times T_2 (seconds)
- Initialization and readout are possible
- Two-qubit gates and scalability?
  (Maybe via photonic entanglement, ...)
- According to the authors' analysis, further improvements seem clearly possible (T_2 on the order of minutes)