N. Sugimoto and N. Nagaosa:
Spin-orbit echo
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Spintronics vs electronics

unlike charge, the spin is not conserved

The continuity equation

$$\partial_t \rho = -\nabla \cdot \mathbf{j},$$

giving

$$\partial_t Q = 0,$$

is of no use for spin,

$$\partial_t \rho_s = -\nabla \cdot \mathbf{j}_s + \ldots,$$

the spin-orbit coupling being among the dissipation sources.

Spin-orbit coupling in 2DEG is special

it is a non-Abelian gauge field

The semiconductor 2DEG spin-orbit coupling,

$$H = \frac{1}{2m}\mathbf{p}^2 + V(\mathbf{x}) + \frac{\hbar}{2ml_\alpha}(p_x\sigma_y - p_y\sigma_x) + \frac{\hbar}{2ml_\beta}(-p_x\sigma_x + p_y\sigma_y),$$

can be written as

$$H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2 + V(\mathbf{x}),$$

with $\mathbf{A} = \sum_i \mathbf{e}_i A^i_j \sigma_j$, and $[A_\mu, A_\nu] = F_{\mu\nu} \neq 0$.

Alternative - the unitary basis transformation
(spin-orbit interaction “removal”)

consider $U = \exp\left(\mathrm{i}e\mathbf{A}\cdot\mathbf{x}/\hbar\right)$, then

$$U^\dagger H(\mathbf{p},\mathbf{x})U = H(\mathbf{p}+e\mathbf{A},\mathbf{x}),$$

so that you can remove the gauge from the kinetic energy

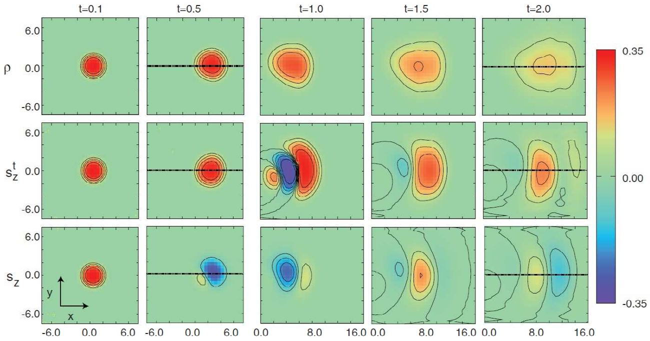
$$H_{\mathrm{eff}} = U^\dagger(H_0 + H_{so})U = H_0 + f(F_{\mu\nu}).$$

This is useless (due to f) except for:

- quasi 0d system where $F_{\mu\nu}$ is “small” ($x, y \ll l_{so}$)
- truly 1d system where $F_{\mu\nu}$ is zero (single vector $\mathbf{e}_i$)
- 2d “compensated” system where $F_{\mu\nu}$ is zero by chance ($l_\alpha = l_\beta$)

Article main result 1: Twisted Spin

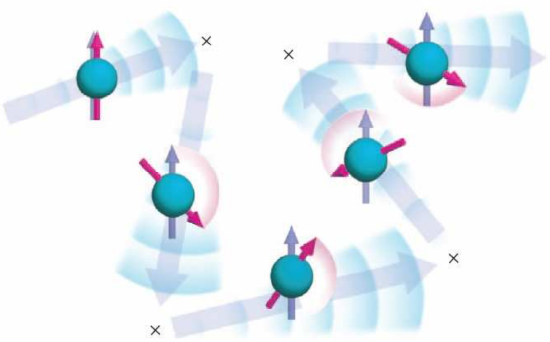
There exists coordinate frame where spin-orbit effects are trivial (=none) for general gauge-like spin-orbit interaction. Going back to a laboratoty frame, there is a conserved quantity.



Article main result 1 $\longrightarrow$

In another words, Dyakonov-Perel relaxation is not really a relaxation (it is reversible, in principle).

A



Special frame / Twisted Spin: $\mathbf{s}^t = (\hbar/2)\Psi^\dagger Y^\dagger \sigma Y \Psi$

$$Y = \lim_{x' \to x} \frac{1}{2} \exp\{\frac{\mathrm{i}}{2} m(\alpha+\beta)\sigma_- x_+\} \times \Big\{ 2\sin^2\left[\frac{\mathrm{i}(\partial_x - \partial_y)}{8\sqrt{2}m(\alpha+\beta)}\right] +$$

$$\exp\{-\frac{\mathrm{i}}{2} m(\alpha-\beta)\sigma_+ x'_-\} \exp\{-\frac{\mathrm{i}}{2}[\frac{\overleftarrow{\partial}_{x'}\sigma_z\partial_y + \overleftarrow{\partial}_{y'}\sigma_z\partial_x}{2m^2(\alpha^2-\beta^2)}]\}$$

$$+\exp\{-\frac{\mathrm{i}}{2}[\frac{\partial_{x'}\sigma_z\partial_y + \partial_{y'}\sigma_z\partial_x}{2m^2(\alpha^2-\beta^2)}]\} \exp\{-\frac{\mathrm{i}}{2} m(\alpha-\beta)\sigma_+ x'_-\} \Big\}$$

properties:

- \- Y depends on electron velocity (reversible in principle, but hopeless in practice)
- \- Y is non-unitary
- \+ twisted spin is really a spin: $U^\dagger_{SU(2)} \mathbf{s} U_{SU(2)} = R_{SO(3)}[\mathbf{s}]$
- \+ persistent helix limit follows as a special case $Y \to \exp(\mathrm{i}e\mathbf{A}\cdot\mathbf{x}/\hbar)$

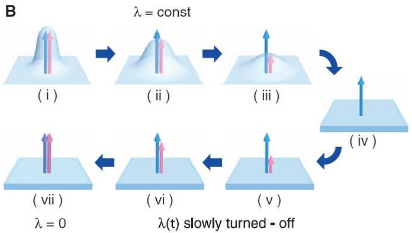
Article main result 2

There is a way to uncover the Twisted Spin: it is an adiabatic invariant.

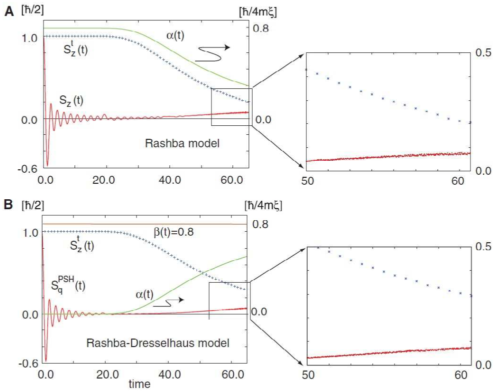
$$\partial_t \mathbf{s}^t = O\{(\partial_t l_{so}^{-1})^{n\geq 2}\}$$

turning off the spin-orbit strength adiabatically, the twisted spin is:

1. conserved,
2. reduced to the “standard” spin



Spin-orbit echo numerical demonstration



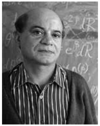
- turning off SOC is not really possible; persistent helix will serve well
- adiabaticity requirement is in odds with inelastic spin dissipation channels (nuclei, EY, BAP, ee, . . .) it is a question of material parameters now

Derivation* (supplementary information): preliminaries

- multiplication of two numbers $\mu : a \otimes b \rightarrow a.b$ ($\sim$3000BC)

- (Emmy) Noether theorem: To every 1-parametric (Sophus) Lie group symmetry of the action there corresponds a conserved quantity (1915AC)

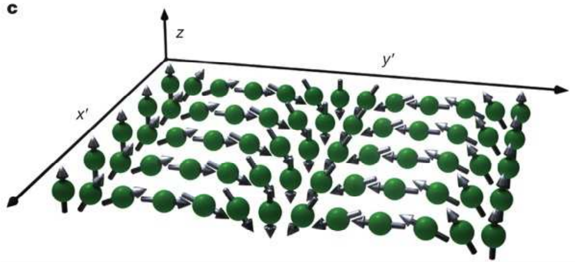
- (Jose Enrique) Moyal star product on a manifold (1949AC)



- (Masud) Chaichian proof of (Hendrik) Lorentz invariance of Quantum Field Theories based on Non-Commutative Geometry(2004AC)

*Fajnšmekers only

Persistent helix



J.D.Koralek et al, Nature **458**, 610 (2009).