Luttinger liquid physics from infinite-system DMRG

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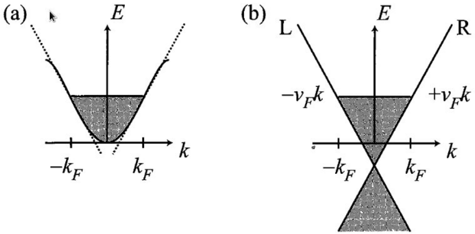
We study one-dimensional spinless fermions at zero and finite temperature T using the density matrix renormalization group. We consider nearest as well as next-nearest neighbor interactions; the latter render the system inaccessible by a Bethe ansatz treatment. Using an infinite-system alogrithm we demonstrate the emergence of Luttinger liquid physics at low energies for a variety of static correlation functions as well as for thermodynamic properties. The characteristic power law suppression of the momentum distribution $n(k)$ function at $T = 0$ can be directly observed over several orders of magnitude. At finite temperature, we show that $n(k)$ obeys a scaling relation. The Luttinger liquid parameter and the renormalized Fermi velocity can be extracted from the density response function, the specific heat, and/or the susceptibility without the need to carry out any finite-size analysis. We illustrate that the energy scale below which Luttinger liquid power laws manifest vanishes as the half-filled system is driven into a gapped phase by large interactions.

Motivation

- Verify Luttinger Liquid (LL) theory for microscopic theory
- What is the energy scale below which LL behavior manifests?
- How can the LL parameters K and v_F be obtained?
- What is the physics away from the zero Temperature limit?
- Avoid finite-size effects.

Luttinger Liquids



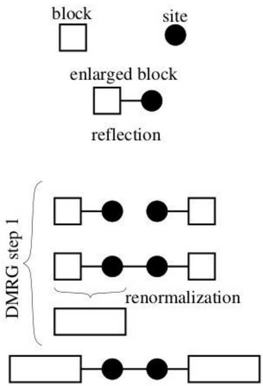
$$
\begin{aligned}
H &= \sum_k \epsilon(k) c_k^\dagger c_k + \frac{1}{L} \sum_{k,k',q} V_q c_k^\dagger c_{k+q} c_{k'}^\dagger c_{k'-q} \\
\longrightarrow H_{LL} &= \frac{v_F}{2\pi} \int \mathrm{d}x \left[K (\partial_x \theta)^2 + \frac{1}{K} (\partial_x \phi)^2 \right]
\end{aligned}
$$

Power laws:

$$
G(x) \sim x^{-\left(\frac{K}{2} + \frac{1}{2K}\right)}, \quad \left| n(k \approx k_F) - 0.5 \right| \sim |k - k_F|^{\left(\frac{K}{2} + \frac{1}{2K} - 1\right)}
$$

Giamarchi (2003)

Infinite-system DMRG



G. De Chiara, M. Rizzi, D. Rossini, S. Montangero (2009)

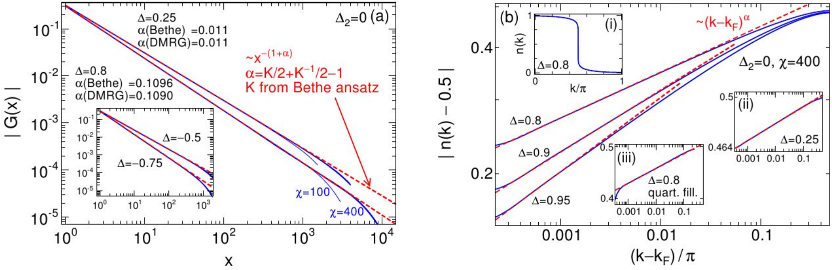
- for 1D lattice models
- start with H for 1 site, add further sites stepwise
- after each step, find density matrix ρ
- diagonalize ρ and neglect redundant DoF
- obtain a new H' with the same dimension but describing more sites
- use H' for the next step

Model of spinless fermions (incl. next-nearest neighb. int.)

$$H = \sum_i \left[\left(-\frac{1}{2} c_i^\dagger c_{i+1} + H.c. \right) + \Delta \left(c_i^\dagger c_i - \frac{1}{2} \right) \left(c_{i+1}^\dagger c_{i+1} - \frac{1}{2} \right) \right.$$
$$\left. + \Delta_2 \left(c_i^\dagger c_i - \frac{1}{2} \right) \left(c_{i+2}^\dagger c_{i+2} - \frac{1}{2} \right) \right]$$

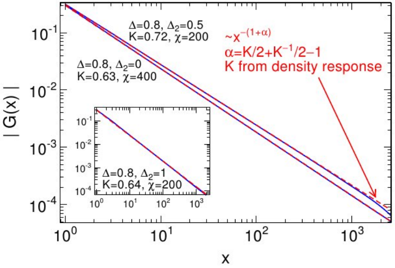
- For $\Delta_2 = 0$ the Bethe ansatz gives an analytic solution and can be used to extract LL parameters K, v_F
- At half filling a gap opens for $\Delta > 1$. Away from half filling or for $|\Delta| < 1$: LL
- Finite Δ_2: no exact solution known. LL for small Δ_2
- expectation values:
  $G(x) = \langle c_{j+x}^\dagger c_j \rangle_T, \quad n(k) = \sum_x e^{-ikx} G(x)$

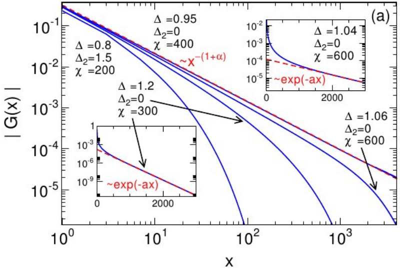
$T = 0$, $\Delta_2 = 0$

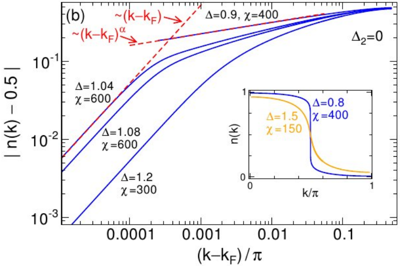


- χ - size of DMRG Hilbert space
- $\alpha = \frac{K}{2} + \frac{1}{2K} - 1$
- everything for half filled system, zero temperature

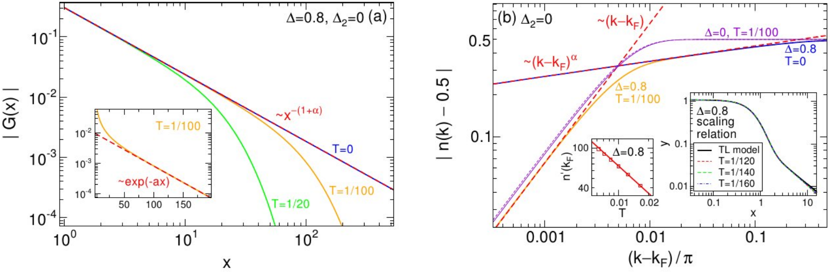
$\Delta_2 \neq 0$, Gapped phase





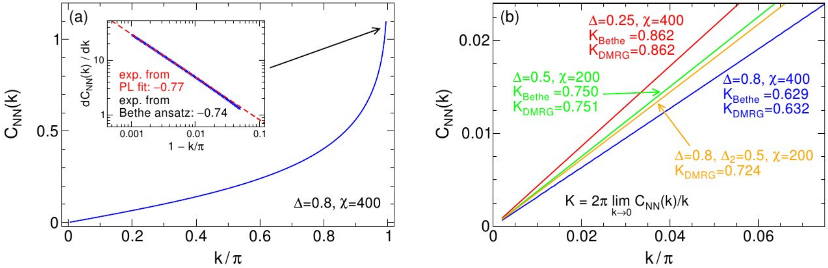


Finite temperature



- T cuts off LL power laws exponentially in the infrared
- Power laws in temperature appear: $\frac{dn(k)}{dk}|_{k_F} \sim T^{\alpha-1}$
- Analytical results yield $\frac{dn(k)}{dk} \sim T^{\alpha-1} F\left[\frac{v_F(k-k_F)}{\pi T}\right]$
- Strong indication that low-energy $n(k, T)$ behavior universal for models in the LL universality class

Instantaneous density response function, extraction of LL parameters



- Instantaneous density response function: $C_{NN}(x) = \langle c^\dagger_{j+x} c_{j+x} c^\dagger_j c_j \rangle_T$
- Calculate the specific heat and susceptibility ($\sim C_{NN}$) in DMRG to extract the LL parameters v_F and K