Adiabatic State Preparation of Interacting Two-Level Systems

R. T. Brierley, C. Creatore, P. B. Littlewood, and P. R. Eastham, Phys. Rev. Lett. 109, 043002

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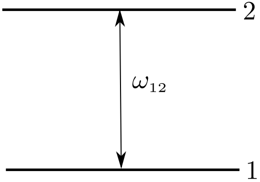
August 7, 2012

Adiabatic rapid passage (ARP)

problem: population transfer in a two-level (n-level) system



"old-school" approach: π-pulse

not robust: sensitive to pulse area, inhomogeneities, etc.

ARP idea: sweep ("chirp") through the resonance!

$\Omega(t) = \mu E(t)$ (Rabi frequency) $\quad \Delta\omega(t) \equiv \omega(t) - \omega_{12}$ (detuning)

$\theta(t) \equiv \tan^{-1}[\Delta\omega(t)/\Omega(t)]$ (phase angle)

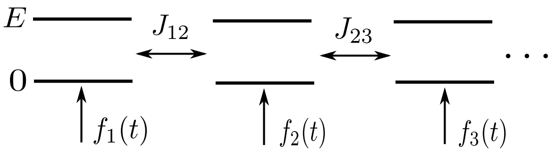
the adiabaticity condition $\boxed{\sqrt{\Omega^2(t) + \Delta\omega^2(t)} \gg |d\theta(t)/dt|}$

guarantees 100% population transfer to the desired excited state!

[regardless of the concrete shape of $E(t)$ and $\omega(t)$!]

What is this paper about?

ARP applied to a collection of interacting two-level systems



with a generic Hamiltonian:

$$\boldsymbol{H = \sum_{i=1}^{N}\left[\frac{E}{2}(\sigma_i^z + 1) + (f_i(t)\sigma_i^+ + \mathrm{H.c.})\right] - \sum_{i,j} J_{ij}\sigma_i^+\sigma_j^-}$$

Goals of this paper:

demonstrate the feasibility of state preparation via ARP;

find the dependence of the pulse shapes of $\boldsymbol{f(t)}$ on the interaction strength

Outcome: ARP efficient here if the pulse bandwidth is sufficient

Chirped pulse and Hamiltonian

decomposition of the driving field into amplitude and frequency:

$$f_i(t) = g_i(t) \exp(i \int \omega(t')dt')$$

eliminating the instantaneous frequency:

$$H = \sum_{i=1}^{N} \left[\frac{E - \omega(t)}{2} (\sigma_i^z + 1) + (g_i(t)\sigma_i^+ + \mathrm{H.c.}) \right] - \sum_{i,j} J_{ij} \sigma_i^+ \sigma_j^-$$

Gaussian, linearly chirped pulse with uniform amplitude:

$g_i(t) = g \exp(-t^2/\tau^2)$, $\omega(t) = E + \alpha t$

α – (linear) chirp

in the special case $\alpha = 0$ one recovers a Rabi pulse with frequency E

Recap: the non-interacting ($J = 0$) case

$$H_{J=0} = \sum_{i=1}^{N} \left[\frac{E - \omega(t)}{2} (\sigma_i^z + 1) + (g_i(t)\sigma_i^+ + \mathrm{H.c.}) \right]$$

$g(t) = 0$: level crossing for $E - \omega(t) = 0$

$g(t) = g$: standard Landau-Zener problem

L-Z: the probability to remain in the adiabatic state (i.e., transfer from the initial ground state to the excited state) is $1 - \exp(-2\pi g^2/\alpha)$

$\Longrightarrow$ final population always increases with reducing α!

ARP: pulses of finite duration, i.e., $g(t) \neq \mathrm{const.}$

the two levels must be coupled long enough $\Rightarrow$ $\alpha \gg 1/\tau^2$

Special case: $1d$ model with n.n. coupling only

coupling $J_{i,j} = J\delta_{j,i+1} \Rightarrow$ JW and F.T. yield

$$H = -\sum_k (\alpha t + J\cos k) c_k^\dagger c_k + \frac{1}{\sqrt{N}} \sum_{k,i} (g_i^* T_i c_k e^{ikr_i} + \mathrm{H.c.})$$

Reminder: the Jordan-Wigner (JW) transformation

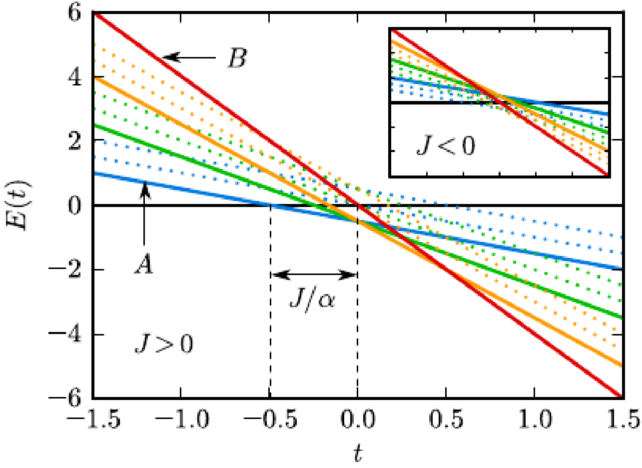
$$\sigma_1^- = c_1 \quad , \quad \sigma_i^- = \underbrace{\exp[i\pi \sum_{l<i} c_l^\dagger c_l]}_{T_i} \, c_i \;\; (i \geq 2)$$

maps H onto a Hamiltonian for free fermions with n.n. hopping and an on-site potential ($\sigma_i^z = 2c_i^\dagger c_i - 1$)

collective pseudospin: $S \equiv \sum_i \sigma_i/2$ $\quad n = S_z + N/2$

for $J = 0$ all the states in the n-th band have energy $-n\alpha t$!

Energy spectrum (for $g = 0, J = 1/2, N = 4$)



interaction ($J \neq 0$) lifts the degeneracy within each band!

different states correspond to different values of $\mathbf{S^2}$ for given S_z!

Possible transitions

To prepare an excited state, e.g., the fully occupied one ($n = N$) we have to go through multiple crossings from $n = 0$ to $n = N$!

...for these crossings to allow adiabatic state transfer, we need them to be avoided (the role of $g \neq 0$)!

simplifying assumption: uniform driving $g_i(t) = g(t) \Rightarrow$ coupling $g(t)\sum_i(\sigma_i^+ + \sigma_i^-) = g(t)(S^+ + S^-)$ commutes with $\mathbf{S^2}$!

$\Rightarrow$ transitions possible only between states with the same $\mathbf{S^2}$ in different bands!

$S^+,\ S^- \Rightarrow$ only $n \to n \pm 1$ direct transitions possible!

Mean-field: Lipkin-Meshkov-Glick Hamiltonian in higher dim.

mean-field replacement: $\sum_{i,j} \sigma_i^+ \sigma_j^- \Rightarrow \sum_i (J_{\text{eff}}\, \sigma_i^+ \langle \sigma_i^- \rangle + \text{H.c.})$

effective mean-field Hamiltonian:

$$H_{\text{MF}} = -J_{\text{eff}}(S^+ S^- + S^- S^+) - \frac{\alpha t}{2} S^z + 2g S^x$$

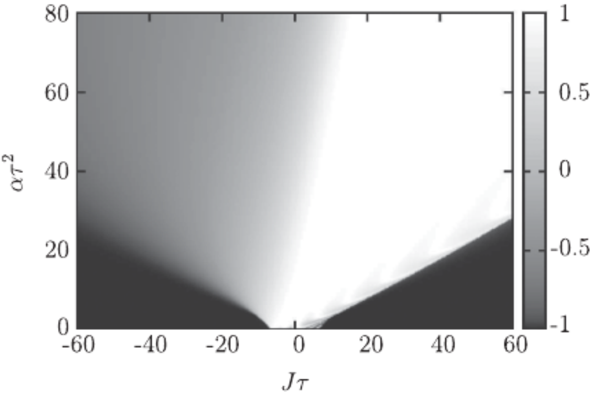
$$H_{\text{MF}} = 2J_{\text{eff}}(S^z)^2 - \frac{\alpha t}{2} S^z + 2g(t) S^x$$

for fixed α occupation increases for $J > 0$ because of icreased separation between crossings, which increases the individual splitings!

due to finite pulse duration, occupation should be very small when the spacing between crossings ($\approx J/\alpha$) becomes larger than the pulse width (τ)!

Average excitation of two-level systems (mean field)

$g\tau = 3$



for fixed α, the occupation increases (decreases) for $J > 0$ ($J < 0$)