Weak Measurements with Orbital-Angular-Momentum Pointer States

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Weak measurements are a unique tool for accessing information about weakly interacting quantum systems with minimal back action. Joint weak measurements of single-particle operators with pointer states characterized by a two-dimensional Gaussian distribution can provide, in turn, key information about quantum correlations that can be relevant for quantum information applications. Here we demonstrate that by employing two-dimensional pointer states endowed with orbital angular momentum (OAM), it is possible to extract weak values of the higher order moments of single-particle operators, an inaccessible quantity with Gaussian pointer states only. We provide a specific example that illustrates the advantages of our method both in terms of signal enhancement and information retrieval.

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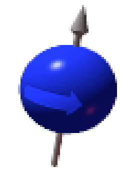
Measurement: weak vs strong

"Meter"

$$H = -g(t) P_x A$$

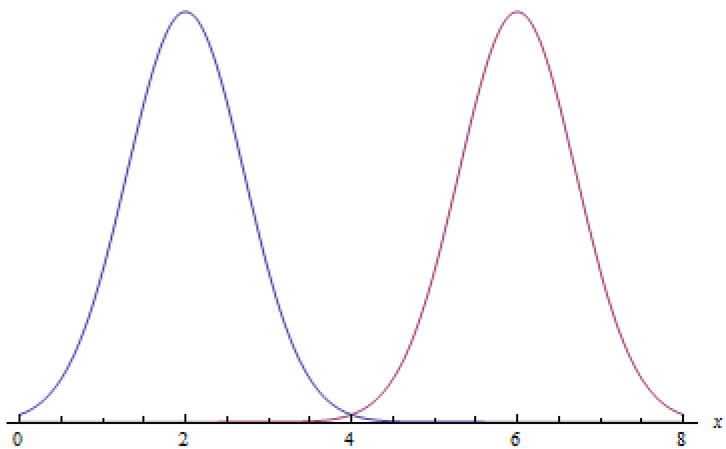
$$U = e^{i P_x A}$$

Measurement Hamiltonian

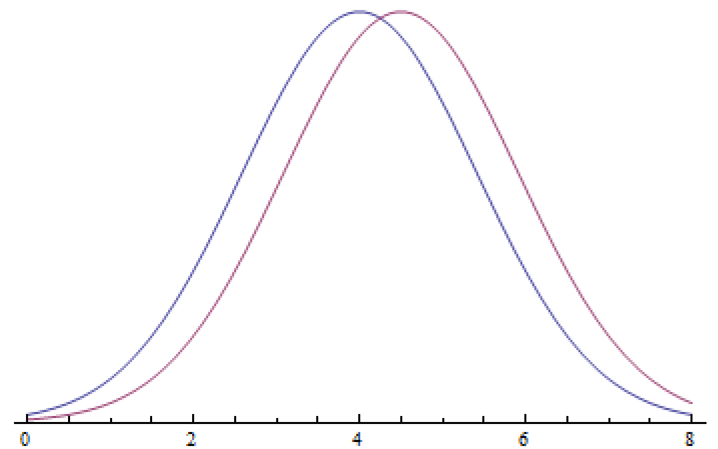


"System"

Strong



Weak



Outcomes are distinguishable in a single measurement. The system is projected onto eigenstates of A: "collapse".

Several runs are necessary to extract information on the system. Weak back-action on the system.

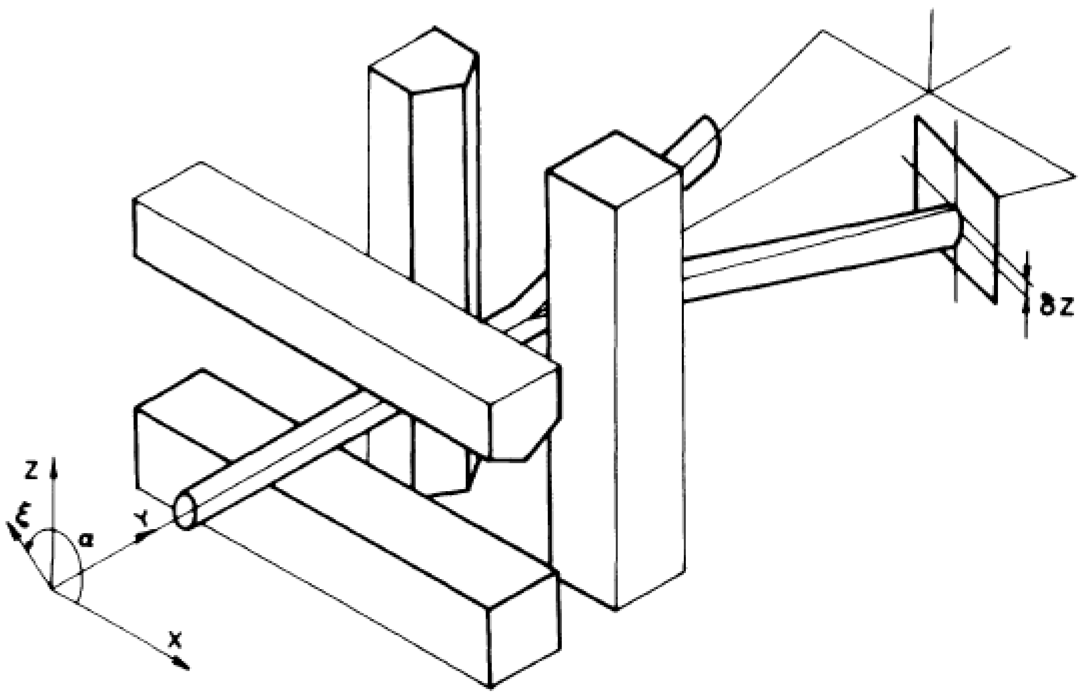
The weak value (WV)

1) Prepare the system in the state $|\psi_i\rangle$ and the meter in the state $|\phi\rangle$

2) Let them interact

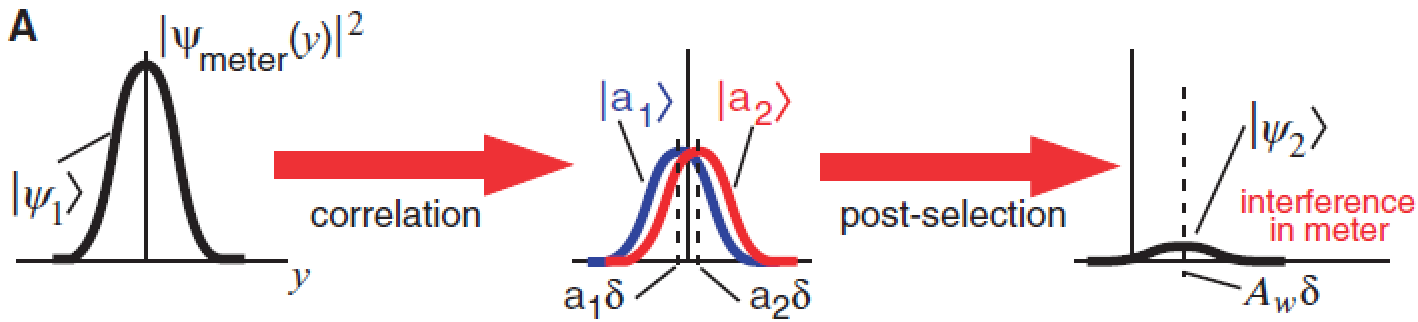
3) Post-selection: keep the result of the meter only when the system is in the final state $|\psi_f\rangle$

$$\langle x\rangle = \frac{\int dx\ x \left|\langle x|\,\langle\psi_f|\,e^{igP_xA}\,|\psi_i\rangle\,|\phi\rangle\right|^2}{\int dx\ \left|\langle x|\,\langle\psi_f|\,e^{igP_xA}\,|\psi_i\rangle\,|\phi\rangle\right|^2} \approx \mathrm{Re}\left(\frac{\langle\psi_f|\,A\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}\right) \equiv \mathrm{Re}\left(\langle A\rangle_w\right)$$



Aharonov, Albert, Vaidman, PRL **60** 1351 (1988)

The weak value (WV)

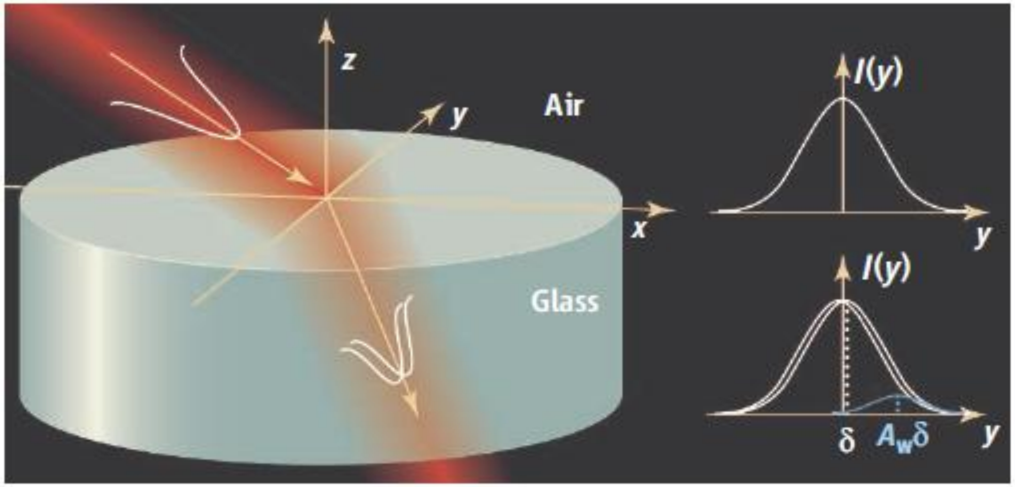


$$\frac{\langle\psi_f|\, A\, |\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \equiv \langle A\rangle_w$$

- *Actual* value read on the meter in weak-measurement experiments with post-selection
- Complex quantity
- “Strange” values: not bounded by the spectrum of A

“Weak-value amplification” : large WVs allow for an amplification of small effects

Applications



Spin Hall Effect of Light, 10’000 fold amplification of a displacement of 1 Å. Science **319**, 787 (2008)

- Deflection of light by a mirror to a precision of 10^{-13} rad. PRL **102**, 173601 (2009)
- Direct measurement of a single-photon wavefunction. Nature **474**, 188 (2011)
- Theoretical proof-of-principle in solid-state devices: charge sensing. PRL **106**, 080405 (2011)

Motivation of the paper

- Strong/conventional measurement of quantities such as $\langle AB \rangle, \langle A^2 \rangle, \ldots$ require non-linear Hamiltonians ➔ hard to implement in practice

- In 2004, Resch and Steinberg (PRL **92**, 043601) figured out a way to obtain $\langle AB \rangle_w$ using a two-dimensional “meter” and the interaction Hamiltonian in the case $[A, B] = 0$

$$H = g_A A P_x + g_B B P_y$$

$\langle AB \rangle_w$ essentially appears in the joint average value $\langle XY \rangle$ of the meter.

- The goal is to extend the approach, and find a way to measure more of those quantities

The idea

Key idea: use a nonfactorizable initial meter wavefunction

$$\phi(x,y) = N\,(x + i\,\mathrm{sgn}(l)\,y)^{|l|}\exp\left(-\frac{x^2+y^2}{4\sigma^2}\right)$$

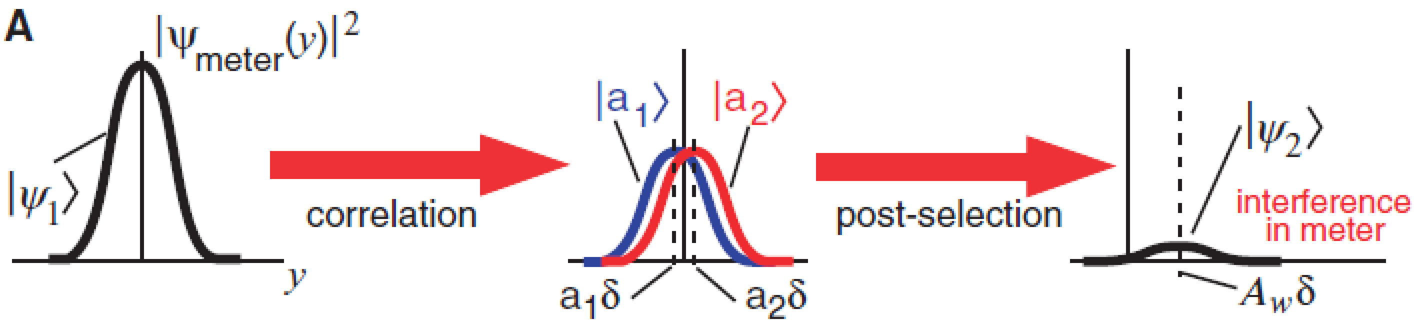
(Laguerre-Gauss modes)

l: orbital angular momentum of the photons
σ: uncertainty of the pointer state

- $l=0$ corresponds to the (factorizable) Gaussian case.
- Experimentally accessible (at least for l up to 3, maybe more)

Results

1) $\phi(x,y) = N\,(x + i\,\mathrm{sgn}(l)\,y)^{|l|}\exp\left(-\frac{x^2+y^2}{4\sigma^2}\right)$



2) $H = g_A A P_x + g_B B P_y$

3) $O_{XY} = |\psi_f\rangle\langle\psi_f|\,XY$

$$\langle XY\rangle = \frac{g_A g_B t^2}{2}\left[\mathrm{Re}(\langle AB\rangle_w) + \mathrm{Re}(\langle A\rangle_w^* \langle B\rangle_w)] + l\,\frac{t^2}{2}[g_A^2 \mathrm{Im}(\langle A^2\rangle_w) + g_B^2 \mathrm{Im}(\langle B^2\rangle_w)\right]$$

Results

• For example, choose $B = 0$ ➔ $\langle XY \rangle = l \, \frac{(g_A t)^2}{2} \mathrm{Im}(\langle A^2 \rangle_w)$

Although Y does not interact with the system in this case, its value is still modified due to the nonfactorizability of the initial meter wavefunction.

• Possible to obtain all the second order joint weak-values.

• By measuring the conjugate variables of the meter, also possible to obtain both real and imaginary parts

• There might be some cases where LG modes perform better than Gaussian modes for use in the weak-value amplification scheme

• They say it is possible to obtain even higher-order weak values by appropriate combinations of such LG modes with high ls but it may be fishy…

Conclusion

- Interesting idea for adding complexity to the initial state, while staying realistic
- Should work well to measure at least second-order weak-values
- But: in the context of optics, where A, B are essentially Pauli matrices, nobody is interested in A^2 or B^2…