Odd-frequency superconducting pairing in topological insulators

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Layout

Odd-frequency superconducting pairing in topological insulators

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We discuss the appearance of odd-frequency spin-triplet s-wave superconductivity, first proposed by Berezinskii [*JETP* **20**, 287 (1974)], on the surface of a topological insulator proximity coupled to a conventional spin-singlet s-wave superconductor. Using both analytical and numerical methods we show that this disorder robust odd-frequency state is present whenever there is an in-surface gradient in the proximity induced gap, including superconductor-normal state (SN) junctions. The time-independent order parameter for the odd-frequency superconductor is proportional to the in-surface gap gradient. The induced odd-frequency component does not produce any low-energy states.

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Anti-Symmetry of the Cooper pair wavefunction under exchange
$\psi(\mathbf{r}, s; \mathbf{r}', s') = f(\mathbf{r} - \mathbf{r}')\chi(s, s')$
$\Rightarrow$ Possibilities

$$\begin{cases} f(-r) = f(r), \quad \text{and} \quad \chi_{s,s'} = -\chi_{s,s'} \quad l = 0, 2, 4 \ldots, \quad \text{and} \quad S = 0 \\ f(-r) = -f(r), \quad \text{and} \quad \chi_{s,s'} = \chi_{s,s'} \quad l = 1, 3, 5 \ldots, \quad \text{and} \quad S = 1 \end{cases}$$

- What about exchange of time coordinates ?
- Conventional superconducting states correspond to the even-frequency pairing
- Possible to have odd-frequency pairing

V. L. Berezinskii, JETP **20**, 287 (1974).

A. Balatsky and E. Abrahams, Phys. Rev. B **45**, 13125 (1992).

Class	Time	Spin	Orbital	Total
ESE	$+$	$-$	$+$	$-$
ETO	$+$	$+$	$-$	$-$
OTE	$-$	$+$	$+$	$-$
OSO	$-$	$-$	$-$	$-$

Table: The symmetry of pair amplitude with respect to the exchange of spins, spatial coordinates, and time variables for possible four classes. ESE, ETO, OTE and OSO denote even-frequency spin-singlet even-parity, even-frequency spin-triplet odd-parity, odd-frequency spin-triplet even-parity, and odd-frequency spin-singlet odd-parity.

- $\mathbf{r_1}$ and $\mathbf{r_2} \rightarrow \mathbf{R} = \frac{1}{2}(\mathbf{r_1} + \mathbf{r_2})$ and $\mathbf{r} = (\mathbf{r_1} - \mathbf{r_2})$
- Parity of Cooper pair defined with respect to $\mathbf{r} \rightarrow -\mathbf{r}$; $\mathbf{r} \rightarrow \mathbf{k}$
- Inhomogeneities break translational symmetry $\Rightarrow$ Parity is no longer a good quantum number
- For SN junctions with spin rotational symmetry, near the interface ($\sim \xi$) mixed parity states can be realized
  ESE $\rightarrow$ ESE+OSO, and ETO $\rightarrow$ ETO+OTE
- For homogeneous system with broken spin rotational symmetry, singlet-triplet pairings are mixed
  ESE $\rightarrow$ ESE+OTE, and ETO $\rightarrow$ ETO+OSO
- What about pairing symmetry in F/S junctions ? All 4 possible ESE $\rightarrow$ ESE+OSO+OTE+ETO,
- ESE induces OTE in TI SN junction, if $\frac{\partial \Delta}{\partial x} \neq 0$

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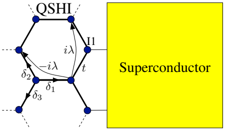
$$\mathcal{H} = H_{KM} + H_{\Delta}$$
$$H_{KM} = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + \mu \sum_i c_i^\dagger c_i + i\lambda \sum_{\langle\langle i,j\rangle\rangle} \nu_{ij} c_i^\dagger \sigma^z c_j$$



- $\nu_{ij} = \pm 1$ left (right) turn to get to the second bond
- $H_\Delta = -\sum_i U_i c_{i\downarrow} c_{i\uparrow} c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$
- Self-consistently for mean field s-wave $\Delta(i) = -U_i \langle c_{i\downarrow} c_{i\uparrow}\rangle$
- $F_t^0(\tau|i) = \langle c_{i\uparrow}(\tau)c_{i\downarrow}(0) + c_{i\downarrow}(\tau)c_{i\uparrow}(0)\rangle/2$
- $F_t^1(\tau|i) = \langle c_{i\uparrow}(\tau)c_{i\uparrow}(0) - c_{i\downarrow}(\tau)c_{i\downarrow}(0)\rangle/2$
- $F_s(\tau|i) = (\langle c_{i\downarrow}(\tau)c_{i\uparrow}(0) - c_{i\uparrow}(\tau)c_{i\downarrow}(0)\rangle)/2$
- $F_t(\tau = 0) = 0$, $\Delta_i = -U_i F_s(\tau = 0|i)$
- $\partial_\tau F_t\,|_0 \equiv$ odd frequency order parameter

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).

Results

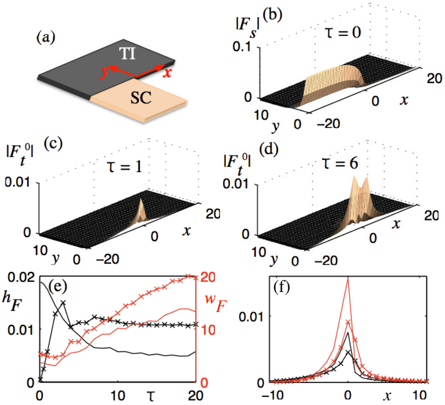


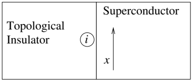
Figure: (a) SN junction along the edge of a 2D TI. (b) Even-frequency pairing $|F_s(0)|$, (c) odd-frequency pairing $|F_t^0(1)|$, and (d) $|F_t^0(6)|$ in a 2D TI SN junction with $\mu_{\rm S} = 0.3$, $\mu_{\rm N} = 0$, and $U = 2.2$, giving $\Delta_{\rm S} = 0.16$. (e) Maximum height h_F (black, left axis) and average width w_F (red, right axis) as function of τ for $|F_t^0|$ (crosses) and $|\partial_x F_s|$ (line). (f) Magnitude of odd-frequency order parameter $\partial_\tau F_t^0 |_0$ (crosses) and $\frac{\partial \Delta}{\partial x}$ scaled by a factor of 0.4 (line) for $\Delta_{\rm S} = 0.16$ (black) and $\Delta_{\rm S} = 0.30$ (red).

$$H_{\mathrm{TI}} = \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\alpha,\mathbf{k}} \mathbf{k} \cdot \sigma_{\alpha\beta} c_{\beta,\mathbf{k}}$$

$$H_{\mathrm{SC}} = \sum_{\mathbf{k},\alpha,\beta} \varepsilon(\mathbf{k}) d^{\dagger}_{\alpha,\mathbf{k}} d_{\alpha,\mathbf{k}} + \sum_{i,\alpha,\beta} \Delta(i)_{\alpha\beta} d^{\dagger}_{\alpha,i} d^{\dagger}_{\beta,i} + \mathrm{H.c.}$$

$$H_T = \sum_{\alpha} T_i c^{\dagger}_{\alpha,i} d_{\alpha,i} + \mathrm{H.c.}.$$

- $\Delta(i) = \Delta_0 + (ia)\frac{\partial \Delta}{\partial x}|_0$
- $F_{\mathrm{TI},\alpha\beta}(\mathbf{k},\mathbf{k}') = -\imath \langle T_\tau c_\alpha(\tau,\mathbf{k}) c_\beta(0,\mathbf{k}') \rangle$
- $\hat{F}_{\mathrm{TI}}(\omega_n|i,i) = -|T|^2 \sum_{j,l} \hat{G}^0(\omega_n|i,j)\hat{F}(j,l)\hat{G}^0(\omega_n|l,i)$



- $\hat{F}_{\mathrm{TI}}(\omega_n|i,i) = -|T|^2 \sum_{j,l} \hat{G}^0(\omega_n|i,j)\hat{F}(j,l)\hat{G}^0(\omega_n|l,i)$
- $\hat{G}^0(\omega|\mathbf{k}) = (\mathbf{k}\cdot\sigma - i\omega_n)/(\mathbf{k}^2+\omega_n^2)$
- $\hat{F}(\omega_n,\mathbf{k}) = \hat{\Delta}(i)[\omega_n^2+\varepsilon^2(\mathbf{k})+\hat{\Delta}(i)^2]^{-1}$
- $\Delta(\mathbf{k}) = \Delta_0\delta_{\mathbf{k},0} + \frac{\partial\Delta}{\partial x}|_0 \imath\partial_{\mathbf{k}_x}$

$$\hat{F}_{\mathrm{TI}}(\omega_n|i=0) = \sum_{\mathbf{k}} \frac{-\imath|T|^2\partial_x\hat{\Delta}|_0\hat{G}^0(\omega_n|\mathbf{k})\partial_{\mathbf{k}_x}\hat{G}^0(\omega_n|-\mathbf{k})}{2[\omega_n^2+\varepsilon^2(\mathbf{k})+\Delta^2(0)]}$$
$$= \sum_{\mathbf{k}} \frac{|T|^2\omega_n\hat{\sigma}\partial_x\hat{\Delta}|_0}{2[\omega_n^2+\varepsilon(\mathbf{k})^2+\Delta^2(0)](\omega_n^2+\mathbf{k}^2)^2}.$$

$$\partial_\tau\hat{F}_{\mathrm{TI}}(\tau|i)|_0 \sim \sum_n |T|^2\sigma^z\frac{\omega_n^2}{|\omega_n|^2}\frac{\partial\Delta}{\partial x} \sim \frac{\partial\Delta}{\partial x}.$$

Adding Rashba term

Add $H_R = i\lambda_R \sum_{\langle i,j\rangle} c_i^\dagger (\mathbf{s} \times \hat{\mathbf{d}}_{ij})_z c_i$; $F_t^{\pm 1} = \langle c_{i,\pm\sigma}(\tau) c_{i,\pm\sigma}(0) \rangle$

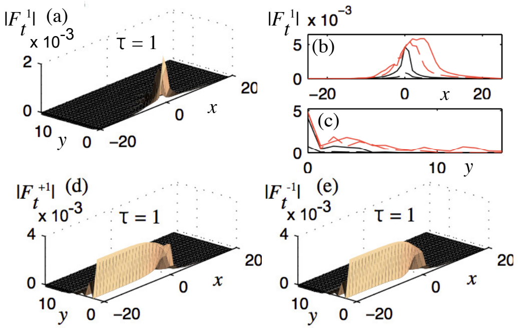


Figure: (a) Odd-frequency pairing $|F_t^1(1)|$ in the SN junction in Fig. 1 with $\lambda_R = 2\lambda$. (b) $|F_t^1|$ along the TI edge and (c) at the SN interface into the TI for $\tau = 0.5$ (dashed black), 2 (black), 8 (dashed red), and 16 (red). (d) Spin components $|F_t^{+1}(1)|$ and (e) $|F_t^{-1}(1)|$ of $F_t^1(1) = (F_t^{+1} - F_t^{-1})/2$.

Layout

- Odd-frequency spin-triplet s-wave pairing in a TI in *zero* magnetic field via proximity effect
- s-wave nature of this pairing guarantees robustness against disorder
- Time independent order parameter for this pairing $\propto$ in-plane gradient of the induced s-wave gap
- Experiments: local tunneling probes, or impact of spin triplet component in NMR

Thank you for your attention