Journal Club 4th September 2012

James Wootton

3D quantum stabilizer codes with a power law energy barrier

Kamil Michnicki

quant-ph/1208.3649

Abstract

We introduce a new primitive, called welding, for combining two stabilizer codes to produce a new stabilizer code. We apply welding to construct surface codes and then use the surface codes to construct solid codes, a variant of a 3-d toric code with rough and smooth boundaries. Finally, we weld solid codes together to produce a $(O(L^3), 1, O(L^{\frac{4}{3}}))$ stabilizer code with an energy barrier of $O(L^{\frac{2}{3}})$, which solves an open problem of whether a power law energy barrier is possible for local stabilizer code Hamiltonians in three-dimensions. The previous highest energy barrier is $O(\log L)$. Previous no-go results are avoided by breaking translation invariance.

Self-correcting quantum memories

- Hamiltonian with degenerate ground state space
- Logical qubit(s) stored in ground state
- Large operators required to map between logical states
- Spin-by-spin performance of logical operators is energetically penalized
- Typically done by energy barrier (minimum over all possible sequences of operators of maximum energy required)
- Ideally, this will increase with system size
- Size of a D-dimensional system with N spins is measured by a linear size $L = O(\sqrt[D]{N})$

Self-correcting quantum memories

- Classical example 1: 1D Ising model
  - Degenerate GS: All up and all down
  - Can be used to encode a bit (all up for 0, all down for 1)
  - Every spin must be flipped to map 0 to 1 or 1 to 0
  - Minimum energy sequence requires flips to be neighbouring
  - Maximum energy cost is that for a domain wall: O(1)
  - Nature easily performs errors and decoheres the memory in O(1) time

- Classical example 2: 2D Ising model
  - Degeneracy and encoding: As for 1D
  - Minimum energy sequence: expand cluster of flipped spins
  - Energy cost = sqrt(# flipped spins)
  - Maximum energy cost = O(L): increases with system size
  - Errors are suppressed, allowing lifetime O(exp[L^2]) below Curie temp. and O(poly L) above [Day, Barrett 2012]

Self-correcting quantum memories

- 2D Ising model provides energy barrier against Z flips but not X flips (or vice-versa)
- Only a good classical memory
- For quantum self-correction, there needs to be an energy barrier for both X and Z flips that increases with L. Is this possible?
- Restricting to stabilizer Hamiltonians:
  - Not possible in 2D [Alicki et al., 2008]
  - Model in 3D known with O(L) barrier for X flips bit O(1) for Z [Castelnovo, Chamon 2007]
  - Model in 3D known with O(log L) energy barrier [Bravyi and Haah, 2011]
  - Model in 4D known with O(L) energy barrier [Alicki et al., 2008]
- This paper constructs the first 3D stabilizer Hamiltonian with power law energy barrier

Code Welding

- Consider Hamiltonians based on CSS codes

- Terms are either tensor products of X or tensor products of Z, and all terms commute

- Codes are characterized by the set of all terms + logical operators

  1D Ising model: {ZZII, IZZI, IIZZ, XXXX}

- To perform an X-type weld of two codes
  - Identify spins on which both codes act
  - Incorporate all Z-type elements from original codes into new code
  - Identify all products of X's that commute with the set of Z elements, and incorporate into new code

    - Example: Weld S1 = {ZZ,XX} and S2 = {ZZ,XX} to form S3

  - Identify second spin of S1 with first of S2: S1 = {ZZI,XXI}, S2 = {IZZ,IXX}
  - Take Z-type elements: S3 = {ZZ1,IZZ,...}
  - Add X type elements that commute S3 = {ZZ1,IZZ,XXX}

- For Z-type weld, interchange X and Z

Solid Codes

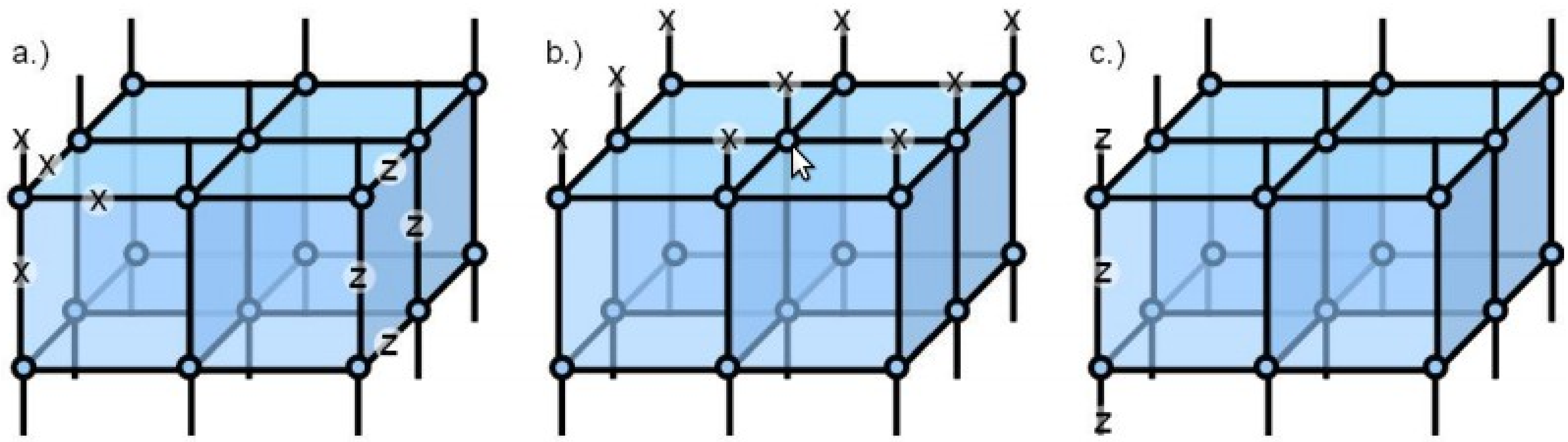

Figure 8: The graph of a small solid code with qubits on the edges. a.) An X-star operator and a Z-plaquette operator are shown. b.) The $\bar{X}$-membrane operator is shown. c.)The $\bar{Z}$-string operator is shown.

- 3D generalization of planar code
- We'll use d to denote linear system size (rather than L, as usual)
- Logical X operator is a membrane, so energy barrier is O(d)
- Logical Z operator is a string, so energy barrier is O(1) for a single domain wall
- Self-correcting classical memory, but not self-correcting quantum memory

Welded Solid Codes

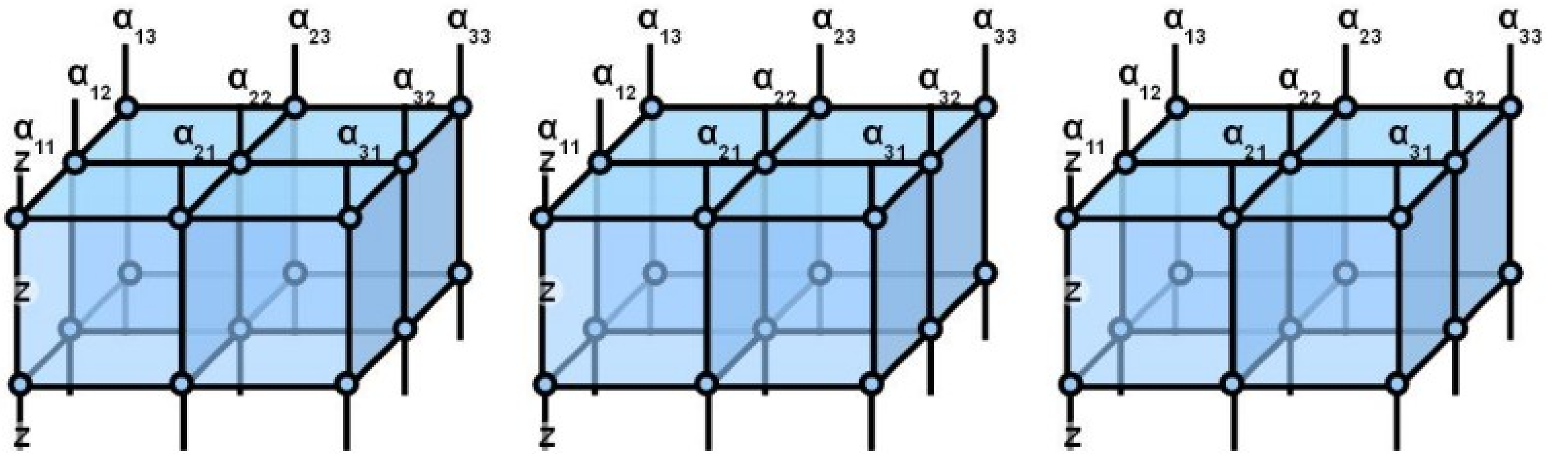

Figure 11: Three small solid codes with a Z-type weld between their upper rough boundary. The α_{ij} symbols label identical qubits.

- Let's weld three of these codes together with Z-type weld on upper boundary
- Result: Logical Z's join together, become Y-shaped rather than a string
- Energy barrier of logical Z increases from that of one domain wall, to two

Welded Solid Codes

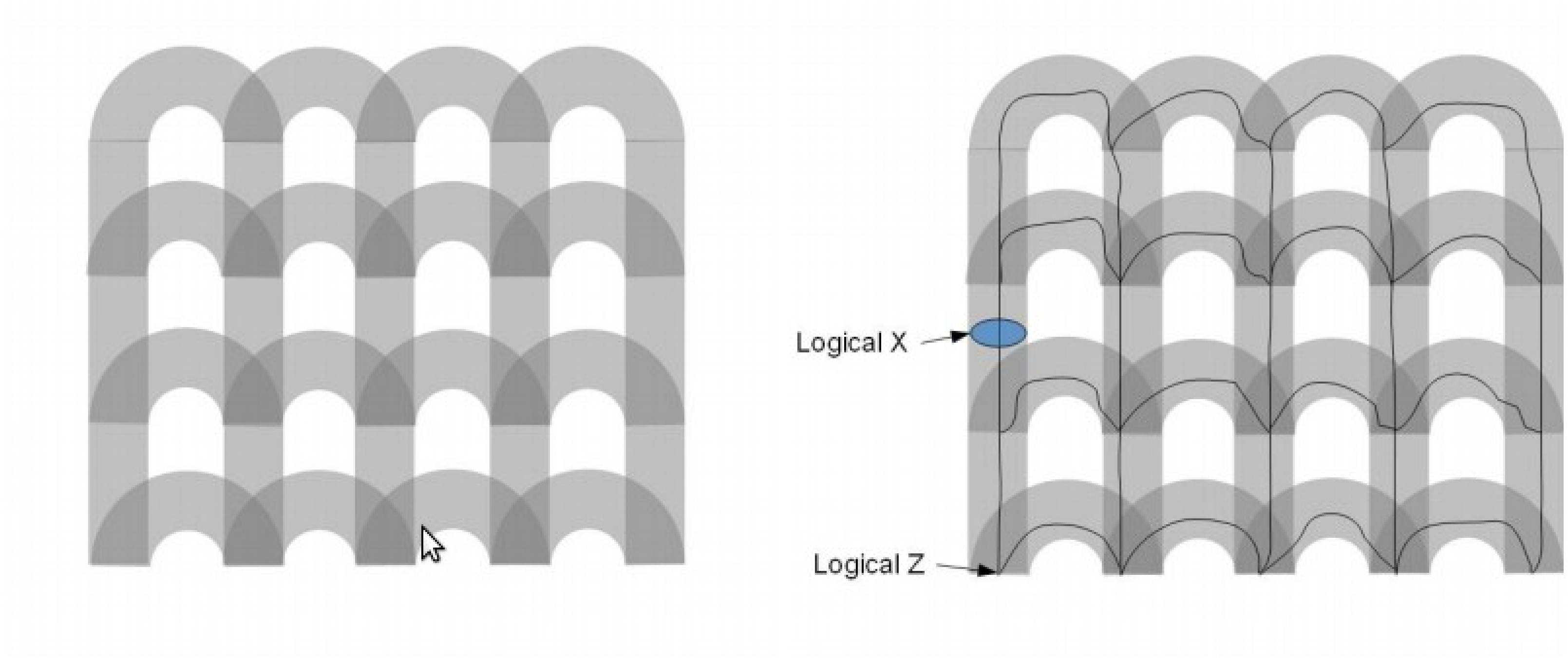

Figure 12: Welded solid code with solids welded together in a 2d square lattice. Notice that the object as a whole is three dimensional.

- Welding codes in an RxR square lattice similarly increases energy barrier of logical Z's to O(R)
- For an RxRxR cubic lattice, barrier increased to O(R^2)
- Energy barrier for logical X's remains O(d)

Welded Solid Codes

- For the cubic lattice of cubic solid codes:

$$N = O(R^3 d^3) = O(L^3)$$
$$\text{Z barrier} = O(R^2)$$
$$\text{X barrier} = O(d)$$

- Let's relate R and d by $d = R^\alpha$

- Let's require that X and Z barriers have same scaling w.r.t. N

- This yields $\alpha = 2$ and a total barrier of $O(N^{9/2}) = O(L^{2/3})$

- Despite this, further (unpublished) work suggests the lack of a phase transition

- Like the Haah-Bravyi model (with log L barrier), the author thinks that this model will not be truly self-correcting, but partially self-correcting

- Model is proof of principle that power law energy barriers are possible with stabilizer Hamiltonians

- But energy landscape seems insufficient to provide actual self-correction (too much entropy)

Thanks for your attention