Suppression of 2π phase-slips due to hidden zero modes in one dimensional topological superconductors

(arXiv:1209.2161)

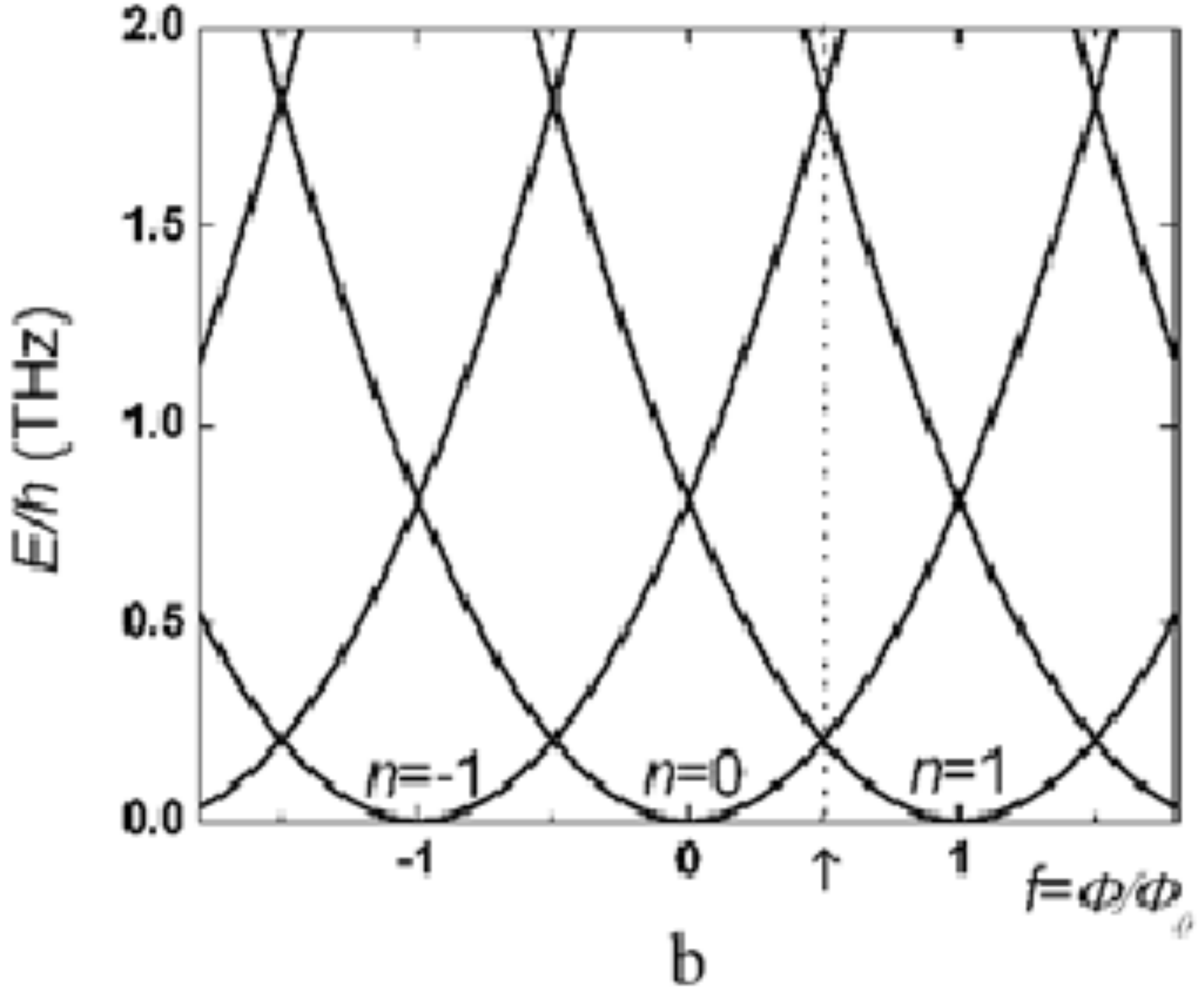
David Pekker
Chang-Yu Hou
Doron L. Bergman
Sam Goldberg
Inanc Adagideli
Fabian Hassler

Main result

Phase slips in topological superconductors are peculiar for the reason that they occur in multiples of 4π (instead of 2π in conventional superconductors)...

...we re-establish this fact via a beautiful analogy to the particle physics concept of dynamic symmetry breaking by explicitly finding a ``hidden" zero mode in the fermion spectrum computed in the background of a $2\pi$ phase-slip (the ``missing meson" problem of quantum chromodynamics)

Quantum Phase Slips (QPS)

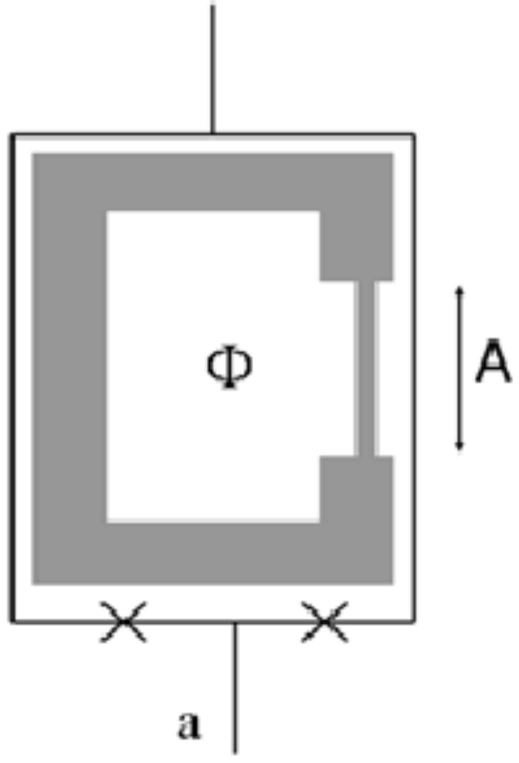


$$n\Phi_0 = \Phi - L_g I_s - L_k I_s$$

$$L_k I_s = \frac{\Phi_0}{2\pi}\gamma$$

$$\gamma = 2\pi(f - n)$$

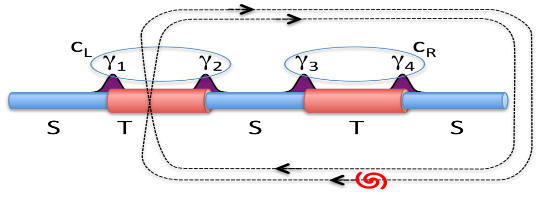
$$E = \frac{\Phi_0^2}{2L_k}(f - n)^2$$



J. E. Mooij and Yu. V. Nazarov, Nature Phys. **2**, 169 (2006)

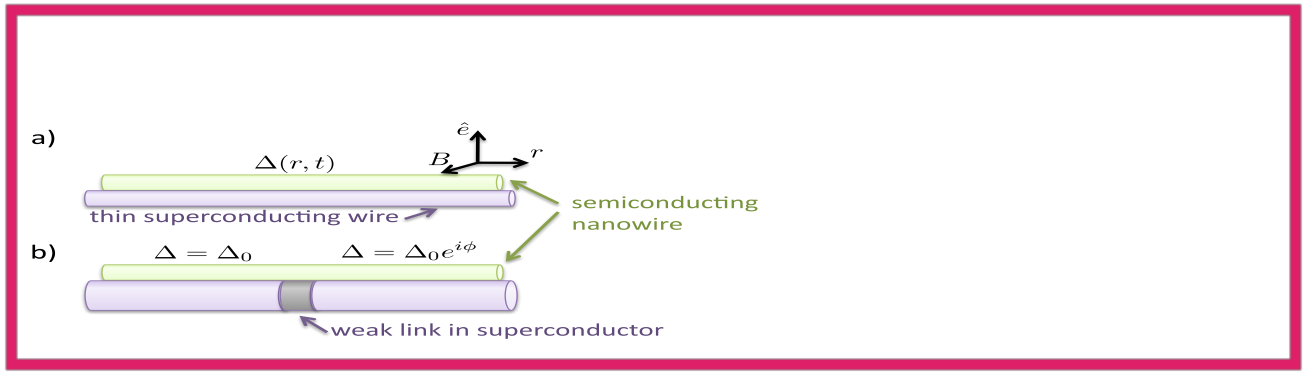
O. V. Astafiev et al., Nature **484**, 355 (19 April 2012)
J. Mooij and C. Harmans, New J. Phys. **7**, 219 (2005)

Phase slips in a topological qubit device



2π-PS decohere the qubit

4π-PS do nothing



Method 1: Integrating out fermion

re N is the number of lattice sites, $c_i^\dagger$ (c_i) is the elec- creation (annihilation) operator at site i, μ_i is the nical potential at site i, $t > 0$ is the hopping matrix nent, and $\Delta_{i,i+1}$ is the complex order parameter, de- d on the link between sites $i, i+1$. This model can be ght of as the large magnetic field regime of the model ribed in Ref. 3. The model supports both topological conventional phases by tuning of the chemical po- ial, with the phase transitions occurring at $|\mu| = 2t$. s, we can model both topological and conventional nents by varying μ_i as a function of position along wire.

o describe the dynamics of the order parameter in superconductor, we need to choose whether we are ribing a Josephson junction or a continuous thin su- conducting wire. As we are interested in the effect he electron degrees of freedom on phase slips, these ils will not be especially important. In the next sec- , we shall focus on the technically simpler problem of se slips at a Josephson junction (weak link).

V. PHASE SLIPS AT A WEAK LINK: THE HIDDEN ZERO MODE

n this section, we construct a theory of phase slips in weak link geometry illustrated in Fig. 3(b): a semi- ducting wire on top of a superconducting wire with gle weak link. We start with this geometry as it in- es fewer degrees of freedom than the continuous wire netry illustrated in Fig. 3(a).

Ve explicitly construct an effective, low energy, model he weak link geometry starting from the Kitaev lel (1) in appendix A. From the point of view of su- conductivity, the weak link geometry is a Josephson tion, that can be characterized by the phase differ- ϕ across the weak link. From the point of view of the trons in the semiconducting nano-wire, the weak link topological-conventional-topological junction. Asso- ed with each topological-conventional interface, there Majorana fermion. By assumption, the weak link is t compared to the Fermi-wavelength in the nanowire, therefore the two Majorana fermions interact to form gle complex fermion c_w, that is localized on the weak . The low frequency effective action involves ϕ and legrees of freedom associated with the weak link and ven by

$$S_J = \int dt \left[\frac{1}{2}\frac{1}{8E_C}(\partial_t\phi)^2 - E_J(1-\cos(\phi)) + c_w^\dagger\left(i\partial_t - E_M\cos(\phi/2)\right)c_w\right]. \quad (2)$$

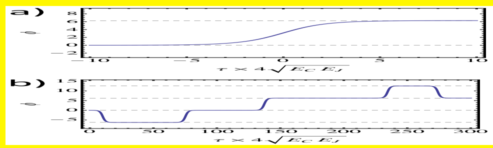


FIG. 4. (a) Instanton trajectory in the sine-Gordon model, Eq. (4). (b) Schematic representation of the dilute instanton gas composed of 2π phase slips and -2π anti-phase-slips.

can be a secondary contribution to the E_J term from the Josephson energy associated with the weak link in the underlying superconductor. The final term describes the sub-gap fermion c_w, localized at the weak link. The energy scale E_M and E_J can be obtained from the Kitaev model, see appendix A.

We begin by sketching the semi-classical dynamics of the phase only (i.e. sine-Gordon) model, without the fermionic term, as described by the real time action

$$S_\phi = \int dt \left[\frac{1}{2}\frac{1}{8E_C}(\partial_t\phi)^2 - E_J(1-\cos(\phi))\right]. \quad (3)$$

The potential energy associated with the second term of this action is 2π periodic, thus we would expect that the phase would be localized near $0, \pm 2\pi, \pm 4\pi \ldots$. However, quantum fluctuations driven by the first term can connect these minima via phase slips. Following the instanton prescription, we can obtain a semiclassical approximation for the tunneling matrix element[25,26]. The prescription states that we must first go to the imaginary

he Lagrangian density of the fermionic part of the action

$$\begin{aligned} S_f(\phi) &= \int d\tau\, c_w^\dagger \mathcal{K}_f(\phi) c_w, \\ &= \int d\tau c_w^\dagger \left[\partial_\tau + E_M\cos(\phi/2)\right] c_w, \end{aligned} \quad (8)$$

ubject to an anti-periodic boundary condition $c_w(\beta) = -c_w(0)$.

To compute the fermionic determinant, we make use f the fact that $\det[\mathcal{K}_f(\phi)] = \prod_n \lambda_n$, where λ_n's come rom the eigenvalue problem

$$\mathcal{K}_f(\phi)u_n(\tau) = \lambda_n u_n(\tau). \quad (9)$$

olving the eigenvalue problem, for arbitrary $\phi(\tau)$, we ind the implicit expression for the eigenfunctions u_n

$$u_n(\tau) = e^{\int_0^\tau (\lambda_n - E_M\cos[\phi(\tau')/2])\, d\tau'}. \quad (10)$$

Vith the anti-periodic boundary conditions, we obtain

$$\lambda_n = \frac{i\pi(2n+1)}{\beta} + \frac{I_1}{\beta} \quad (11)$$

vhere $I_1 = \int_0^\beta d\tau\, E_M\cos(\phi/2)$ and n is an integer. Using few well known identities as in Ref. 29, we now find

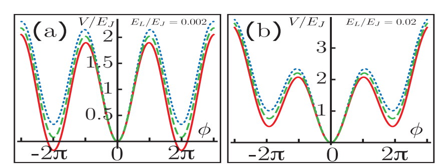
$$\begin{aligned} \det[\mathcal{K}_f(\phi)] &= \prod_n \left(\frac{i\pi(2n+1)}{\beta} + \frac{I_1}{\beta}\right), \\ &= \left[\prod_n \left(\frac{i\pi(2n+1)}{\beta}\right)\right]\cosh(I_1/2). \end{aligned} \quad (12)$$

Thus we find that the partition function becomes

$$Z \propto Z_{\text{eff}} = \int \mathcal{D}\phi\, \cosh(I_1/2) e^{-\tilde{S}_\phi}. \quad (13)$$

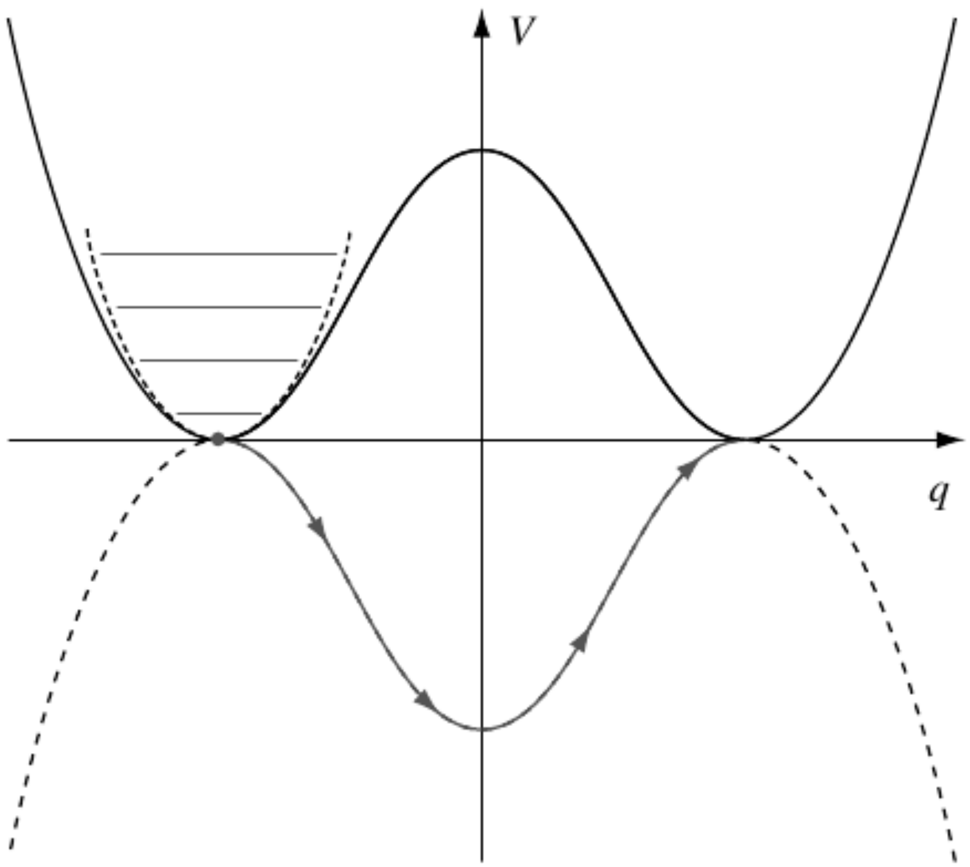
Ve interpret this partition function as follows. The ermion can be in one of two states (either even or odd arity), since there are no terms in the Hamiltonian hat connect these states, the partition function splits nto two parts: one part for even parity and the other art for odd parity, manifested in $\cosh(I_1/2)e^{-\tilde{S}_\phi} = \frac{1}{2}\left[e^{-\tilde{S}_\phi - I_1/2} + e^{-\tilde{S}_\phi + I_1/2}\right]$. The even and odd parity tates are separated by the energy $E_M\cos(\phi/2)$, and the ffective action becomes

$$\tilde{S}_{\phi-\text{eff}} = \int_0^\tau \left[d\tau\,\frac{1}{2}\frac{1}{8E_C}(\partial_t\phi)^2 - E_J(1-\cos(\phi)) \pm \frac{E_M}{2}\cos(\phi/2)\right], \quad (14)$$



in Fig. 8 with a shift to make all $V^{\pm}(0) = 0$. The initial condition is prepared such that the superconducting wire is at its ground state for $\phi = 2\pi$. Therefore, with $E_M > 0$, the effective action should take the sector S_{eff}^{+}, which will be assumed throughout the following discussions. We note that the effective potential $V^{+}(\phi)$ behaves qualitatively different depending on E_L is greater or smaller than $E_M/4\pi^2$. When the inductance energy is dominates, $E_L > E_M/4\pi^2$, the potential has a global minimum at $\phi = 0$ and two local minima at $\phi = \pm 2\pi$.

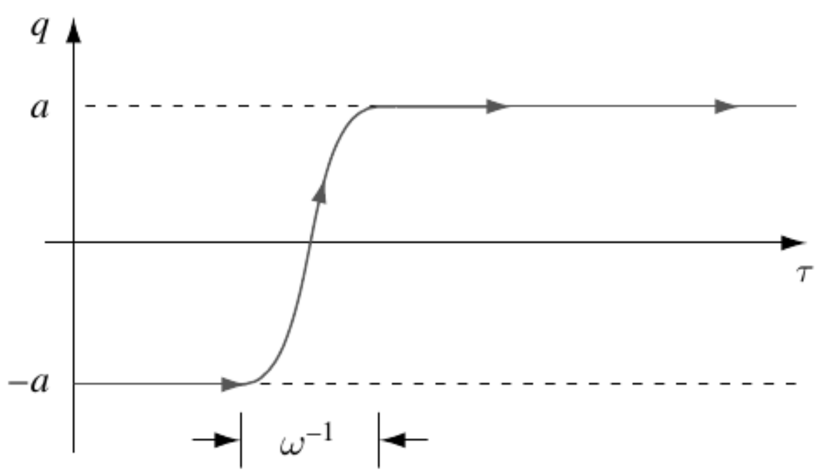
Tunneling and instanton gas



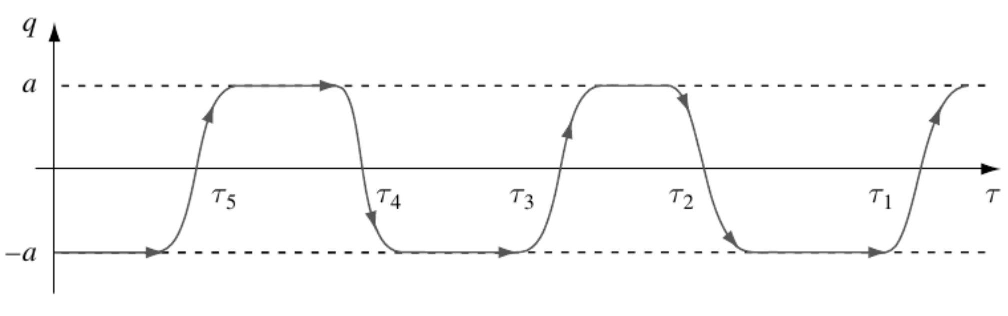
$$G(-a, a, \tau) =$$
$$\int Dq \exp\left[-\frac{1}{\hbar}\int_0^\tau \left(\frac{m}{2}\dot{q}^2 + V(q)\right)\right]$$

$$-m\ddot{q} + V'(q) = 0$$

$$S_{\text{in}} = \int_{-a}^{a} dq \sqrt{2mV(q)}$$



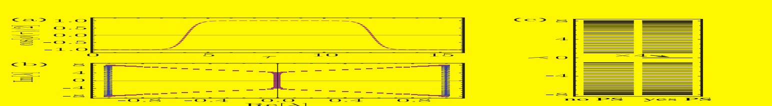
$$E_S = 2\left(\int d\tau m \dot{q}_{\text{in}}^2 / 2\pi\right)^{1/2} (D')^{-1/2} e^{-\mathcal{S}_{\text{in}}}$$

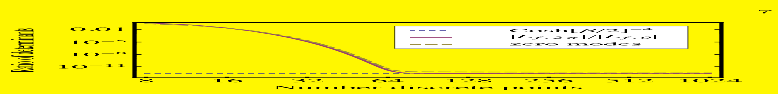


$$D' = \det{}'(\delta^2 \mathcal{S}/\delta\phi^2|_{\text{in}}) / \det(\delta^2 \mathcal{S}/\delta\phi^2|_0)$$

R. Rajaraman, *Solitons and Instantons*

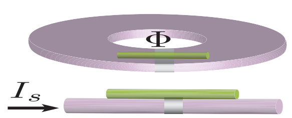
Method II: The hidden zero-mode



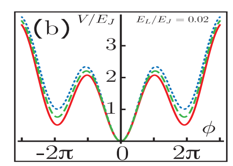


Topological superconductig devices

8



9



in Fig. 8 with a shift to make all $V^{\pm}(0) = 0$. The initial condition is prepared such that the superconducting wire is at its ground state for $\phi = 2\pi$. Therefore, with $E_M > 0$, the effective action should take the sector $S^{+}_{\rm eff}$, which will be assumed throughout the following discussions. We note that the effective potential $V^{+}(\phi)$ behaves qualitatively different depending on E_L is greater or smaller than $E_M/4\pi^2$. When the inductance energy is dominates, $E_L > E_M/4\pi^2$, the potential has a global minimum at $\phi = 0$ and two local minima at $\phi = \pm 2\pi$.

Suppresion of 2π tunneling by MF

9

a shift to make all $V^{+}(0) = 0$. The ini- is prepared such that the superconducting ground state for $\phi = 2\pi$. Therefore, with effective action should take the sector $S^{+}_{2\pi}$, assumed throughout the following discus- te that the effective potential $V^{+}(\phi)$ be- ively different depending on E_L is greater n $E_M/4\pi^2$. When the inductance energy $E_L > E_M/4\pi^2$, the potential has a global $\phi = 0$ and two local minima at $\phi = \pm 2\pi$. hen the Majorana fermion energy becomes $E_L < E_M/4\pi^2$, there are two degenerate $\pm 2\pi$ and a local minimum at $\phi = 0$. potential profiles in the $E_L < E_M/4\pi^2$ d that a phase slip from $\phi = 2\pi$ to $\phi = 0$ is unfavorable as $V^{+}(0) > V^{+}(2\pi)$. Instead, of 4π, tunneling between $\phi = \pm 2\pi$, would le state. As discussed earlier, such a phase change the states of a qubit based on this

ime where $E_L > E_M/4\pi^2$, an initial state relax to $\phi = 0$ state since now $V^{+}(0) <$ relaxation rate is given by $\Gamma_{2\pi\to 0} = K e^{-S'_0}$ esponds to the attempt rate for the tun- is the adjusted action evaluated along the ectory that starts from the initial energy $\approx 2\pi$ to the bouncing point ϕ_b and then Here, the adjusted action is defined by $d\tau V^{+}(\phi_i)$ such that the corresponding po- $= V^{+}(\phi) - V^{+}(\phi_i)$ vanishes at the potential As a rough first approximation, we can as- is not affected by the presence of Majorana plays no role for our discussion. the relaxation rates, $\Gamma^{M}_{2\pi\to 0}$ (Majorana ent) and $\Gamma^{NM}_{2\pi\to 0}$ (Majorana fermions ab- now compute the S'_0 for both cases. As the ectory is a stationary path of the equation can show that

$$S'_0 = \frac{1}{\sqrt{E_C}} \int_{\phi_i}^{\phi_b} d\phi \sqrt{V'(\phi)}. \qquad (27)$$

9

shift to make all $V^{+}(0) = 0$. The ini- prepared such that the superconducting ound state for $\phi = 2\pi$. Therefore, with ective action should take the sector $S^{+}_{2\pi}$, sumed throughout the following discus- that the effective potential $V^{+}(\phi)$ be- ely different depending on E_L is greater $E_M/4\pi^2$. When the inductance energy $_L > E_M/4\pi^2$, the potential has a global $= 0$ and two local minima at $\phi = \pm 2\pi$. n the Majorana fermion energy becomes $< E_M/4\pi^2$, there are two degenerate $\pm 2\pi$ and a local minimum at $\phi = 0$. tential profiles in the $E_L < E_M/4\pi^2$ that a phase slip from $\phi = 2\pi$ to $\phi = 0$ is favorable as $V^{+}(0) > V^{+}(2\pi)$. Instead, 4π, tunneling between $\phi = \pm 2\pi$, would state. As discussed earlier, such a phase hange the states of a qubit based on this

e where $E_L > E_M/4\pi^2$, an initial state elax to $\phi = 0$ state since now $V^{+}(0) <$ laxation rate is given by $\Gamma_{2\pi\to 0} = K e^{-S'_0}$ ponds to the attempt rate for the tun- the adjusted action evaluated along the tory that starts from the initial energy 2π to the bouncing point ϕ_b and then Here, the adjusted action is defined by $V^{+}(\phi_i)$ such that the corresponding po- $V^{+}(\phi) - V^{+}(\phi_i)$ vanishes at the potential a rough first approximation, we can as- not affected by the presence of Majorana ays no role for our discussion. the relaxation rates, $\Gamma^{M}_{2\pi\to 0}$ (Majorana t) and $\Gamma^{NM}_{2\pi\to 0}$ (Majorana fermions ab- ow compute the S'_0 for both cases. As the tory is a stationary path of the equation an show that

$$S'_0 = \frac{1}{\sqrt{E_C}} \int_{\phi_i}^{\phi_b} d\phi \sqrt{V'(\phi)}. \qquad (27)$$

$= E_M = 0$, we have $\phi_i = 2\pi$ and $\phi_b = 0$, s $S'_0 = 4\sqrt{2E_J/E_C}$. When $E_L/E_J \lesssim 1$, $\simeq 2\pi$ and $\phi_b = 0$, and we can approximate C. Qualitatively, the presence of a small gy $E_L/E_J \lesssim 1$ increases the relaxation ly, i.e., decreasing the action such that

at the suppression of tunneling rate due fermions is given by $e^{-\delta S'_0}$, where

$$\delta S'_0 = S'_0 - S'_0|_{E_M=0}, \qquad (28)$$

$\delta S'_0$ in Eq. (28) is evaluated numerically and curve as a function of E_L/E_J with $E_C/E_J = 1$ $= 0.05$. The red curve is the approximate result (30).

$$\left(\frac{E_L}{E_J}, \frac{E_M}{E_J}\right) \approx \frac{E_M}{2\sqrt{2E_J}} \ln(E_J/E_L) \qquad (29)$$

to

$$\delta S'_0 \approx \frac{E_M}{2\sqrt{2E_C E_J}} \ln(E_J/E_L). \qquad (30)$$

10

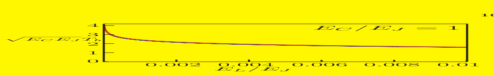


FIG. 10. The time interval T_b as a function of E_L/E_J is evaluated numerically with $E_C/E_J = 1$, c.f. Eq. (33), and shown in the blue curve. The red curve is the approximated result in Eq. (35).

i.e., from $\phi > \pi$ to $\phi < \pi$ or vise versa. For a bouncing event, similar to the phase trajectory depicted in Fig. 5(a), the superconducting phase $\phi(\tau)$ passes through π twice, separated by a time interval of T_b. Therefore, the zero energy eigenvalues at $\phi = \pi$ split to finite energies $\delta\lambda = \pm E_M e^{-E_M T_b}/2$. When $E_L > 2E_J/(3\pi^2)$ and hence $\phi_b < \pi$, the imaginary time interval of T_b can be readily evaluated from

Conclusions

The fact that phase-slips in topological wires occur in multiples of 4π is well known.

We show an alternative explanation of this fact by a beautiful analogy to spontaneous symmetry breaking of the theta vacuum in quantum chromodynamics. For the case of QCD, t'Hooft found that in the background of the instanton of the gauge field, there is a zero mode in the fermionic determinant. This zero mode results in the vanishing of the transition rate between configurations of the vacuum with different winding numbers.

Similarly, we find that in the background of a 2π phase slip, the fermion determinant contains a ``hidden" zero mode, that results in the vanishing (suppression) of the rate of 2π phase slips.

THE END