Phase diagram of spin 1 antiferromagnetic Bose-Einstein condensates

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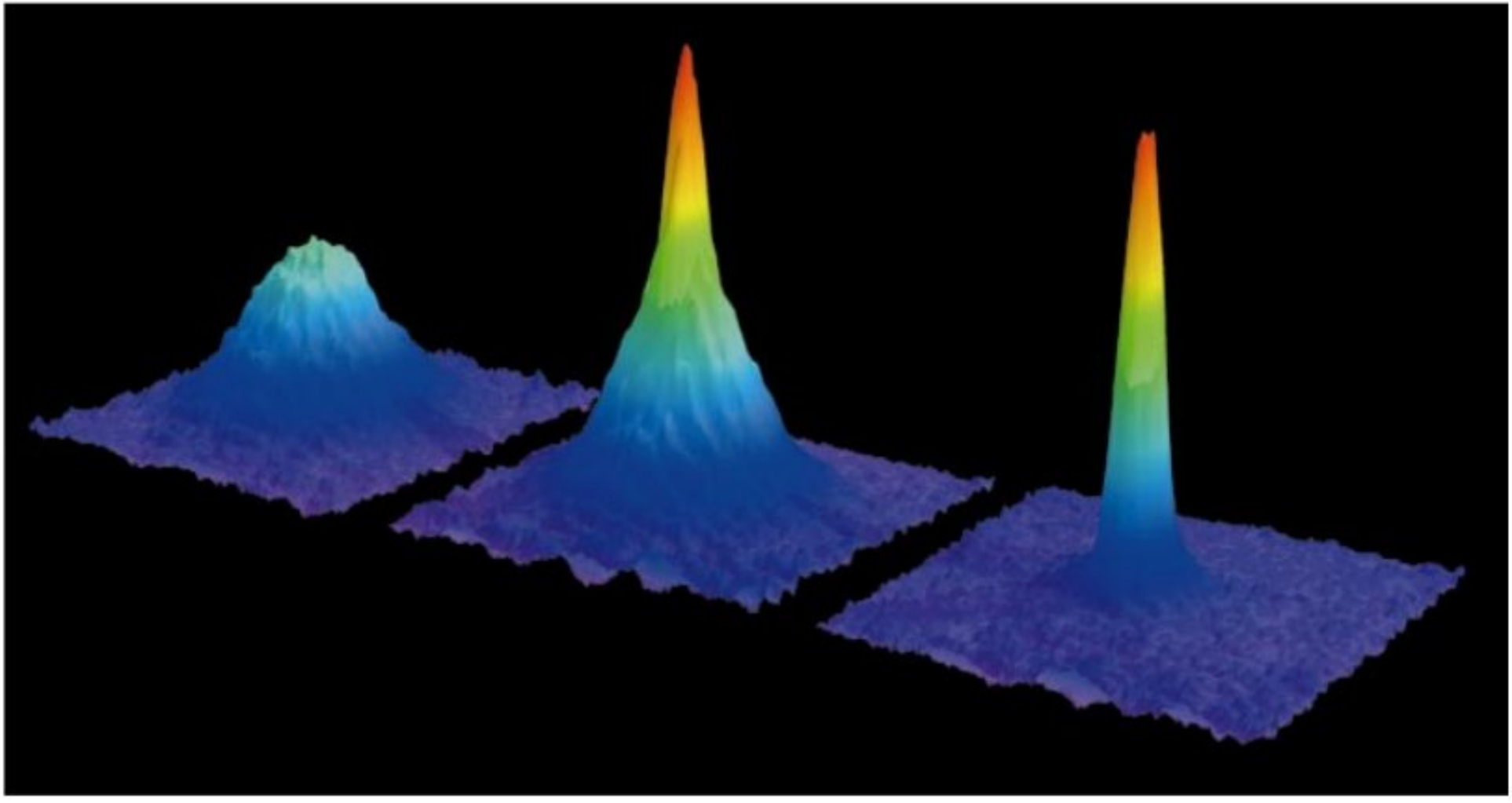
We study experimentally the equilibrium phase diagram of a spin 1 Bose-Einstein condensate with antiferromagnetic interactions, in a regime where spin and spatial degrees of freedom are decoupled. For a given total magnetization m_z, we observe for low magnetic fields an "antiferromagnetic" phase where atoms condense in the $m = \pm 1$ Zeeman states, and occupation of the $m = 0$ state is suppressed. Conversely, for large enough magnetic fields, a phase transition to a "broken axisymmetry" phase takes place: The $m = 0$ component becomes populated and rises sharply above a critical field $B_c(m_z)$. This behavior results from the competition between antiferromagnetic spin-dependent interactions (dominant at low fields) and the quadratic Zeeman energy (dominant at large fields). We compare the measured B_c as well as the global shape of the phase diagram with mean-field theory, and find good quantitative agreement.

Overview

- Examine a quantum liquid with spin $\rightarrow$ Spinor Condensates
- Competition between spin-dependent interactions and quadratic Zeeman effect drives an quantum phase transition
- Compare the measurements with mean-field theory

First Realization of a Bose-Einstein Condensate



Spinor Condensates

Ultracold atoms in 2. Quantization:

$$\begin{aligned}\hat{H} &= \int d^3\vec{x}\ \hat{\psi}^\dagger(\vec{x}) \left(-\frac{\hbar^2}{2m}\ \nabla^2 + V(\vec{x}) \right) \hat{\psi}(\vec{x}) \\ &+ \frac{1}{2}\frac{4\pi a_s \hbar^2}{m} \int d^3x\ \hat{\psi}^\dagger(\vec{x})\hat{\psi}^\dagger(\vec{x})\hat{\psi}(\vec{x})\hat{\psi}(\vec{x})\end{aligned}$$

- V is the trapping potential
- only s-wave scattering and contact interactions

Now spin carrying atoms are trapped by purely optical means:

$$\hat{H} = \int d^3r \left\{ \hat{\psi}_m^\dagger \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} \right) \hat{\psi}_m + \frac{g_{ij,kl}}{2} \hat{\psi}_i^\dagger \hat{\psi}_j^\dagger \hat{\psi}_k \hat{\psi}_l \right\}$$

Spinor Condensates

Interactions are rotationally symmetric $\Rightarrow$ Spin is conserved

$$\hat{V}(r_1, r_2) = \sum_{S=0,2} g_S \ \hat{P}_S \ \delta(r_1 - r_2),$$

where $g_S = \frac{4\pi\hbar^2 a_S}{m}$

Rewrite the interaction potential in terms of spin operators:

$$\begin{aligned}
(\mathbf{F}_1 \cdot \mathbf{F}_2)|\phi\rangle &= (\mathbf{F}_1 \cdot \mathbf{F}_2)(\hat{P}_0 + \hat{P}_2)|\phi\rangle \\
&= \frac{1}{2}\left[(\mathbf{F}_1 + \mathbf{F}_2) - \mathbf{F}_1^2 - \mathbf{F}_2^2\right](\hat{P}_0 + \hat{P}_2)|\phi\rangle \\
&= (-2\hat{P}_0 + \hat{P}_2)|\phi\rangle,
\end{aligned}$$

$$\Rightarrow \quad \hat{V}(r_1, r_2) = \left(\frac{g_0 + 2g_2}{3} + \frac{g_2 - g_0}{3}\ \mathbf{F}_1 \cdot \mathbf{F}_2\right)\ \delta(r_1 - r_2).$$

T.-L. Ho, Phys. Rev. Lett. **81** 742 (1998)

Spinor Condensates

Write down the interaction terms in detail:

$$c_0\hat{\psi}_m^\dagger\hat{\psi}_j^\dagger\hat{\psi}_j\hat{\psi}_m + c_2\ \hat{\psi}_m^\dagger(F_\nu)_{mj}\hat{\psi}_j\cdot\hat{\psi}_l^\dagger(F_\nu)_{lk}\hat{\psi}_k =$$

$$(c_0+c_2)\hat{\psi}_1^\dagger\hat{\psi}_1^\dagger\hat{\psi}_1\hat{\psi}_1 + c_0\hat{\psi}_0^\dagger\hat{\psi}_0^\dagger\hat{\psi}_0\hat{\psi}_0 + (c_0+c_2)\hat{\psi}_{-1}^\dagger\hat{\psi}_{-1}^\dagger\hat{\psi}_{-1}\hat{\psi}_{-1}$$

$$+\quad 2(c_0+c_2)\hat{\psi}_1^\dagger\hat{\psi}_0^\dagger\hat{\psi}_1\hat{\psi}_0 + 2(c_0+c_2)\hat{\psi}_{-1}^\dagger\hat{\psi}_0^\dagger\hat{\psi}_{-1}\hat{\psi}_0 + 2(c_0-c_2)\hat{\psi}_1^\dagger\hat{\psi}_{-1}^\dagger\hat{\psi}_1\hat{\psi}_{-1}$$

$$+\quad 2c_2\hat{\psi}_0^\dagger\hat{\psi}_0^\dagger\hat{\psi}_1\hat{\psi}_{-1} + 2c_2\hat{\psi}_1^\dagger\hat{\psi}_{-1}^\dagger\hat{\psi}_0\hat{\psi}_0.$$

- three self-scattering terms
- three cross-scattering terms
- two spin relaxation terms

Meanfield Ansatz

Plug in the expression for the contact interaction:

$$\begin{aligned}\hat{H} &= \int d^3r \Bigg[\hat{\psi}_m^\dagger \left(-\frac{\hbar^2}{2M}\nabla^2 + V_{ext}\right)\hat{\psi}_m + \frac{c_0}{2}\hat{\psi}_m^\dagger \hat{\psi}_j^\dagger \hat{\psi}_j \hat{\psi}_m \\ &+ \frac{c_2}{2}\,\hat{\psi}_m^\dagger (F_\nu)_{mj}\hat{\psi}_j \cdot \hat{\psi}_l^\dagger (F_\nu)_{lk}\hat{\psi}_k \Bigg],\end{aligned}$$

Use $\hat{\vec{\psi}} = \sqrt{n}\vec{\xi}$ (where $|\vec{\xi}|^2 = 1$ and n is the atomic density)

$$\begin{aligned}E[\{\phi_i\}] &= \int d^3r \Bigg\{ \frac{\hbar^2}{2m}\left(\nabla\sqrt{n}\right)^2 + \frac{\hbar^2}{2m}\left(\nabla\vec{\xi}\right)^2 n + (V_{ext} - \mu) \\ &+ \frac{n^2}{2}\left(c_0 + c_2\langle\vec{F}\rangle^2\right)\Bigg\}\end{aligned}$$

where $\langle\vec{F}\rangle = \sum_{kl}\xi_k^*\vec{F}_{kl}\xi_l$

Detection of Different Hyperfine Components

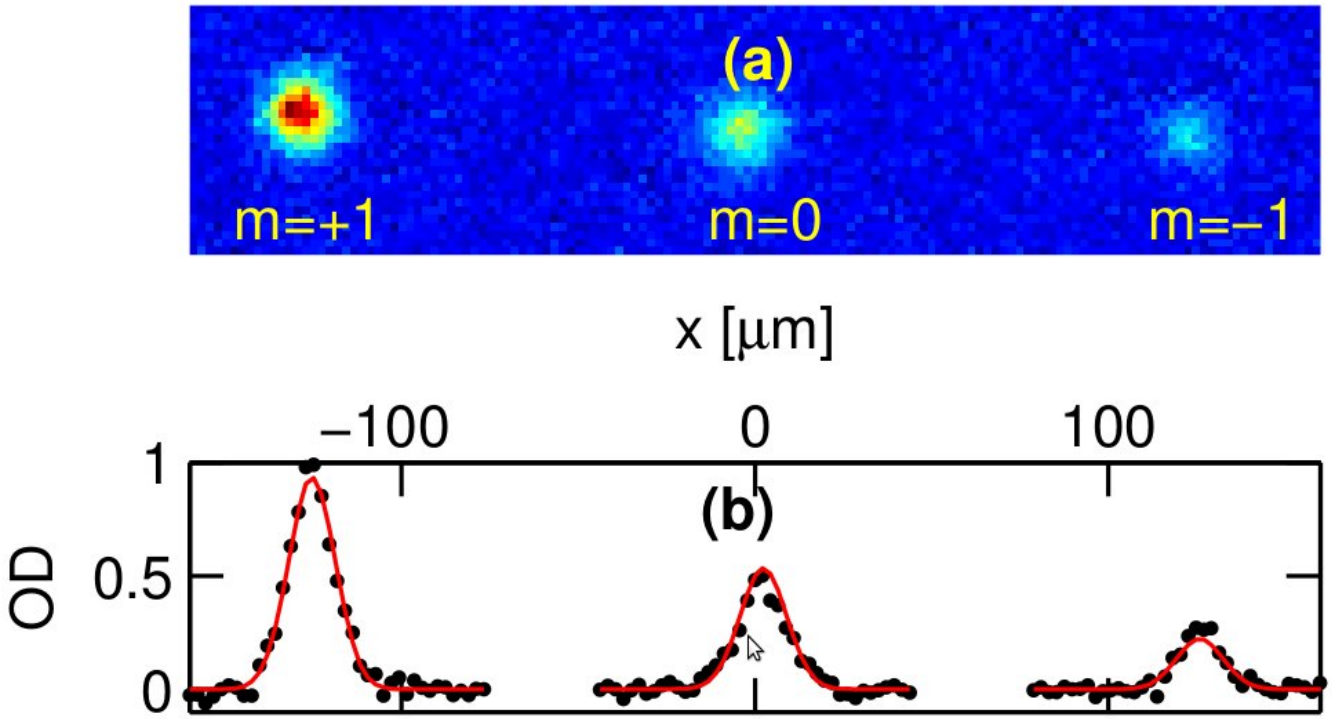
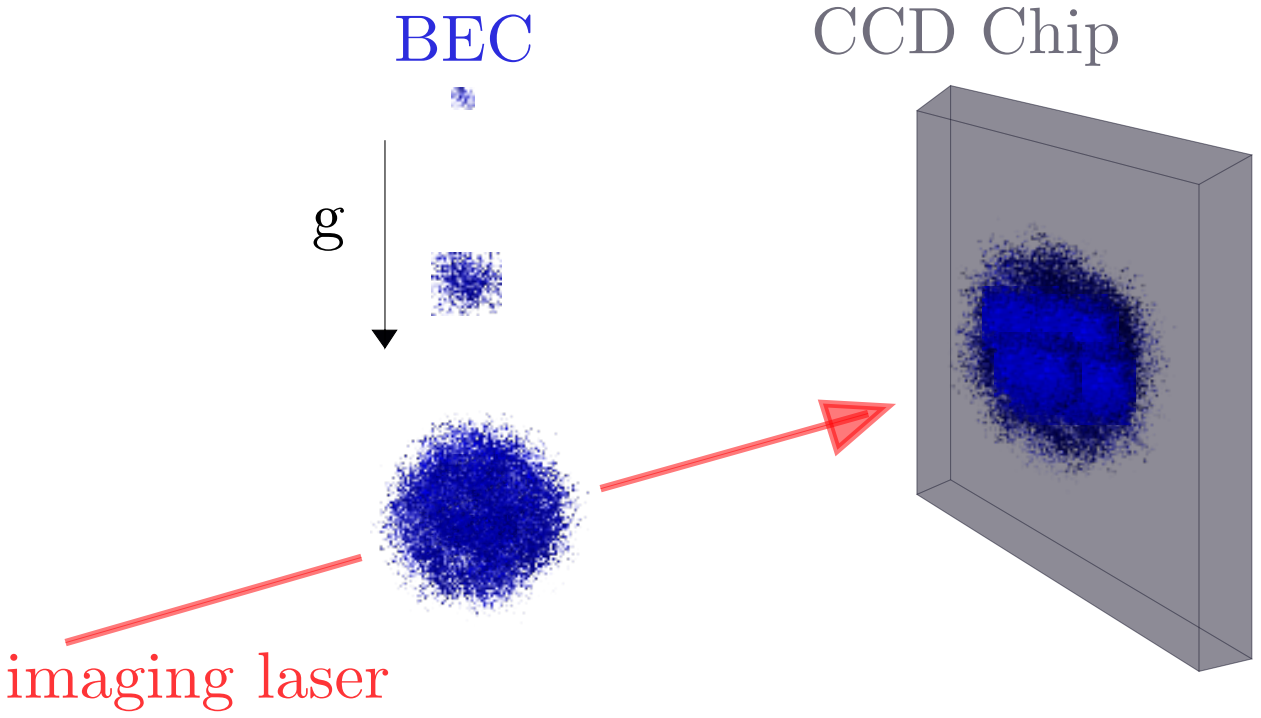


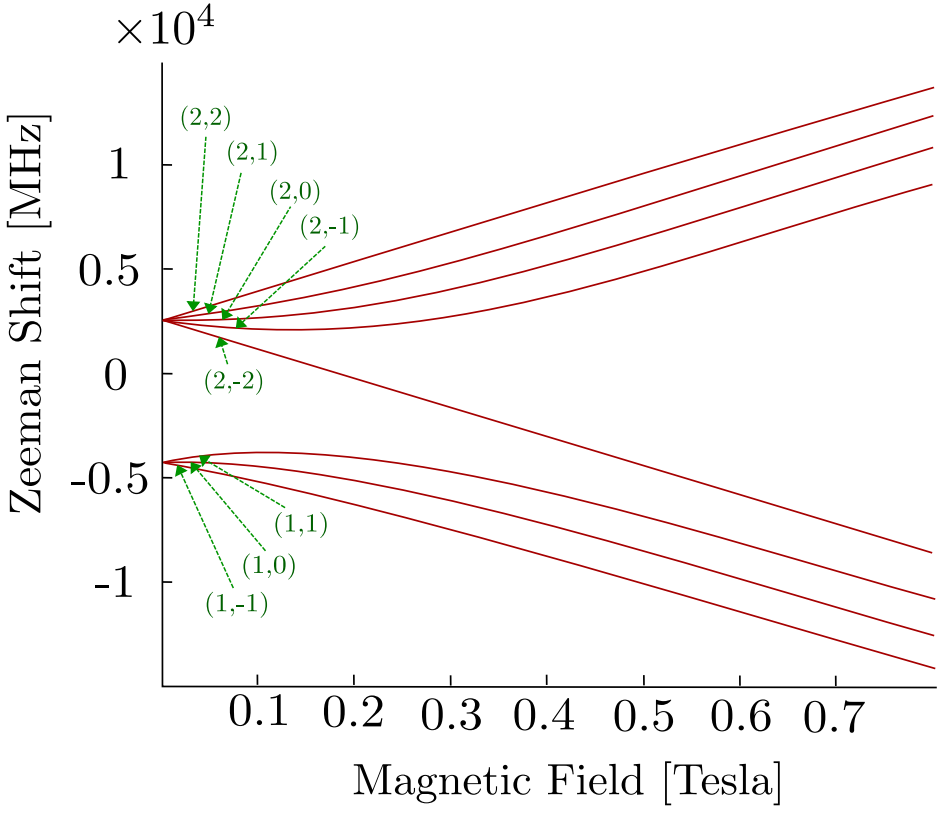
FIG. 1: (Color online) **a:** Absorption image of a spin 1 BEC after expansion in a magnetic gradient. **b:** Horizontal cuts through the images in (**a**). The same function (shown by straight lines), only recentered and reweighted, is used to fit the density profile of each Zeeman state.

Time-of-Flight Imaging

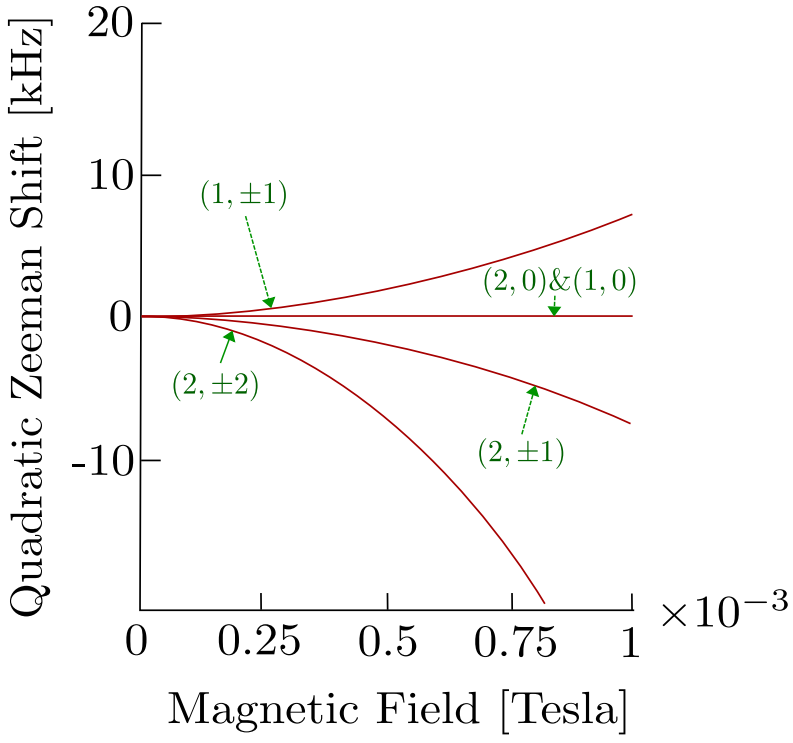


Effects of Magnetic Fields

(a)



(b)



Effects of Magnetic Fields

$$E[\{\phi_i\}] = H_S + \frac{c_2}{2}\langle\vec{F}\rangle^2 - p\,\langle F_z\rangle + q\,\langle F_z^2\rangle$$

where $p = p_0 B$ and $q = q_0 B^2$

Express this in terms of occupancies of the hyperfine components:

$$\begin{aligned}\frac{E[\{n_i\}]}{N} &= \frac{H_S}{N} + \frac{c_2 n}{2}\left((n_1 - n_{-1})^2 + 2n_0\left(\sqrt{n_1} \pm \sqrt{n_{-1}}\right)\right) \\ &- p\,(n_1 - n_{-1}) + q\,(n_1 + n_{-1})\end{aligned}$$

Minimize this energy functional under the constraints:

- Particle number N is conserved $\Rightarrow n_1 + n_0 + n_{-1} = 1$
- Magnetization $m = n_1 - n_{-1}$ is conserved

Effects of Magnetic Fields

Minimize

$$\mathcal{F}(x) = c_2 n \left(x - \sqrt{x^2 - m^2}\right)(1 - x) + q\, x$$

where $x = n_1 + n_{-1}$

Introduce $q_c = c_2 n \left(1 - \sqrt{1 - m^2}\right)$

- For $q < q_c$: $x_0 = 1 \Leftrightarrow n_0 = 0 \Rightarrow$ Antiferromagnetic Phase
- For $q > q_c$: $x_0 < 1$ & $n_0 \neq 0 \Rightarrow$ Broken Axisymmetry Phase

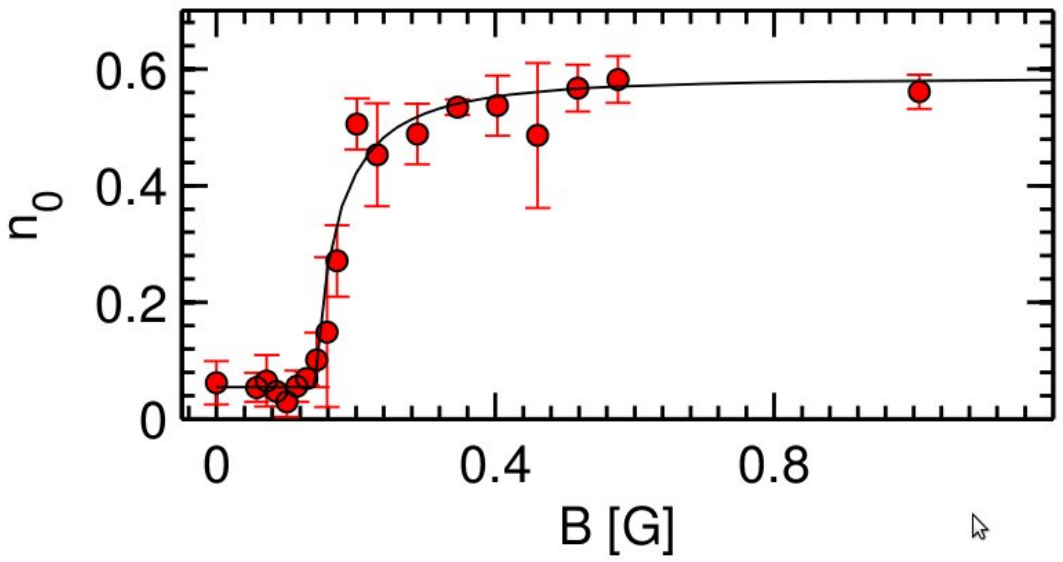


FIG. 2: (Color online) Sample data showing the population n_0 of the $m = 0$ Zeeman state versus applied magnetic field B, for a magnetization $m_z \approx 0.4$. The solid line is a fit to the data using Eq. (5). Vertical error bars show statistical uncertainties on the measured values (one standard deviation).

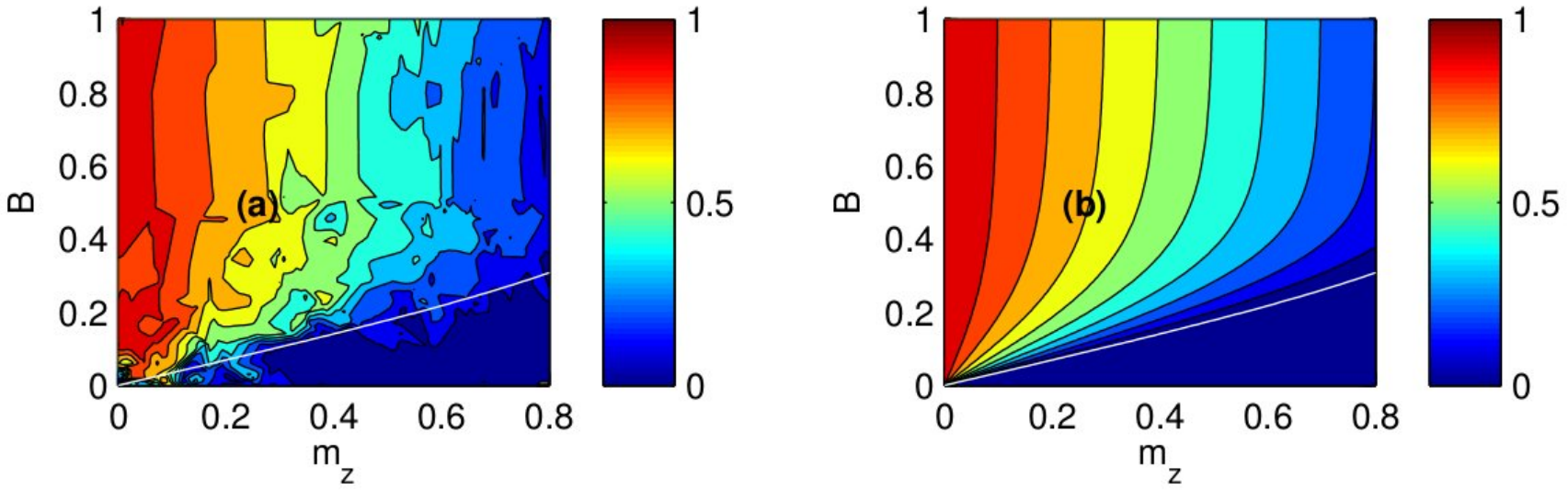


FIG. 3: (Color online) **a:** Experimental phase diagram showing the population n_0 of the $m = 0$ Zeeman state versus magnetization m_z and applied magnetic field B. The plot shows a contour interpolation through all data points, with magnetization ranging from 0 to 0.8. The white line is the predicted critical field B_c separating the two phases, deduced from Eq. (4) by $q_c = q_B B_c^2$. **b:** Theoretical prediction for n_0 at $T = 0$ K.

Summary

- "We explored experimentally the phase diagram of spin 1 BECs with antiferromagnetic interactions"
- "Two phases are found, reflecting the competition between the spin-dependent interactions and the quadratic Zeeman energy"
- "The measurements are in quantitative agreement with mean-field theory"

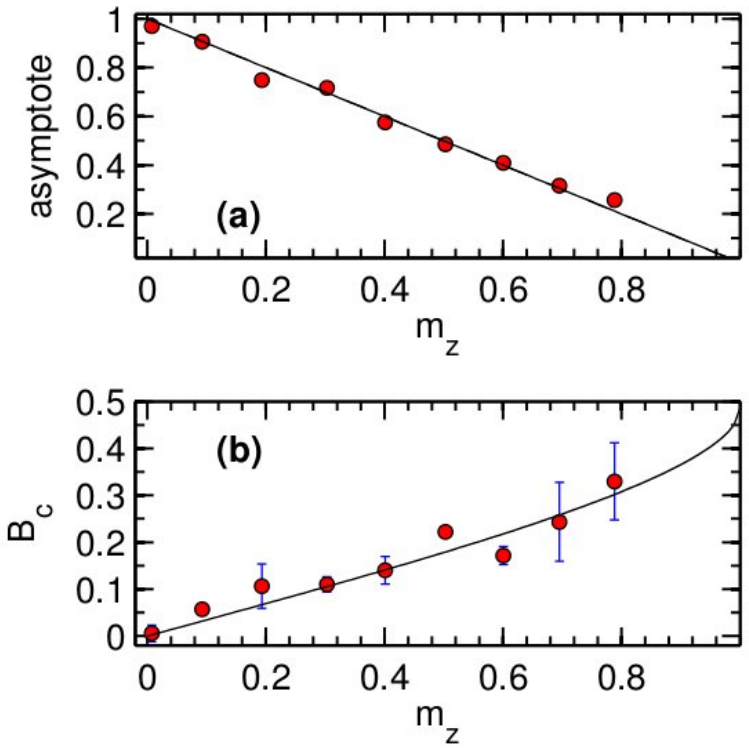


FIG. 4: (Color online) **a:** Asymptotic value of n_0 for large q (determined from $A_0 + A_1$ in Eq. 5). The solid line shows the value $1 - m_z$ expected at zero temperature. **b:** Measured critical field B_c versus magnetization. The solid line shows the values expected from Eq. (4) and $q_c = q_B B_c^2$, using $U_s/h \approx 65.6$ Hz. The gray area show the uncertainty on the theoretical value of B_c, dominated by the 15 % uncertainty on the spin-dependent scattering length a_s. For both plots, vertical error bars show statistical uncertainties on the measured values (one standard deviation).

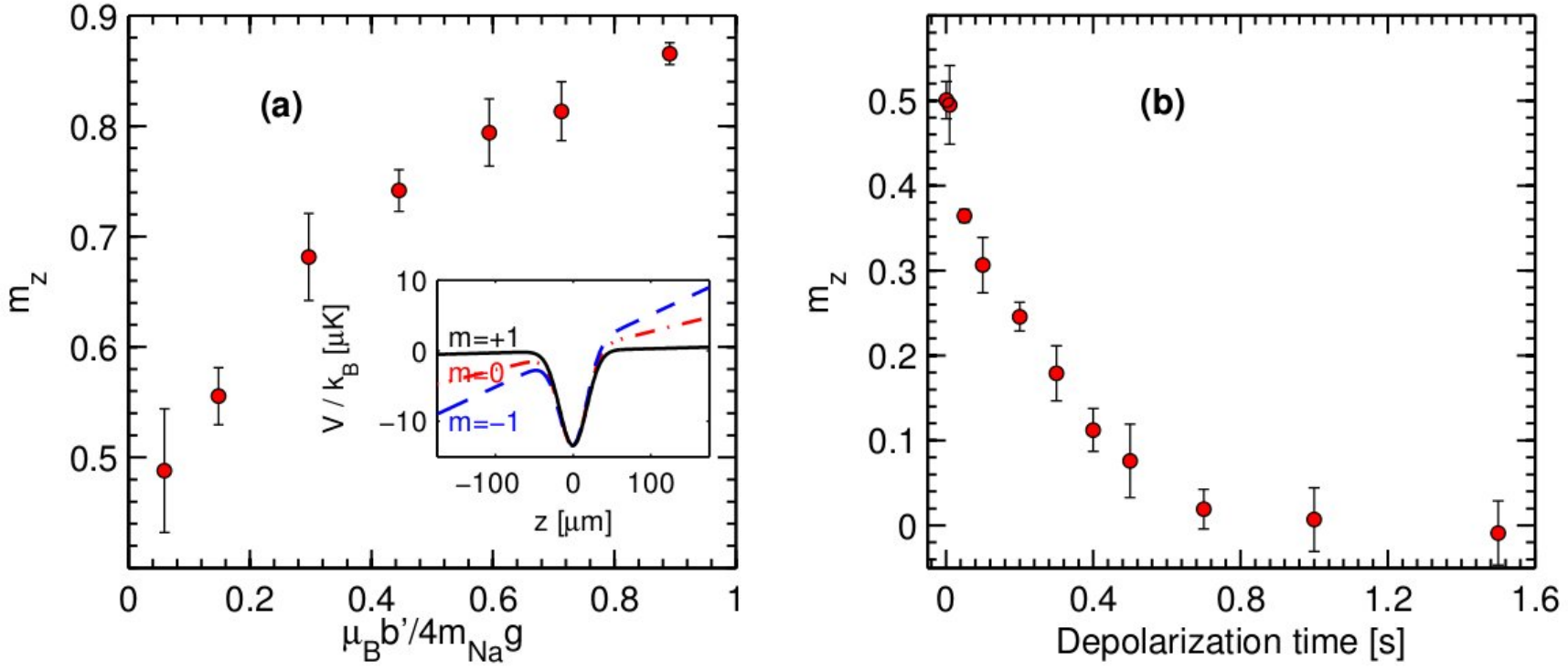


FIG. 5: **Supplementary Material-** (Color online) **(a):** Spin distillation to prepare samples with high magnetization ($m_z > 0.5$). The plot shows the magnetization measured for cold clouds, as a function of the magnetic gradient b' in units of $m_{Na}g/\mu_B$, with g the acceleration of gravity and μ_B the Bohr magneton. The inset shows a sketch of the potential energies for each Zeeman state along the vertical axis z. The potential drop is exaggerated for clarity, and is smaller than depicted in the actual experiment. **(b):** Depolarization to prepare samples with low magnetizations ($m_z < 0.5$). The time shown corresponds to the length of a radio-frequency pulse at the Larmor frequency.