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arXiv:1209.6334v1 [quant-ph]

Observation of Radiation Pressure Shot Noise

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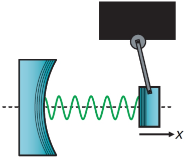


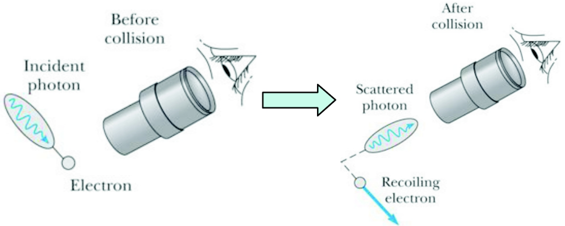
Outline

- Standard quantum limit on measurement precision
- Quantum limit on the added noise
- Experimental setup and parameters
- Theoretical description of the device
- Ingredients for the observation of RPSN
- Experimental results

Standard quantum limit on measurement precision

Heisenberg microscope:

$$\Delta x_{imp} \sim \lambda,$$
$$\Delta p_{BA} \sim h/\lambda,$$
$$\Delta x_{imp}\, \Delta p_{BA} \geq \frac{h}{2}.$$

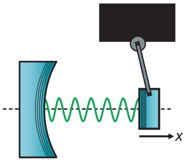


Weak continuous measurement: imprecision vs back-action noise.

Optomechanical setup: radiation pressure force: $\hat{F} = -\hbar G \hat{a}^\dagger \hat{a}$,

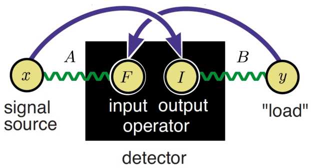
$$\hat{H}_{int} = -\hat{x} \cdot \hat{F}$$

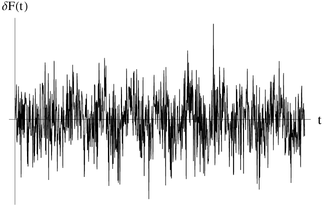
- $\hat{x}$ is measured via the cavity phase shift.
- Photon shot noise (fluctuating force $\hat{F}$) causes **back-action**.



What is the equivalent of Heisenberg uncertainty relation for weak continuous measurement?

Constraint on the detector input and output noises





Noise is characterized by its power spectral density:

$$\bar{S}_{AB}[\omega] = \frac{1}{2}\int dte^{i\omega t}\left\langle\left\{\delta\hat{A}(t),\delta\hat{B}(0)\right\}\right\rangle,$$

Quantum constraint on noise detector:

$$\bar{S}_{FF}[\omega]\bar{S}_{II}[\omega] - \left|\mathrm{Re}\left(\bar{S}_{IF}[\omega]\right)\right|^2 \geq \frac{\hbar^2}{4}\left|\mathrm{Re}\left(\chi_{IF}[\omega]\right)\right|^2$$

where $\chi_{IF}(t) = -\frac{i}{\hbar}\Theta(t)\left\langle\left[\hat{I}(t),\hat{F}(0)\right]\right\rangle$ is the gain of the detector.

Quantum limit on the added noise

Detector output: signal + back-action + imprecision,

$$I_{meas} = A\chi_{IF}\,(x + \delta x_{BA}) + \delta I_{imp} = A\chi_{IF}\,(x + \delta x_{add})\,,$$

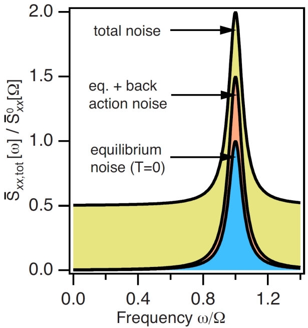
$$\delta x_{add} = \delta x_{BA} + \delta x_{imp}, \qquad \delta x_{imp} = \delta I_{imp}/(A\chi_{IF}), \qquad \delta x_{BA} = A\chi_{xx}\delta F.$$

Spectral density of $x_{meas} = I_{meas}/(A\chi_{IF})$

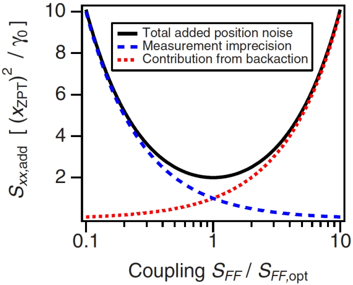
$$\bar{S}_{xx}^{meas}[\omega] = \bar{S}_{xx}^{eq}[\omega, T] + \bar{S}_{xx}^{add}[\omega].$$

Quantum limit on the added noise

$$\boxed{\bar{S}_{xx}^{add}[\omega] \geq \bar{S}_{xx}^{eq}[\omega, T = 0].}$$



SQL : optimal working point of a detector, where the added noise is minimized.



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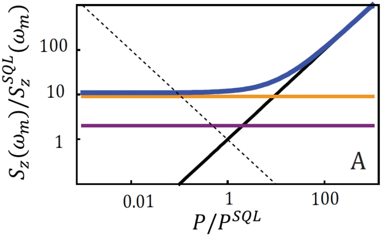
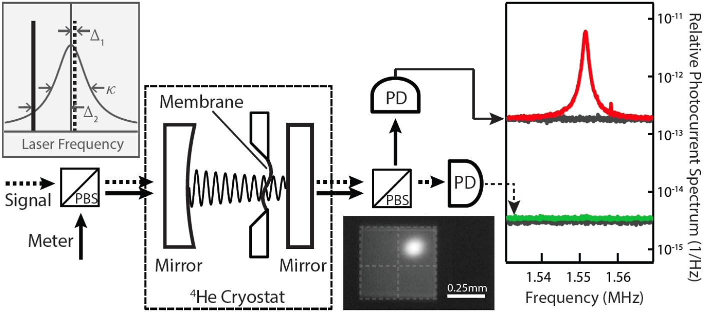


Reported in this work: first experiment where radiation-pressure shot noise is the dominant driving force of a solid object (RPSN comparable to thermal forces).

[1]A. A. Clerk et al. *Rev. Mod. Phys.* **82** (2010)

Experimental setup and parameters

Micromechanical SiN membrane inside a Fabry-Perot optical cavity.



Mechanical mode:

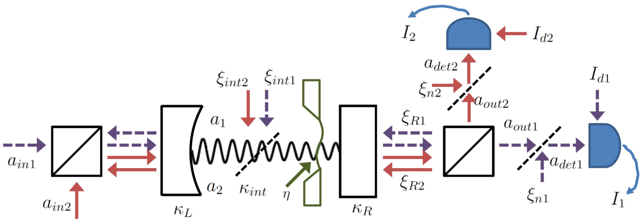
- $\omega_m/2\pi = 1.55$ MHz
- $\Gamma_0/2\pi < 1$ Hz

Optical modes:

- cavity linewidth: $\kappa/2\pi \sim 1$ MHz
- $\Delta_S \sim 0$ and $\Delta_M \simeq \omega_m$

Single-photon optomechanical coupling: $g/2\pi = 17$ Hz

Theoretical description of the device



Hamiltonian: $H = H_0 + H_\kappa + H_\Gamma$

$$H_0 = \hbar\omega_m c^\dagger c + \hbar\Delta_1 a_1^\dagger a_1 + \hbar\Delta_2 a_2^\dagger a_2 + \hbar g_1 a_1^\dagger a_1 (c + c^\dagger)$$
$$+ \hbar g_2 a_2^\dagger a_2 (c + c^\dagger) + \epsilon_{d,1}(a_1 + a_1^\dagger) + \epsilon_{d,2}(a_2 + a_2^\dagger)$$

Heisenberg-Langevin equations of motion:

$$\dot{a}_1 = -\frac{i}{\hbar}[a_1, H_0] - \frac{\kappa}{2}a_1 + \sqrt{\kappa_L}(\xi_{\mathrm{L1}} + dx_1) + \sqrt{\kappa_{\mathrm{int}}}\xi_{\mathrm{int1}} + \sqrt{\kappa_R}\xi_{\mathrm{inR1}},$$
$$\dot{a}_2 = -\frac{i}{\hbar}[a_2, H_0] - \frac{\kappa}{2}a_2 + \sqrt{\kappa_L}\xi_{\mathrm{L2}} + \sqrt{\kappa_{\mathrm{int}}}\xi_{\mathrm{int2}} + \sqrt{\kappa_R}\xi_{\mathrm{R2}},$$
$$\dot{c}(t) = -\frac{i}{\hbar}[c, H_0] + \sqrt{\Gamma_0}\eta$$

Regime of linearized optomechanics

Linearize the eom's around classical coherent field amplitudes

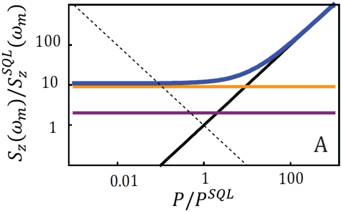
$$a_1(t) = \bar{a}_1 + d_1(t), \qquad a_2(t) = \bar{a}_2 + d_2(t)$$

and solve in the frequency-domain for the quantum operators d_1, d_2 (optical) and $z = Z_{\text{zpf}}(c + c^\dagger) - \bar{z}$ (displacement).

Enhanced optomechanical coupling:

$$g_i a_i^\dagger a_i (c + c^\dagger)$$
$$\downarrow$$
$$\tilde{g}_i (d_i + d_i^\dagger)(c + c^\dagger)$$

where $\tilde{g}_i = g_i |\bar{a}_i| = g_i \sqrt{N_i} \;\propto\; \epsilon_{d,i}$



Increasing the driving power increases the measurement strength.

Mechanical spectrum

$S_{zz}(\omega) = \langle z(-\omega)z(\omega)\rangle$ can be calculated from the noise correlators:

$$\left\langle \xi_i(\omega)\xi_j^\dagger(\omega')\right\rangle = \delta_{ij}\delta(\omega+\omega'), \qquad \left\langle \xi_i^\dagger(\omega)\xi_j(\omega')\right\rangle = 0$$

$$\left\langle \eta(\omega)\eta^\dagger(\omega')\right\rangle = (n_{th}+1)\delta(\omega+\omega'), \qquad \left\langle \eta^\dagger(\omega)\eta(\omega')\right\rangle = n_{th}\delta(\omega+\omega')$$

$$\langle dx_1(\omega)dx_1(\omega')\rangle = B_1$$

$$\frac{S_{zz}(\omega)}{Z_{\mathrm{zpf}}^2} = \frac{1}{|\mathcal{N}(\omega)|^2}\left(\frac{\Gamma_0(n_{\mathrm{th}}+1)}{|\chi_m(\omega)|^2} + \frac{\Gamma_0 n_{\mathrm{th}}}{|\chi_m(-\omega)|^2} + 4\omega_m^2\kappa|\bar{a}_1 g_1\chi_{c1}(-\omega)|^2\right.$$
$$\left. + 4\omega_m^2\kappa|\bar{a}_2 g_2\chi_{c2}(-\omega)|^2 + 4\omega_m^2\kappa_L|\bar{a}_1 g_1\left(\chi_{c1}(\omega)+\chi_{c1}^*(-\omega)\right)|^2 B_1\right)$$

In particular $S_{zz}(\omega)$ is used to calculate the change in the mechanical response, needed to access the BA noise ($\delta x_{BA} = A\chi_{xx}\,\delta F$)

Ingredients for the observation of RPSN

Signal (mode 1)

- provides the RPSN
- its transmitted shot intensity fluctuations constitute a record of the optical force on the resonator.
- driven on resonance $\Delta_S \sim 0$,
- measurement strength $(g_1|\bar{a}_1|)$ modulated via driving stength $N_S = |\bar{a}_1|^2 \simeq (\epsilon_{d,1}/\kappa)^2$

Meter (mode 2):

- more weakly driven $(N_M \ll N_S)$ on the first red sideband $(\Delta_M \simeq \omega_m)$
- provides cooling $\Gamma_M \gg \Gamma_0, \Gamma_S$
- The resonator's displacement spectrum is imprinted in the transmitted intensity spectrum of this laser.

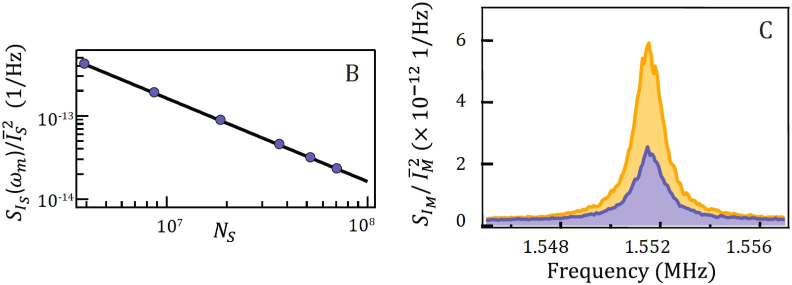
Effective phonon occupation:

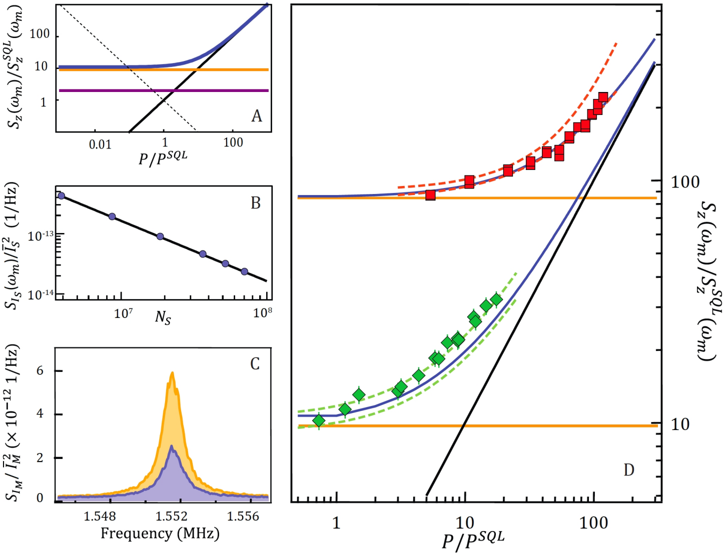
$$n_m = \frac{n_{th}\Gamma_0 + n_S\Gamma_S + n_M\Gamma_M}{\Gamma_m}, \qquad \Gamma_m = \Gamma_0 + \Gamma_S + \Gamma_M$$

Regime where RPSN dominates:

$$R_S = \frac{C_S}{n_{th}(1 + (2\omega_m/\kappa)^2)} > 1$$

$C_S = 4N_S g^2/\kappa\Gamma_0$ is the multiphoton cooperativity.





Device A (Device B): $N_M = 3.4 \times 10^6$ (7.0×10^6), $N_S^{\max} = 1.2 \times 10^8 (4.4 \times 10^8)$, $\omega_m/2\pi = 1.575$ MHz (1.551 MHz), $\Gamma_m/2\pi = 3$ kHz (1.43 kHz).

Correlation between signal and the meter beam photocurrents

Cross-correlation function:

$$S_{I_{SM}}(\omega) = \langle I_S^*(\omega) I_M(\omega) \rangle$$

Individual photocurrents:

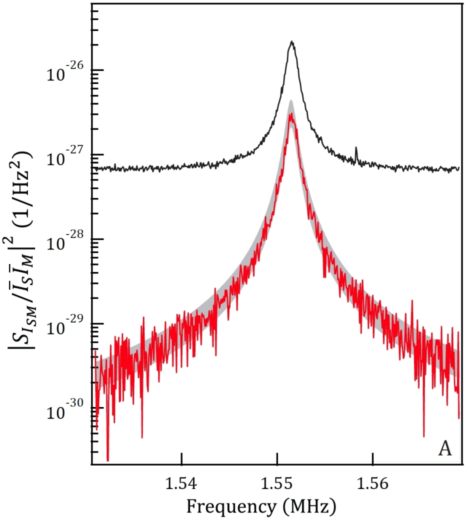
$$S_{I_{S,M}}(\omega) = \langle |I_{S,M}(\omega)|^2 \rangle$$

Peak correlation:

$$\mathcal{C}(\omega_m) = \frac{|S_{I_{SM}}(\omega_m)|^2}{S_{I_S}(\omega_m) S_{I_M}(\omega_m)}$$

$$\propto \frac{R_S}{1+R_S} = 0.4 \pm 0.03,$$

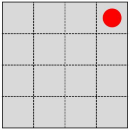
indicates the fraction of displacement spectrum due to RPSN.



Red: $|S_{I_{SM}}(\omega)|^2$ Black: $S_{I_S}(\omega) \times S_{I_M}(\omega)$

Control of laser absorbtion heating

(4,4) mode



(2,2) mode