Topological Kondo Effect with Majorana Fermions

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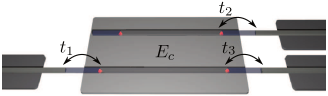
Journal Club

Daniel Becker
16 October 2012



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- non-local Kondo impurity realized by spin-1/2 algebra formed by 3 of 4 Majorana modes
- experimentally observe non-locality of Majoranas in conductance signatures of topological Kondo effect
- estimate: charging energy E_C and induced SC gap Δ_{NW} of order $0.5 - 1K$; Kondo temperature $T_K \lesssim 0.1K$

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topological Kondo effect

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- robust non-Fermi liquid (NFL) behavior for very small temperatures
- topological Kondo impurity states correspond to logical qubit states

Majorana Fermions in 1D Nanowires (Essentials)

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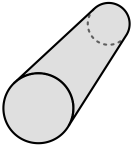

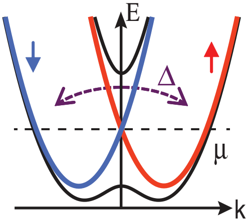

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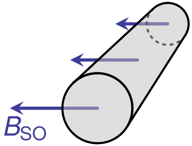

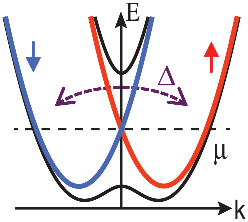

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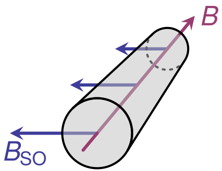

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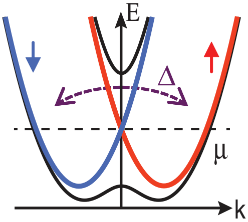

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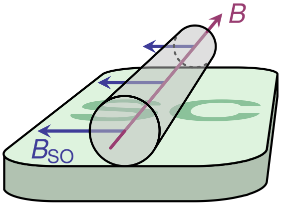

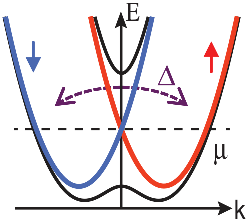

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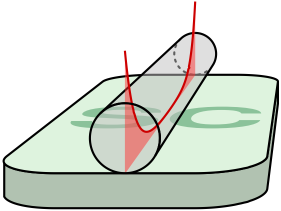

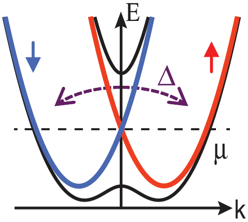

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4. s-wave superconductor close by (induce coupling between k and $-k$ electrons, proximity effect)
5. zero-energy Majorana end-modes $\gamma_1 = \hat{f}^\dagger + \hat{f}$ and $\gamma_2 = \boldsymbol{i}(\hat{f}^\dagger - \hat{f})$ (spin-less, charge-less)

fermionic zero mode

$\hat{f}^\dagger = (\gamma_1 - \boldsymbol{i}\gamma_2)/2$ with $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$

Logical Qubit needs Four Majorana Modes

two Majoranas $\rightarrow$ two-fold degenerate space

- states $|0\rangle$ and $|1\rangle = \hat{f}^\dagger|0\rangle$ with $\hat{f}^\dagger = (\gamma_1 - \boldsymbol{i}\gamma_2)/2$
- spin algebra: $\hat{\sigma}_x = \gamma_1$, $\hat{\sigma}_y = -\gamma_2$, and $\hat{\sigma}_z = \boldsymbol{i}\gamma_1\gamma_2$

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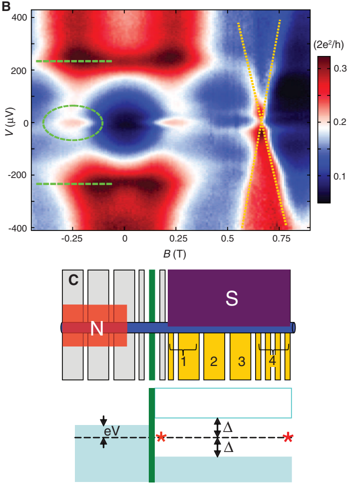
spin-$1/2$ algebra from at least three Majoranas

- Pauli matrices are bilinear products of the γ_i
- for example: $\hat{\sigma}_x = -\boldsymbol{i}\gamma_2\gamma_3$, $\hat{\sigma}_y = \boldsymbol{i}\gamma_1\gamma_3$, and $\hat{\sigma}_z = -\boldsymbol{i}\gamma_1\gamma_2$

Signatures at Zero Bias Voltage

Mourik et al., Science **336**, 1003 (2012)



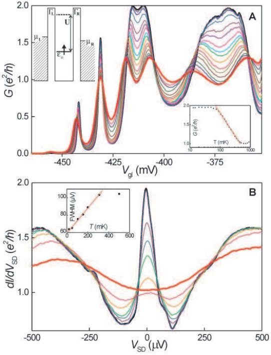
- zero-bias peak (ZBP) in conductance may indicate presence of Majorana mode
- ZBP rather stable against change of magnetic field and gate voltage
- non-locality not explicitly used/tested in experiment
- non-topological ZBP (e.g. due to disorder) might be similar (stability, appearance/disappearance)

The “Traditional” Kondo Effect

e.g. A. Hewson, The Kondo Problem to Heavy Fermions (Cambridge 1997)

in quantum dots:

- arise for coupling of itinerant electrons to degenerate state manifold of quantum spin $\boldsymbol{S}$ (Kondo impurity)
- conduction electrons screen Kondo impurity for small temperatures $T \lesssim T_K$
- opening of zero-bias conducting channel in Coulomb blockade regime



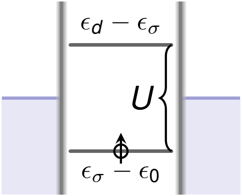
Wiel et al., Science **289**, 2105 (2000)

Kondo model

$$H = \sum_{k\sigma} \epsilon_k \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma} + \frac{J}{2} \sum_{kk'\sigma\sigma'} \left(\hat{c}^\dagger_{k\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} \hat{c}_{k'\sigma'} \right) \cdot \boldsymbol{S}$$

Schrieffer-Wolf transformation

map singly-occupied, spin-degenerate Anderson dot
$H = H_{\text{leads}} - U/2 \sum_\sigma \hat{d}^\dagger_\sigma \hat{d}_\sigma + U \hat{d}^\dagger_\uparrow \hat{d}_\uparrow \hat{d}^\dagger_\downarrow \hat{d}_\downarrow$
to effective Kondo model (for $U \gg T$)



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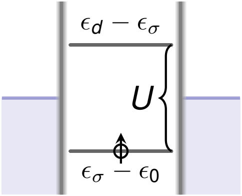

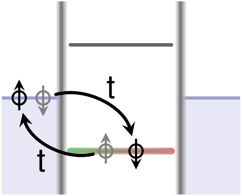

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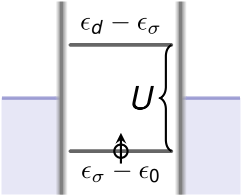

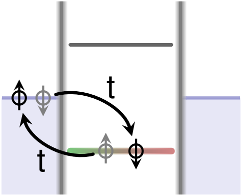

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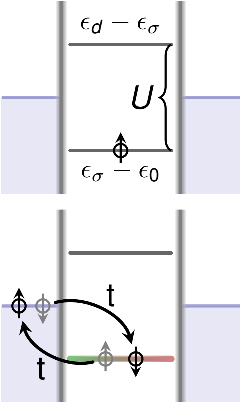

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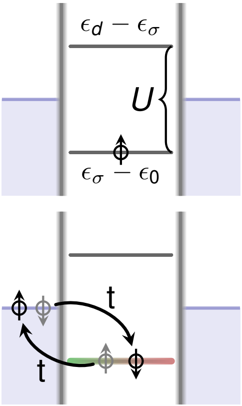
- effective anti-ferromagnetic Heisenberg Hamiltonian



Poor Man's Scaling

P.W. Anderson, JoPC: Sol. St. Phys. **3**, 2436 (1970)

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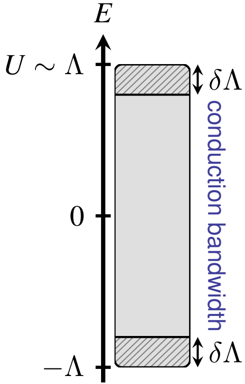

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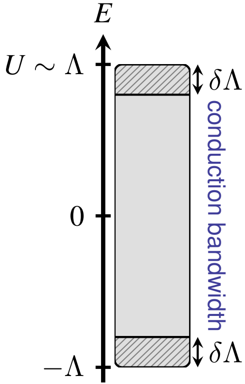

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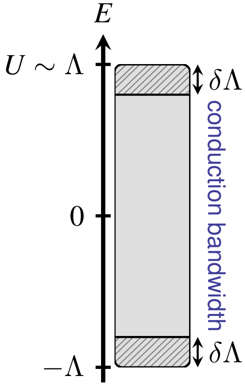

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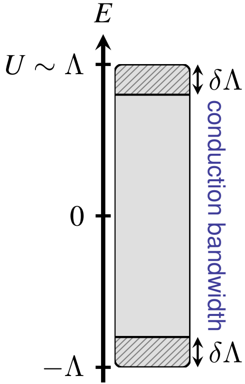

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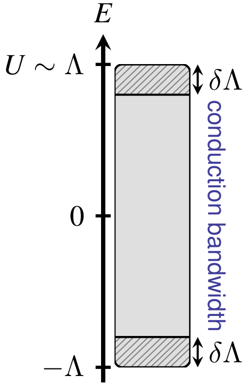

rescaled interaction strength J

$$\delta J = -2\rho J^2 \frac{\delta\Lambda}{\Lambda}$$

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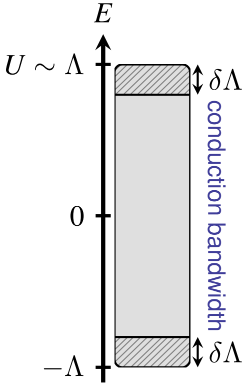

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- integration yields inverse logarithmic scaling of $J(\Lambda)$ for $\Lambda \gtrsim T_K$

$$J(\Lambda) \sim \frac{1}{\ln(\Lambda/T_K)} \text{ with } T_K \sim U\, e^{-1/(\rho J_{\text{bare}})}$$

Topological Kondo Hamiltonian

idea

replace spin-1/2 with topologically degenerate zero-energy state space of three Majorana modes

- charging energy E_C conserves particle number N (and parity) on SC island (corresponds to U)
- three different leads weakly tunnel coupled to separate Majorana modes
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effective Hamiltonian ($H = H_{\text{leads}} + H_{\text{eff}}$)

$$H_{\text{eff}} = \sum_{i \neq j} \lambda_{ij}^{+} \gamma_i \gamma_j \hat{\psi}_i^{\dagger} \hat{\psi}_j - \sum_i \lambda_{ii}^{-} \hat{\psi}_i^{\dagger} \hat{\psi}_i$$

- couplings $\lambda_{ij}^{\pm} = (1/U_{+} \pm 1/U_{-}) t_i t_j$

Emergence of Kondo Problem

non-local term of effective Hamiltonian

$$H_{\mathsf{NL}} = \sum_{i \neq j} \lambda^{+}_{ij} \gamma_i \gamma_j \hat{\psi}^{\dagger}_i \hat{\psi}_j$$

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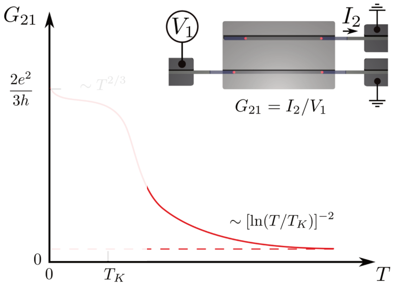
final Kondo Hamiltonian

$$H_{\mathrm{NL}} = \frac{1}{2} \sum_{\alpha} \lambda_\alpha \hat{\sigma}_\alpha \hat{J}_\alpha$$

- coupling constants $\lambda_\alpha = \sum_{ab} |\epsilon_{\alpha ab}| \lambda_{ab}^{+} = 2\gamma_{ab}^{+}$
- "non-local spin-1 object" $\hat{J}_\alpha = \boldsymbol{i} \sum_{ab} \epsilon_{\alpha ba} \hat{\psi}_a^{\dagger} \hat{\psi}_b$

Kondo Signatures in Linear Conductance

1 for $T_K < T$: inverse logarithmic scaling in weak coupling regime



Kondo temperature $T_K \sim E_C\, e^{-1/(\rho\lambda_{\text{bare}})} \lesssim 0.1K$

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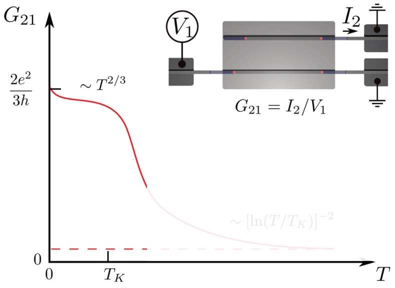
2. for $T \ll T_K$: power-law scaling with $G_{12} = 2e^2/(3h) - |c_{12}|\, T^{2/3}$, non-Fermi liquid behavior



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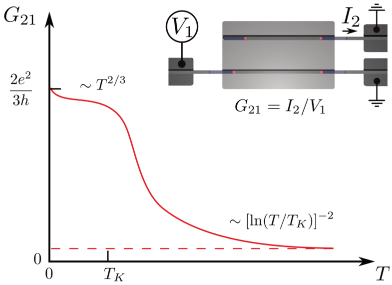
3. dashed line: conductance G_{12} for lead three decoupled $\Longrightarrow$ Ising Hamiltonian



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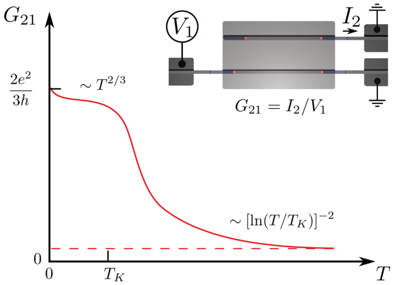
3. dashed line: conductance G_{12} for lead three decoupled $\Longrightarrow$ Ising Hamiltonian
4. flow to isotropic Kondo coupling $\lambda_\alpha/\lambda_\beta \to 1$ for $T \to 0$



Kondo temperature $T_K \sim E_C\, e^{-1/(\rho\lambda_{\text{bare}})} \lesssim 0.1K$

Kondo Signatures in Linear Conductance

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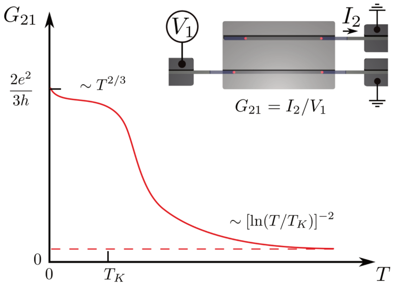

5. diagonal conductance $(4e^2/3h)$ enhanced due to “Andreev reflection fixed point”

Kondo temperature $T_K \sim E_C\, e^{-1/(\rho\lambda_{\text{bare}})} \lesssim 0.1K$

Possible Issues

- very abstract effective model as starting point
- how are Majoranas from different wires coupled?
- topological protection of non-local state might be weak for small island (large E_C)
- larger island with longer wires $\rightarrow$ smaller E_C $\rightarrow$ direct tunneling into $N \pm 1$ states?
- induced gap Δ_{NW} in nanowire large enough?
- effects of disorder, ...