Collectively Enhanced Interactions in Solid-state Spin Qubits

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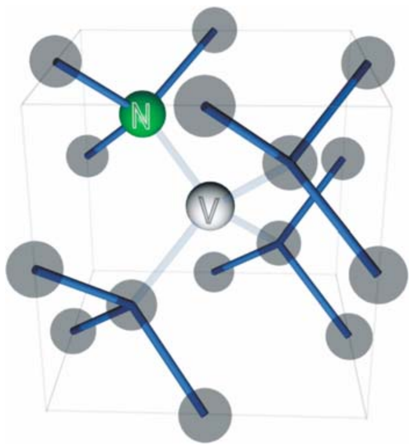

[Jelezko & Wrachtrup, Phys. Stat. Sol. (a) **203**, 3207 (2006)]

We propose and analyze a technique to **collectively enhance interactions** between solid-state quantum registers composed from **random networks of spin qubits**. In such systems, disordered dipolar interactions generically result in localization. Here, we demonstrate the emergence of a single collective delocalized eigenmode as one turns on a transverse magnetic field. The **interaction strength** between this symmetric collective mode and a remote spin qubit **is enhanced by square root of the number of spins** participating in the delocalized mode. Mediated by such collective enhancement, long-range quantum logic between remote spin registers can occur at distances consistent with optical addressing. A specific **implementation utilizing Nitrogen-Vacancy defects in diamond** is discussed and the effects of decoherence are considered.

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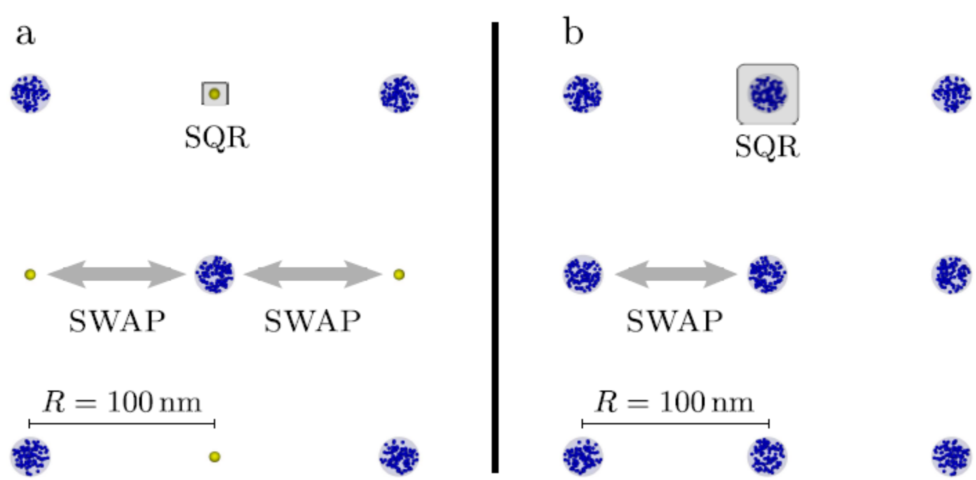
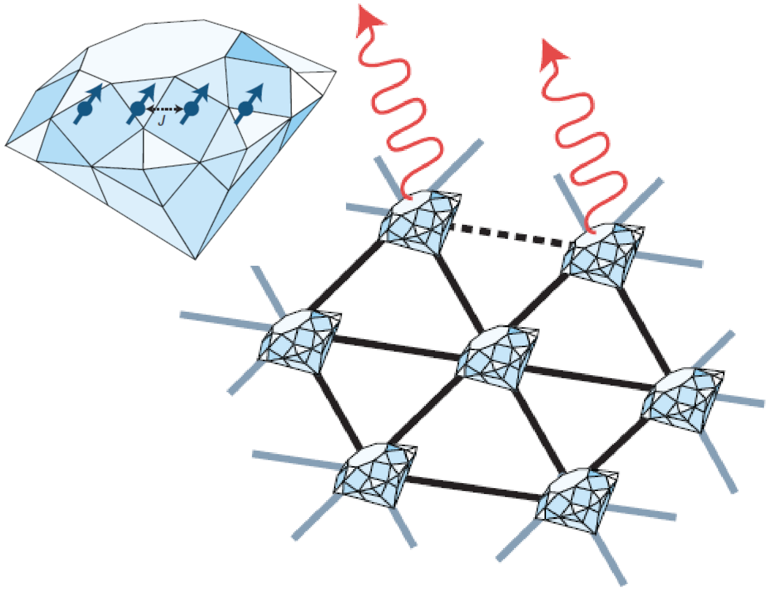


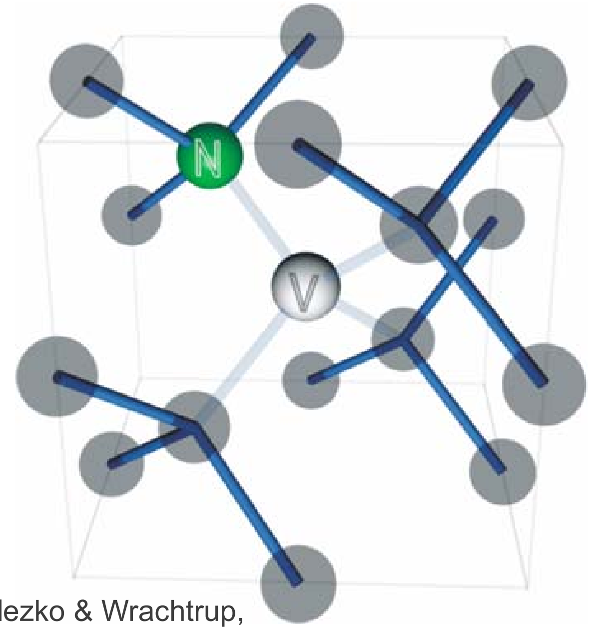


[Morton, Nat. Phys. **2**, 365 (2006)]

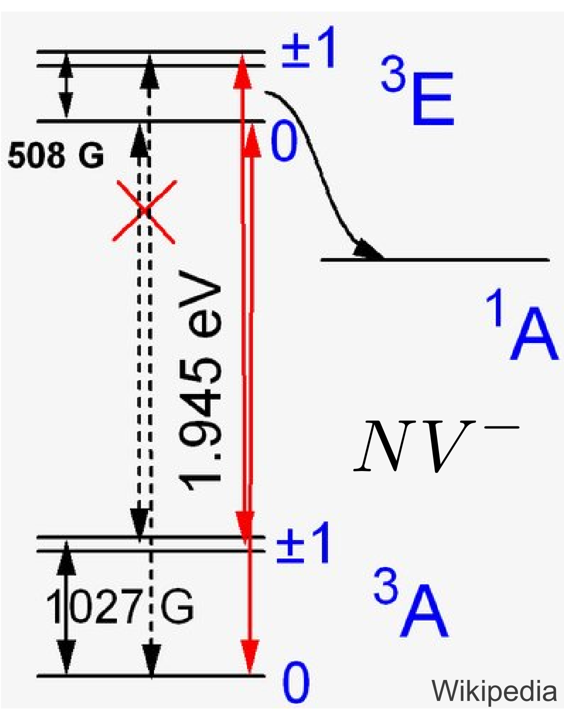
Outline

- Hamiltonian of a NV spin triplet
- **Qualitative picture**
  - Second order perturbation: collective spin state
- **Spin ensemble & qubit coupling – magnetic dipolar coupling**
- Numerical calculation to support qualitative picture
  - “Exact” diagonalization shows collective state
  - **Time-evolution shows enhanced Rabi oscillation frequency**
- Decoherence
- Summary

Nitrogen-Vacancy Color Centers in Diamond



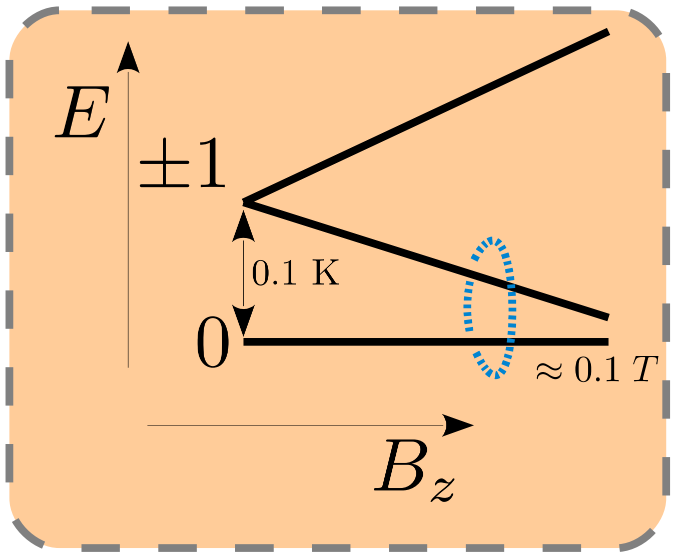
[Jelezko & Wrachtrup, Phys. Stat. Sol. (a) **203**, 3207 (2006)]



Ground state ^{3}A is a triplet (S=1) with a zero field splitting Δ

$$H = \gamma_c \mathbf{B} \cdot \mathbf{S} + D S_z^2$$

[Manson, Harrison, Sellars, Phys. Rev. B **74**, 104303 (2006)]
[Gali, Fyta, Kaxiras, Phys. Rev. B **77**, 155206 (2008)]



$$D = 11.869\ \mu eV$$

$$\gamma_c = 116\ \mu eV/T$$

Negligible nuclear spin corrections

$$+\mathbf{S} \cdot A \cdot \mathbf{I} + Q I_z^2 - \gamma_n \mathbf{B} \cdot \mathbf{I}$$

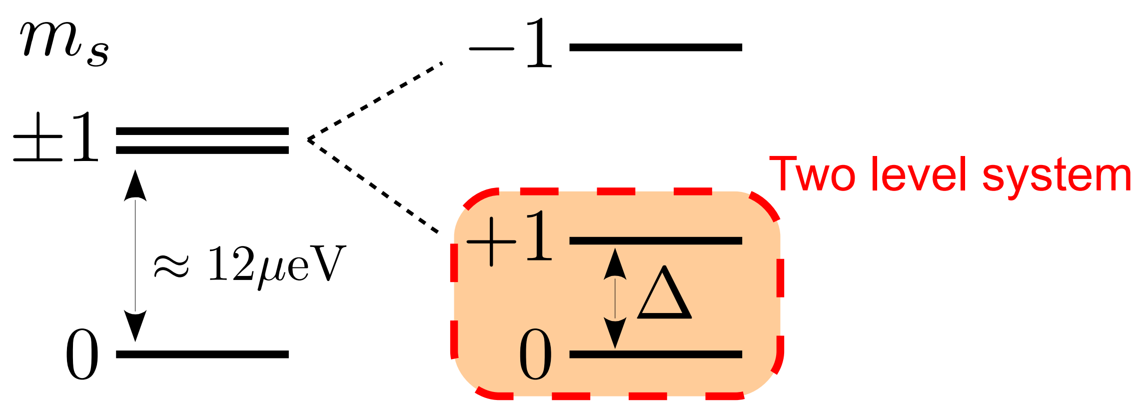
[Fuchs, Burkard, Klimov, & Awschalom, Nat. Phys. **7**, 789 (2011)]

$$A = -0.010\ \mu eV$$

$$Q = -0.02\ \mu eV$$

$$\gamma_n = 0.012\ \mu eV/T$$

Qualitative picture



zero-field splitting

field along NV symmetry axis

N spins ensemble

m = number of NV spins on the +1 state is a good quantum number

$$m = \sum_i m_s^{(i)}$$

$$m_s^{(i)} \in \{0, +1\}$$

$$\downarrow \quad \uparrow$$

m = {0, +1} subspace

$$H = \frac{\Delta}{2} \sum_i \sigma_z^{(i)}$$

Any tensor product state is eigenstate, and collective states as well

$$|1_j\rangle = |0 \ldots 1_j \ldots\rangle \qquad |W\rangle = \frac{1}{\sqrt{N}} \sum_j |0 \ldots 1_j \ldots\rangle$$

Transversal magnetic field couples +1 and 0

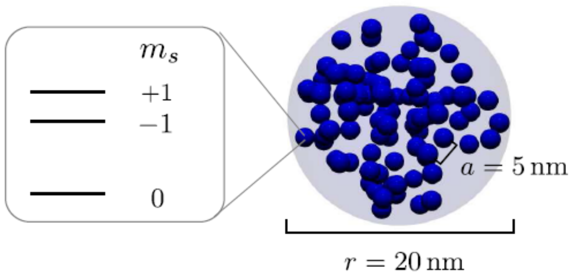
$$\delta H = \Omega \sum_i \sigma_x^{(i)}$$

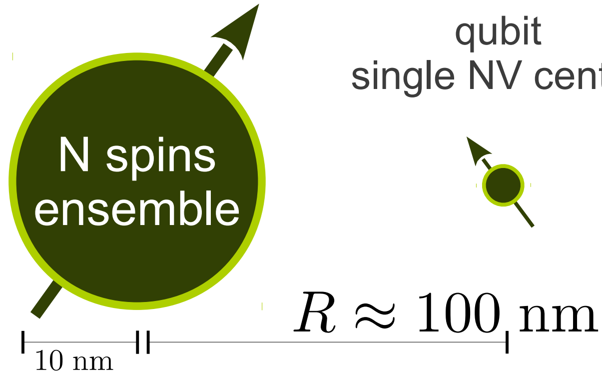
Second order perturbation theory

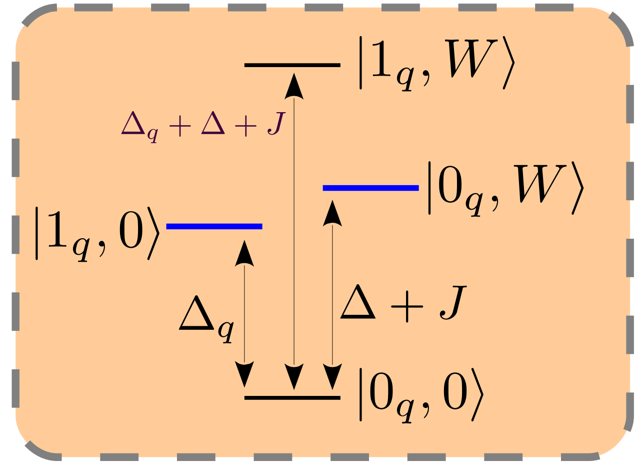
$$H_{eff} = -\Delta|0\rangle\langle 0| + J|W\rangle\langle W|$$

$$J = N\Omega^2/\Delta$$

Dipolar coupling: ensemble – qubit







Dipolar coupling:

{i,j} runs among ensemble spins & qubit

$$V_{i,j} = \left(1 - 3\cos^2\theta_{i,j}\right) \frac{\mu^2}{|r_i - r_j|^3} \left\{ \frac{1}{4}\left[1 + \sigma_z^{(i)}\right]\left[1 + \sigma_z^{(j)}\right] - \sigma_+^{(i)}\sigma_-^{(j)} - \sigma_-^{(i)}\sigma_+^{(j)} \right\}$$

On the collective state approximation: $|r_q - r_W| \approx R$ $\quad \theta_{q,W}$: constant

$$V_{eff} = \left(\frac{3\cos^2\theta_{q,W} - 1}{2}\right)\sqrt{N}\,\frac{\mu^2}{R^3}\left(|1_q, 0\rangle\langle 0_q, W| + \text{h.c.}\right)$$

Rabi frequency $\propto \sqrt{N}$

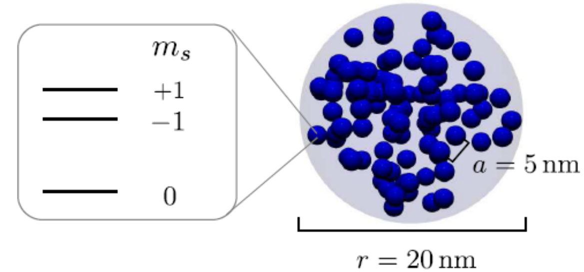
$|1_q, 0\rangle$ ⇄ $|0_q, W\rangle$

$$V_{eff} = \sqrt{N}\,\frac{\mu^2}{R^3}\left(|1_q, 0\rangle\langle 0_q, W| + \text{h.c.}\right)$$

“Exact” Diagonalization – $m_s = \{0,+1\}$ subspace

To support the qualitative picture

$$H = \frac{\Delta}{2}\sum_i \sigma_z^{(i)} + \Omega \sum_i \sigma_x^{(i)} + \sum_{i<j} V_{i,j}$$



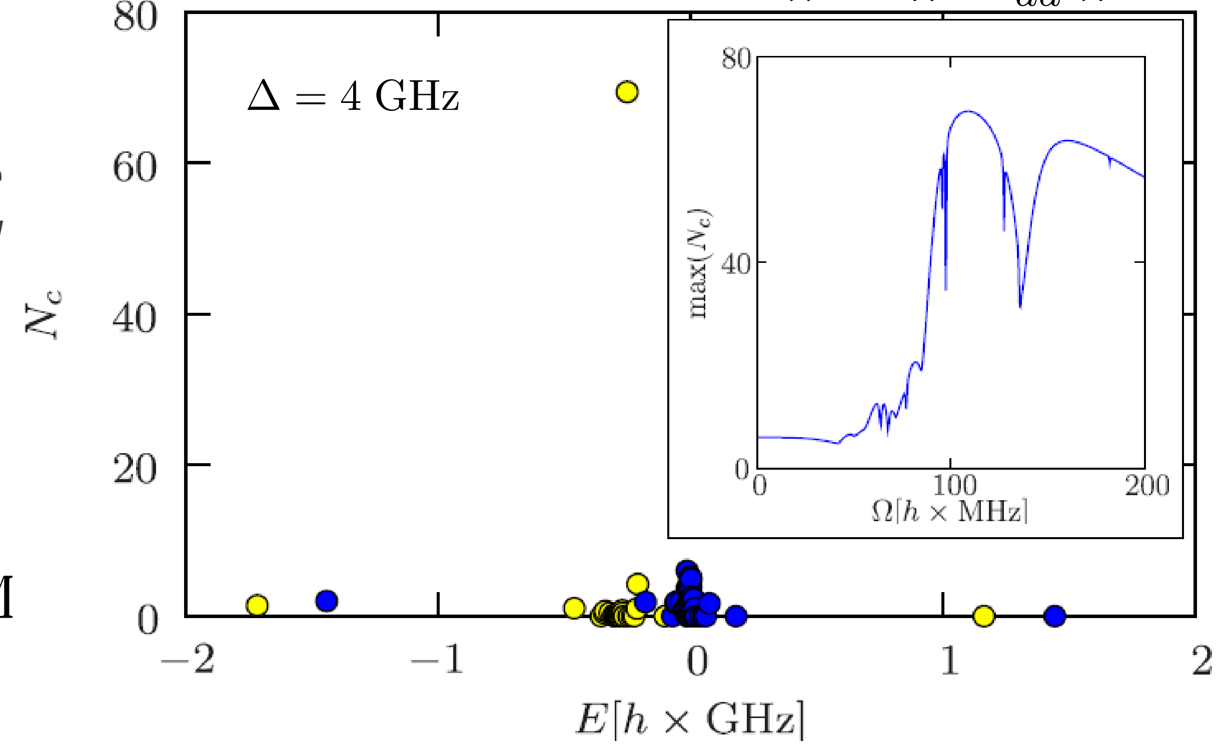
$$V_{i,j} = \left(1 - 3\cos^2\theta_{i,j}\right)\frac{\mu^2}{|r_i - r_j|^3}\left\{\frac{1}{4}\left[1+\sigma_z^{(i)}\right]\left[1+\sigma_z^{(j)}\right] - \sigma_+^{(i)}\sigma_-^{(j)} - \sigma_-^{(i)}\sigma_+^{(j)}\right\}$$

Enhancement factor defined as

$$N_C = \left(\sum_i^N \langle 0\ldots 1_i \ldots |\phi\rangle\right)^2$$

“which essentially characterizes the number of ensemble spins participating in the eigenmode”

$$\Delta \gg J \gg V_{dd} \gg \Omega$$

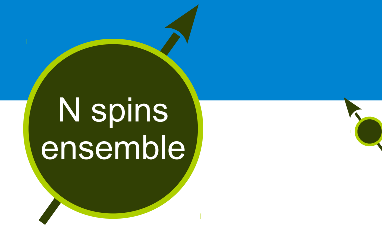


- No transversal field $\Omega = 0$
  - disorder localizes all eigenstates

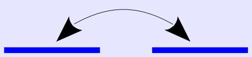
$$N_c \ll N$$

- Small transverse field $\Omega = 100$ M
  - One collective state $N_c \approx N$

Combined ensemble + quBit system



Rabi frequency $\propto \sqrt{N}$

$|1_q, 0\rangle$  $|0_q, W\rangle$

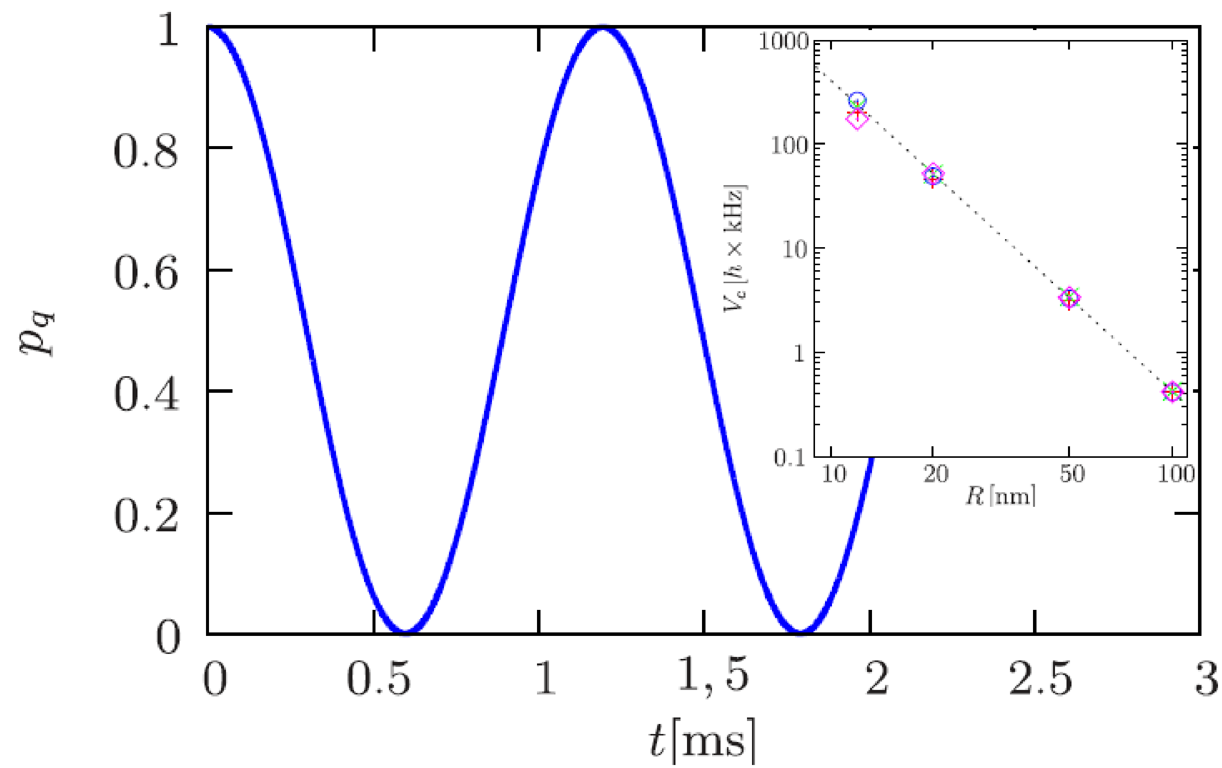
- Initialized to $|1_q, 0\rangle$
- Time-evolution: *not explained in the text*

$$U(t, t+\delta t) = \exp\frac{-iH\delta t}{\hbar}$$

$$\psi(t+\delta t) - \psi(t) = \frac{-i\delta t}{\hbar} H\psi(t)$$

$$p_q = |\langle 1_q, 0|\phi\rangle|^2 = \cos^2\left(\frac{\pi t}{2t_\pi}\right)$$

$$V_c = \frac{h}{4t_\pi} = \sqrt{N}\frac{\mu^2}{R^3}$$



Environment

Dephasing

"leaking out into non-symmetric states"

$$\frac{p_{\bar{W}}}{p_{T_2}} = \left[1 - \left|\langle W|\sigma_z^{(i)}|W\rangle\right|^2\right] = \frac{4}{N}\left(1 - \frac{1}{N}\right)$$

Probability to leave the state W

p_{T_2} ↓ single spin dephasing rate

$$p_{\bar{W}} \propto \frac{4}{T_2}$$

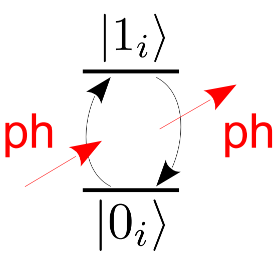
Total error probability after single T_2 event,

$$\sum_i \rightarrow 4\left(1 - \frac{1}{N}\right) \approx 4$$

⇒ Does not scale with N

Depolarization

"phonon-induced spin depolarization processes"



$$\frac{p_{W\rightarrow 0}}{p_{T_1^{1\rightarrow 0}}} = \left|\langle 0|\sigma_-^{(i)}|W\rangle\right|^2 = \frac{1}{N} \xrightarrow{\sum_i} 1$$

⇒ Does not scale with N

$p_{T_1^{0\rightarrow 1}}$: action of $\sigma_+^{(i)}|W\rangle$

- drives the state into the m=2 subspace
- tunes out of resonance (ensemble – quBit)
- single event is already destructive

$$p_{T_1} \propto \frac{N}{T_1}$$

⇒ Scales with N

Environment

Despite the scaling

$$p_{\bar{W}} \propto \frac{4}{T_2} \qquad p_{T_1} \propto \frac{N}{T_1}$$

Typically $T_1 \gg T_2$

thus, proposal is still useful as long as $\frac{T_1}{N} > T_2$

decoherence dominated by dephasing → $p_{\bar{W}} > p_{T_1}$ → Does not scale with N

NV centers in diamond

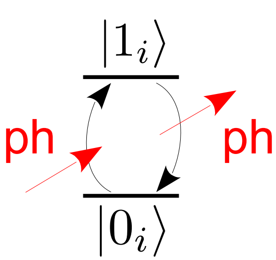
"The dephasing of the NV originates from fluctuating magnetic fields as neighboring pairs of dipoles flip-flop."

[G. Balasubramanian et al., Nature Mater. **8**, 383 (2009)]

"Assuming an external magnetic field parallel to the z axis of the nitrogen-vacancy defect, the Ixy part of the nuclei interaction Hamiltonian (which causes dynamics of the ^{13}C nuclear spin bath) then leads to flip-flop processes, where two nuclei exchange their Iz components. This causes a fluctuating magnetic field that is responsible for dephasing of the electron spin."

[Bar-Gill, Pham, Belthangady, Le Sage, Cappellaro, Maze, Lukin, Yacoby, Walsworth, Nature Comm. **3**, 858 (2012)]

"Room-Temperature Quantum Bit Memory Exceeding One Second" [Maurer, Kucsko, Latta, Jiang, Yao, Bennett, Pastawski, Hunger, Chisholm, Markham, Twitchen, Cirac, Lukin, Science **336**, 1283 (2012)]



$T_2 \rightarrow$ miliseconds

T_1: Orbach spin-phonon process

can be suppressed at low temperatures

$$T_1 \gg 1\text{ s}$$

Dynamical decoupling (e.g., WAHUHA) to further increase T_1

Summary

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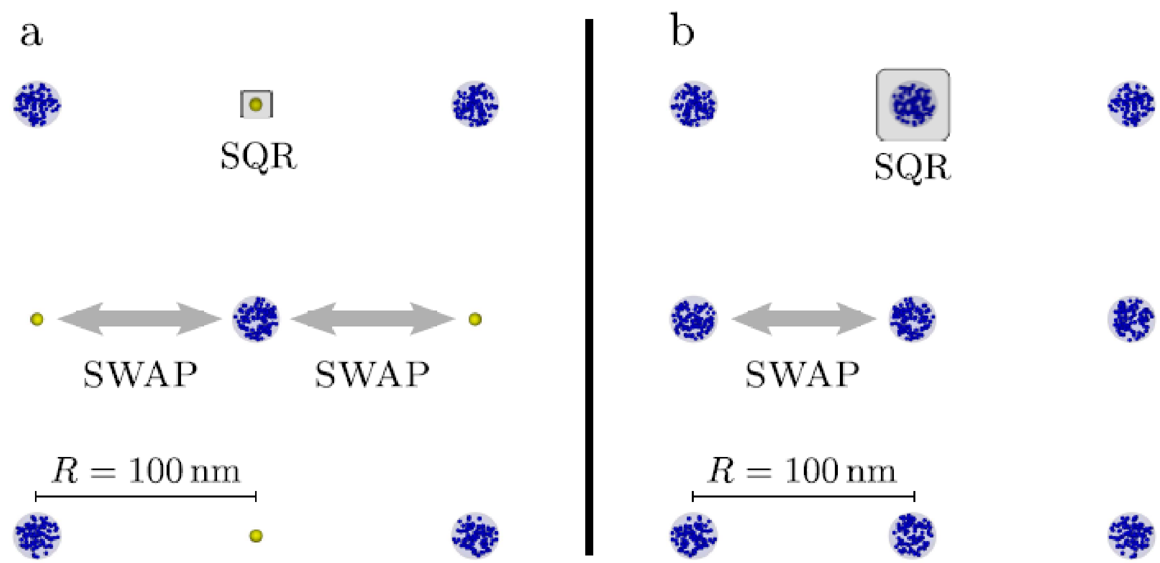
Authors define the error of a gate operation (4 SWAP operations) as:

$$\varepsilon = 1 - \exp\left[-(4t_\pi/T_2^{eff})^3\right]$$

$$\varepsilon = 10^{-2} \rightarrow T_2^{eff} = 11 \text{ ms}$$



SWAP operation

$$p_q = |\langle 1_q, 0|\phi\rangle|^2 = \cos^2\left(\frac{\pi t}{2t_\pi}\right)$$

$$\frac{1}{t_\pi} = \sqrt{N}\frac{4\mu^2}{hR^3}$$

- NV^- spin ensemble coupled to single NV spin quBit
  - Enhanced interaction (Rabi oscillation frequency)
  - Collective state stabilized by transverse magnetic field

Effective Hamiltonian

$$H = -\Delta + \frac{\Delta}{2}\sum_i [1 + \sigma_z^{(i)}]$$

$$\delta H = \Omega \sum_i \sigma_x^{(i)}$$

Hamiltonian and perturbation

Some definitions

$$|0\rangle = |0\ldots0\ldots\rangle$$
$$|1_j\rangle = |0\ldots1_j\ldots\rangle$$
$$|1_i 1_j\rangle = |0\ldots1_i\ldots1_j\ldots\rangle$$
$$|W\rangle = \frac{1}{\sqrt{N}}\sum_i |1_i\rangle$$

Diagonal terms

$$H|0\rangle = -\Delta$$
$$H|1_j\rangle = 0|1_j\rangle$$
$$H|1_i 1_j\rangle = \Delta|1_i 1_j\rangle$$

$$\varepsilon_0 = -\Delta$$
$$\varepsilon_W = \frac{N\Omega^2}{\Delta}$$

leading order

$$|\psi_0\rangle = |0\rangle \qquad |\psi_W\rangle = |W\rangle$$

	$\lvert 0\rangle$	$\lvert W\rangle$	$\lvert “2”\rangle$	$\lvert \cdots\rangle$
$\langle 0\rvert$	$-\Delta$	$\sqrt{N}\Omega$	0	0
$\langle W\rvert$	$\sqrt{N}\Omega$	0	$\sqrt{2N-2}\Omega$	0
$\langle “2”\rvert$	0	$\sqrt{2N-2}\Omega$	Δ	$\ddots$
$\langle \cdots\rvert$	0	0	$\ddots$	$\ddots$

Non-diagonal terms

$$\delta H|0\rangle = \Omega\sum_i |1_i\rangle = \sqrt{N}\Omega|W\rangle$$

$$\delta H|W\rangle = \sqrt{N}\Omega|0\rangle + \frac{\Omega}{\sqrt{N}}\sum_j\sum_{i\neq j}|1_i 1_j\rangle$$