# Surface code with decoherence: An analysis of three superconducting architectures

Joydip Ghosh,[1,*] Austin G. Fowler,[2,†] and Michael R. Geller[1,‡]

[1] *Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602, USA*
[2] *Centre for Quantum Computation and Communication Technology, School of Physics, The University of Melbourne, Victoria 3010, Australia*

(Dated: October 23, 2012)

We consider a realistic, multi-parameter error model and investigate the performance of the surface code for three possible fault-tolerant superconducting architectures. We map amplitude and phase damping to a diagonal Pauli "depolarization" channel via the Pauli twirl approximation, and obtain the logical error rate as a function of the qubit $T_{1,2}$ and intrinsic state preparation, gate, and readout errors. A numerical Monte Carlo simulation is performed to obtain the logical error rates and a leading order analytic model is constructed to estimate their scaling behavior below threshold. Our results suggest that large-scale fault-tolerant quantum computation should be possible with existing superconducting devices.

PACS numbers: 03.67.Lx, 03.67.Pp, 85.25.-j

**arXiv:1210.5799**

**Journal Club Nov. 6, 2012.**
**Adrian Hutter**

# Outline

1. Surface code reminder
2. Decoherence model, error sources
3. Three superconducting archidectures
4. Threshold-$T_1$

# Surface code quantum computing

- Based on Kitaev’s toric code
  [Kitaev, Annals Phys. **303**, 2 (2003)]
- Requires only nearest-neighbor CNOT and single-qubit control to allow for ***fault-tolerant universal quantum computing***
  [Groszkowski et al, Phys. Rev. B **84**, 144516 (2011)]
- Threshold error rate per gate is ***1%*** with MWPM based error correction
  [Wang et al, Phys. Rev. A **83**, 020302 (2011)]

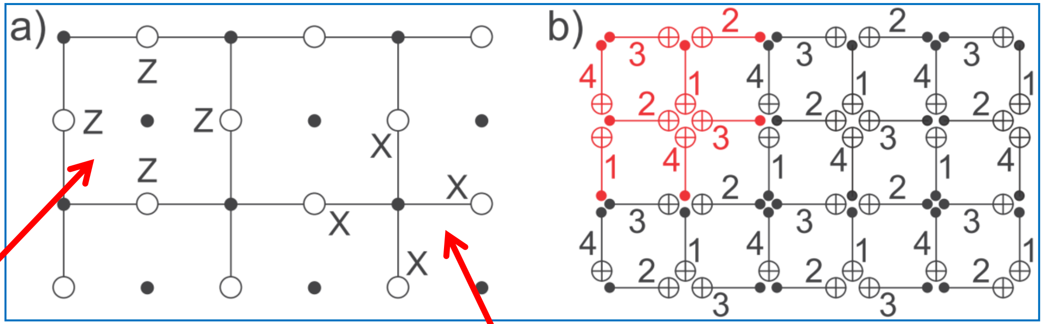
# Surface code quantum computing

- Below threshold, the probability of a *logical* error (i.e. the probability of a failure of error correction) is

$$\sim p^{\frac{d+1}{2}}$$

- *p*: physical error probability
- *d*: <u>distance</u> of the code (minimal number of single-qubit errors to create one logical error)

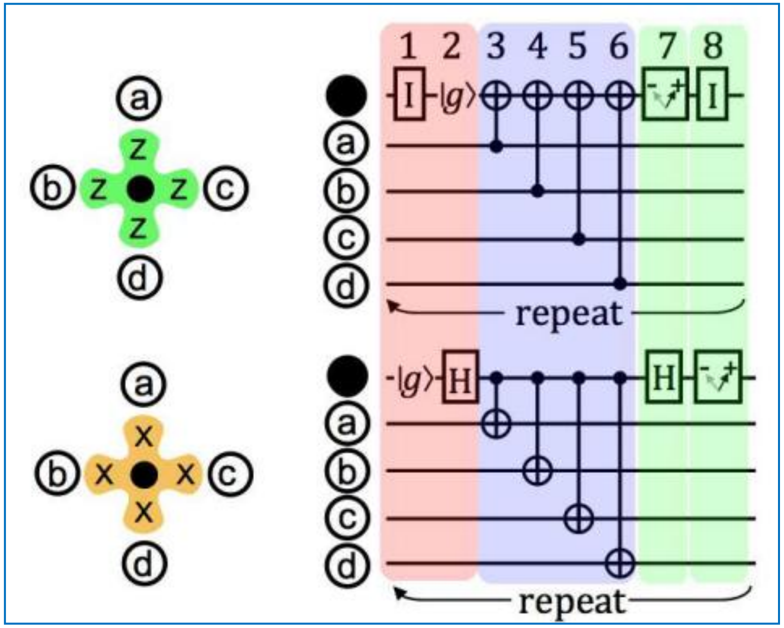
# Error correction cycle



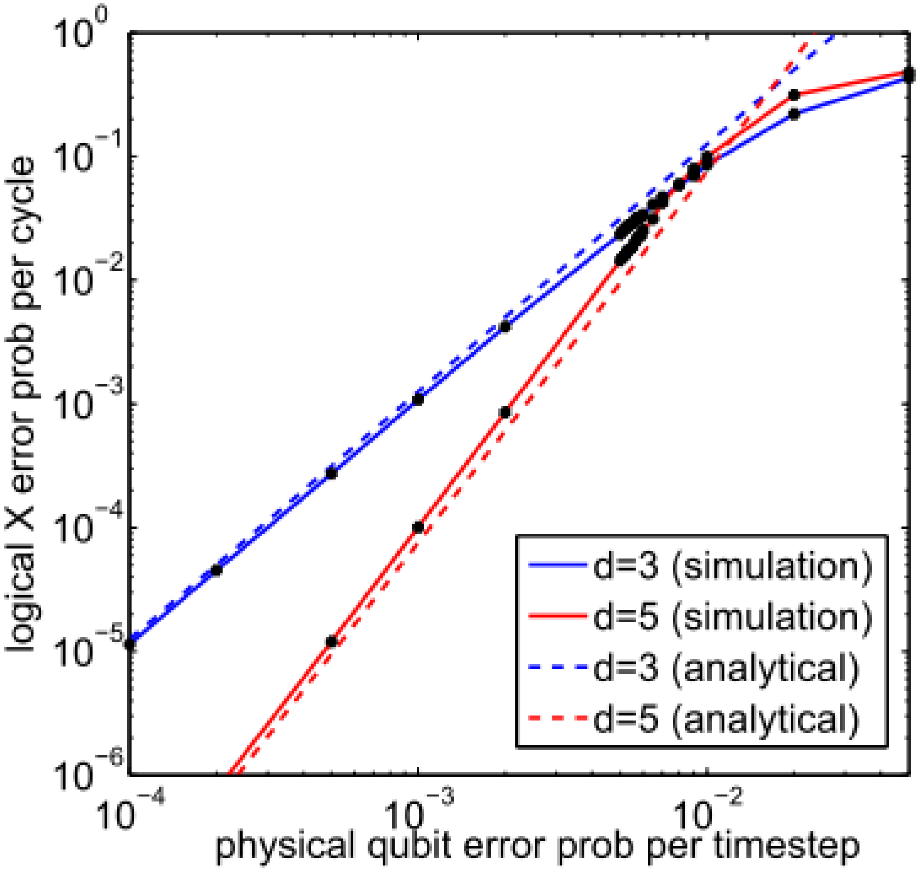

$$\sigma^z\sigma^z\sigma^z\sigma^z|\psi\rangle = |\psi\rangle$$

$$\sigma^x\sigma^x\sigma^x\sigma^x|\psi\rangle = |\psi\rangle$$

Figure taken from
[Wang et al, PRA (2011)]



# Error rates: analytics vs simulation




- Monte Carlo simulation: include effects of imperfect CNOT's → error propagation
- Analytical:
  - assume perfect four-qubit measurements
  - Failure probability is dominated by misidentifying $\frac{d+1}{2}$ errors as $\frac{d-1}{2}$ errors (100%) or as inequivalent $\frac{d+1}{2}$ errors (50%)

  $\rightarrow p_L \sim p^{\frac{d+1}{2}}$

$\Rightarrow$ Excellent approximation for $p \ll p_c$ and small $d$

# Error sources

Assume Markovian, uncorrelated, and independent errors

- Decoherence (c.f. next slide)
- Leakage (projection out of computational subspace)
- Unitary rotation errors

# Decoherence model

- Amplitude damping: spontaneous emission of energy to environment (photon emission) → probability $p_{AD}$
- Phase damping: random phase kicks on a single qubit → probability $p_{PD}$

$$1 - p_{AD} = e^{-t/T_1}$$

$$\sqrt{(1 - p_{AD})(1 - p_{PD})} = e^{-t/T_2}$$

# Single-qubit evolution

Actual decoherence:

$$\mathfrak{E}(\rho) = \begin{pmatrix} 1 - \rho_{11} e^{-t/T_1} & \rho_{01} e^{-t/T_2} \\ \rho_{01}^{*} e^{-t/T_2} & \rho_{11} e^{-t/T_1} \end{pmatrix}$$

Aysmmetric depolarization channel (ADC):

$$\mathfrak{E}_{ADC}(\rho) = (1 - p_x - p_y - p_z)\rho + p_x X\rho X + p_y Y\rho Y + p_z Z\rho Z$$

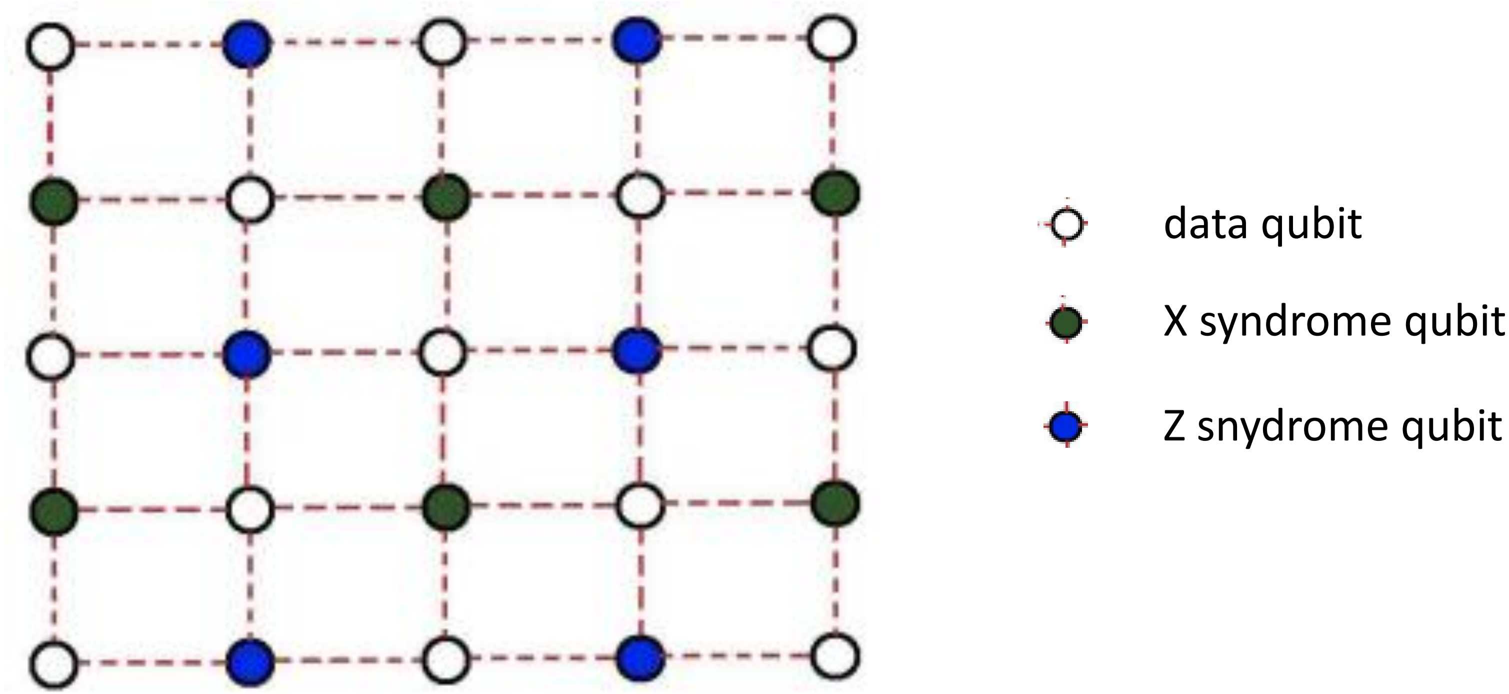
ADC can be simmulated efficiently on a classical computer (c.f. *Gottesmann-Knill-Theorem*), but $\mathfrak{E}(\rho) \neq \mathfrak{E}_{ADC}(\rho)$ !

→ «Pauli Twirl Approximation»: simply remove all off-diagonal terms like $X\rho Y$

→ $p_x(\mathrm{t}) = p_y(\mathrm{t}) = \frac{1-e^{-t/T_1}}{4}$ and $p_z(\mathrm{t}) = \frac{1-e^{-t/T_2}}{2} - \frac{1-e^{-t/T_1}}{4}$

# Architecture I: Textbook

## Distance-3 surface code (textbook architecture)



# Architecture I: Textbook

- Transmon qubits arranged in 2D square lattice
  [Schoelkopf group, Yale]
- Nearest-neighbor tunable coupling (infinite on-off ratio is assumed)
- Single-qubit gates by use of DRAG pulses
  [Motzoi et al, Phys. Rev. Lett. **103**, 110501 (2009)]
- State preparation via ideal projective measurements + local rotation

# Architecture I: Textbook

- Tunable couplers have been demonstrated.
- It is unknown whether they are practical for use in a large-scale qc because of the additional associated harware complexity.
- The textbook archidecture likely provides a bound on the performance of any possible superconducting surface code implementation.

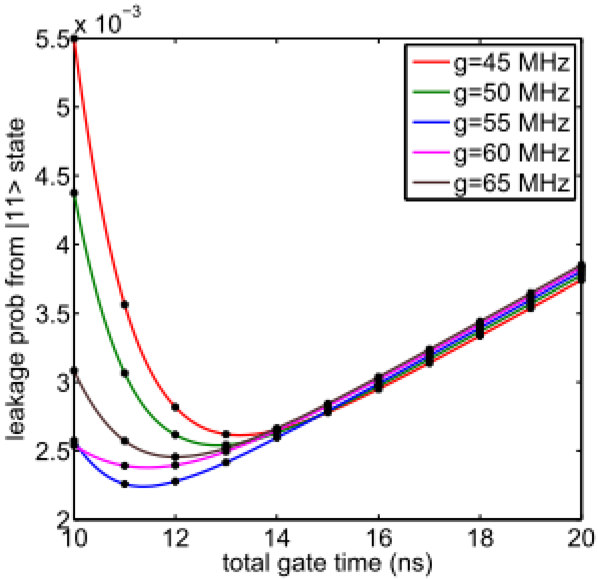
# Qubit-coupling under decoherence

Three-level qubits $q_1$ and $q_2$

**controlled-Z** between $q_1$ and $q_2$

$$H(t) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega_1(t) & 0 \\ 0 & 0 & 2\omega_1(t) - \eta \end{pmatrix}_{q_1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & 2\omega_2 - \eta \end{pmatrix}_{q_2} + g \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i\sqrt{2} \\ 0 & i\sqrt{2} & 0 \end{pmatrix}_{q_1} \otimes \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i\sqrt{2} \\ 0 & i\sqrt{2} & 0 \end{pmatrix}_{q_2}$$

$$\omega_1(t=0) = 8\,GHz, \quad \omega_2 = 6\,GHz, \quad \eta = 300\,MHz$$
$$T_1 = T_2 = 10\mu s$$




→ leakage is minimal for $g = 55\,MHz$ and $t_{CZ} = 11\,ns$

→ **intrinsic error rate** for CNOT is $\mathbf{1.23 \cdot 10^{-4}}$

$$CNOT = (\mathbb{I} \otimes H)\, CZ\, (\mathbb{I} \otimes H)$$

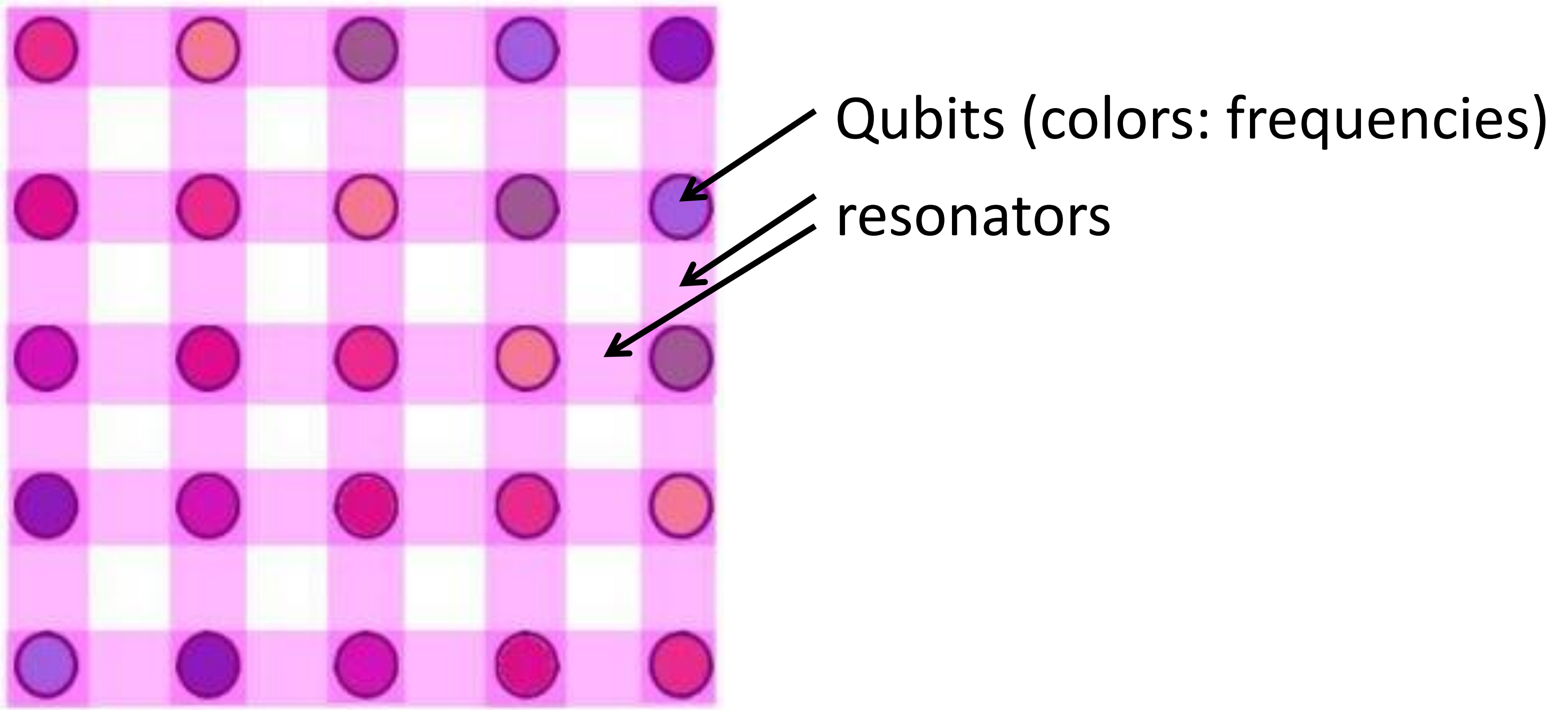
Single-qubit Hadamard gates need 5 $ns$

→ $\boldsymbol{t_{CNOT} = 21\,ns}$

# Architecture II: Helmer

[Helmer et al, EPL **85**, 50007 (2009)]

Distance-3 surface code (Helmer architecture)



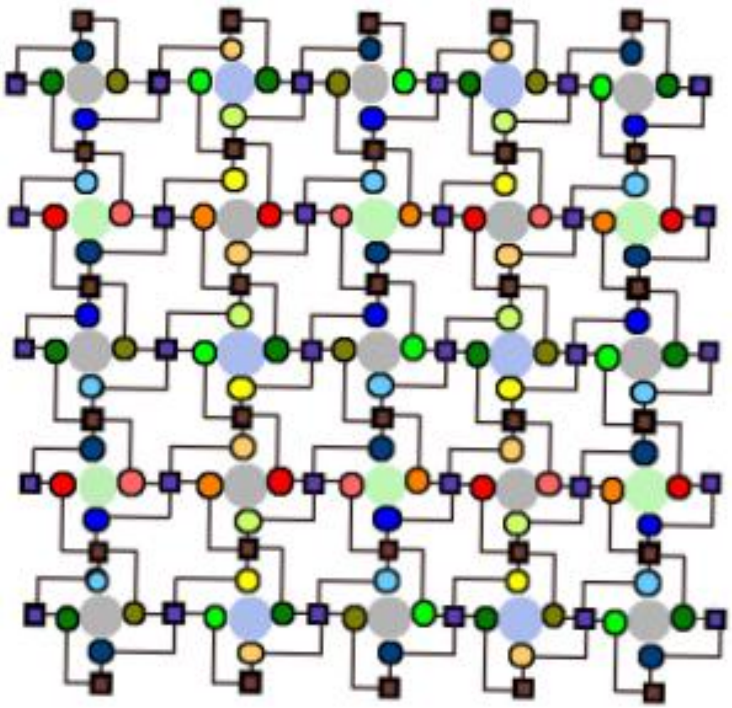
# Architecture II: Helmer

- Each qubit in a 2D lattice is coupled to a horizontal as well as a vertical cavity.
- Hor. and vert. cavities are maintained at different frequencies, qubit frequencies are varied between them.
- CNOT between adjacent qubits via effective two-qubit flip-flop interaction
- *NOT* scalable (required frequency range grows with numer of qubits)

# Architecture III: DiVincenzo

[DiVincenzo, Phys. Script. **2009**, 014020 (2009)]

## Distance-3 surface code (DiVincenzo archidecture)



Bounded circles: qubits
Squares: resonators
Colors: frequencies

Unbounded circles:
: data qubit block

: Z syndrome block

: X syndrome block

# Architecture II: DiVincenzo

- *Scalable* (number of required qubit frequencies is indep. of number of qubits)
- Each qubit is dispersively coupled to two resonators
- Every *data* or *syndrome* qubit consists of four physical qubits
- Qubit and resonator frequencies are fixed
- CNOT gates via a cross-resonance protocol using microwaves

# Parameters for the three archidectures

TABLE I. Parameters used for the three architectures.

| Notation | Description | Architectures: textbook | Helmer | DiVincenzo |
|---|---|---|---|---|
| $T_1$ | qubit relaxation time | 1-10 $\mu s$ | 1-10 $\mu s$ | 1-40 $\mu s$ |
| $T_2$ | qubit dephasing time | $T_1$ | $T_1$ | $2T_1$ |
| $t_{\text{QSP}}$ | state preparation time | 40 ns | 40 ns | 40 ns |
| $t_{\text{loc}}$ | local rotation time | 5 ns | 5 ns | 5 ns |
| $t_{\text{meas}}$ | measurement time | 35 ns | 35 ns | 35 ns |
| $t_{\text{CNOT}}$ | CNOT gate time | 21 ns | 20 ns | 20 ns |
| $t_{\text{cycle}}$ | time duration of a single cycle | 164 ns | 160 ns | 400 ns |
| $p_{\text{intr}}$ | leakage probability for CNOT | $10^{-4}$ | $10^{-3}$ | $10^{-3}$ |
| $p_{\text{meas}}$ | measurement error probability | $10^{-2}$ | $10^{-2}$ | $10^{-2}$ |
| $p_{\text{QSP}}$ | state preparation error probability | $10^{-2}$ | $10^{-2}$ | $10^{-2}$ |

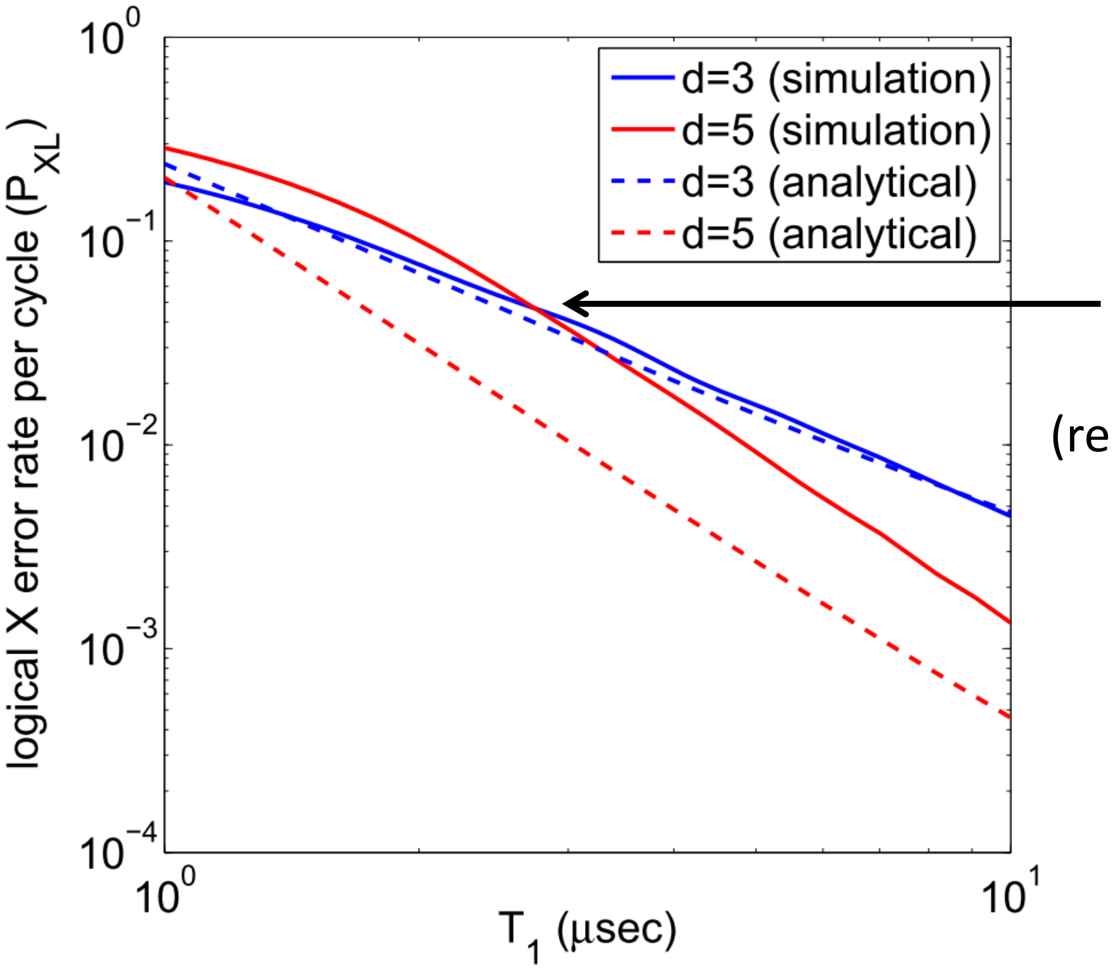
**estimates**

For **textbook** and **Helmer**:

- Additional source of dephasing in tunable transoms $\rightarrow$ assume $T_1 = T_2$
- $t_{\text{cycle}} = t_{\text{QSP}} + t_{\text{loc}} + 4 \cdot t_{\text{CNOT}} + t_{\text{meas}}$

# Finding threshold-$T_1$

For **textbook**:




Threshold: 2.6 $\mu s$

(reminder: $t_{\text{cycle}} = 164$ ns)

# Summary of threshold-$T_1$

| Architecture | Thresholds | |
|---|---|---|
| | logical X error | logical Z error |
| Textbook | 2.6 $\mu s$ | 2.6 $\mu s$ |
| Helmer | 2.8 $\mu s$ | 2.8 $\mu s$ |
| Divincenzo | 10 $\mu s$ | 5 $\mu s$ |

- For $T_1$ above threshold: going from $L = 3$ to $L = 5$ helps, otherwise it hurs.

- **Textbook** & **Helmer**: $T_2 = T_1 \rightarrow$ symmetry between X and Z
- **DiVincenzo**: $T_2 = 2T_1 \rightarrow$ higher prob. for X errors than Z errors

# Conclusions

- For an estimated intrinsic error probability ($10^{-4}$ resp. $10^{-3}$), decoherence times of a few $\mu s$ are sufficient for small-distance surface code quantum computing.
- The threshold values are «within the reach of current state-of-the-art design of superconducting qubits».
- «The time requirement for qubit state preparation and read-out is, however, yet to be achieved experimentally up to the order assumed in this work.»