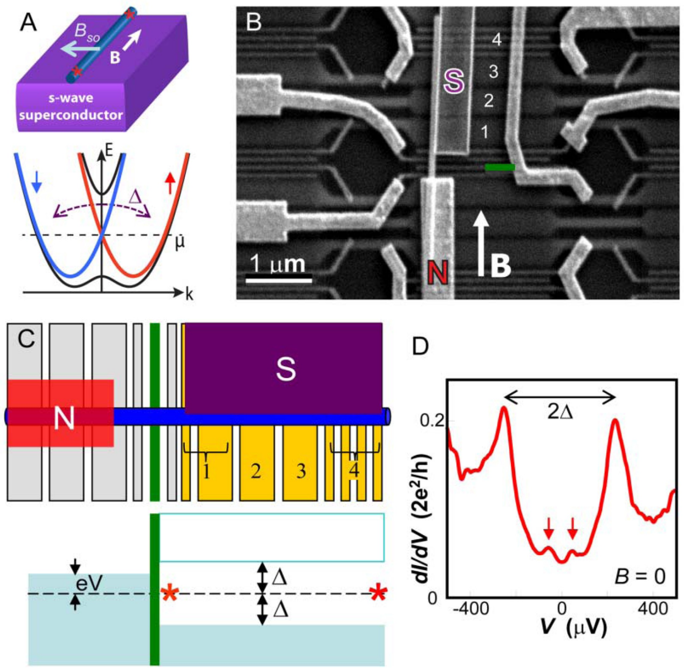
The soft superconducting gap in semiconductor Majorana nanowires

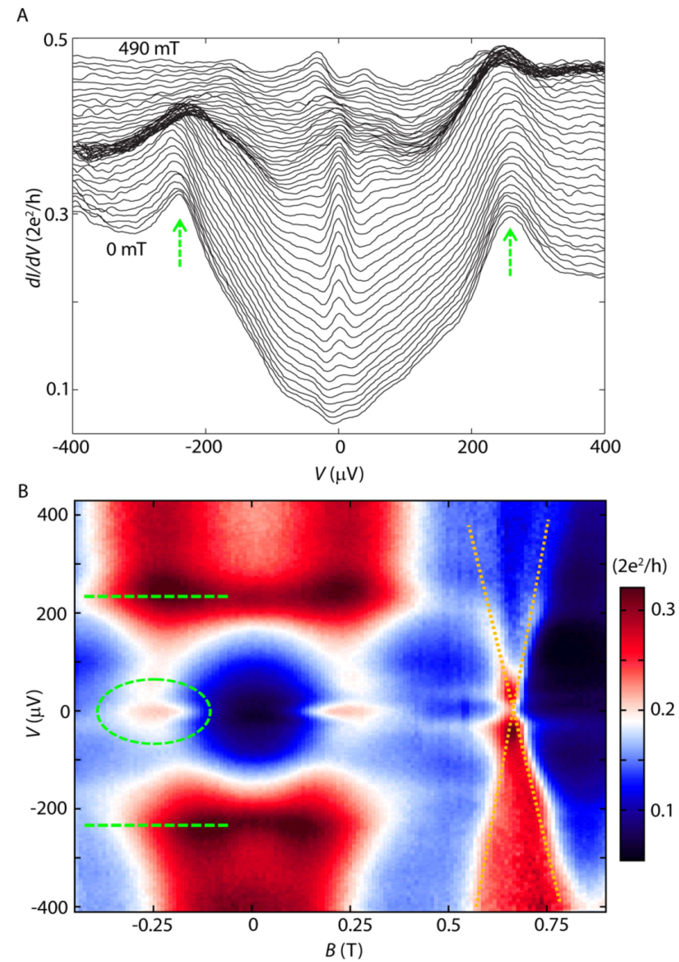
So Takei, Benjamin Fregoso, Hoi-Yin Hui, Alejandro M. Lobos, and S. Das Sarma

arXiv:1211.1029

Journal Club *13.11.12* *Jelena Klinovaja*

Delft experiment





$l_{\rm so} \approx 200$ nm

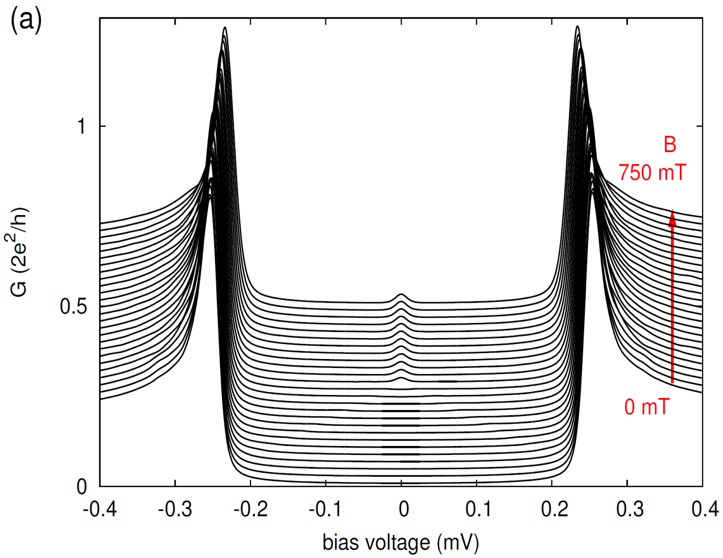
$\alpha \approx 0.2$ eV•Å

$\Delta \approx 250$ μeV

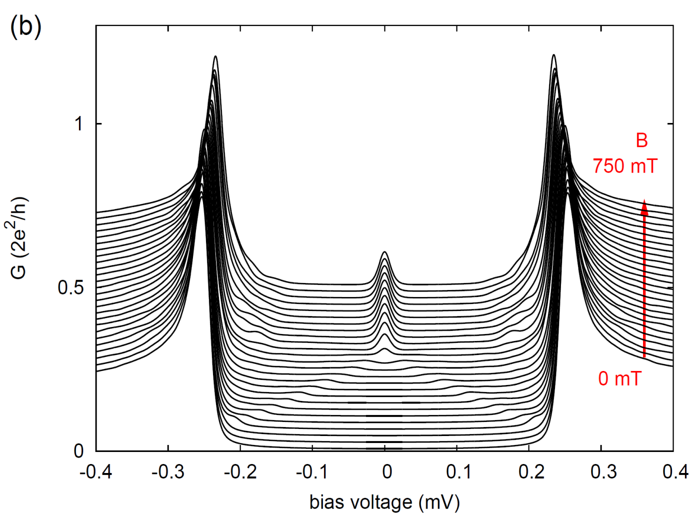
no closing of the topological gap!!!

V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, L. P. Kouwenhoven, Science **336**, 1003 (2012).

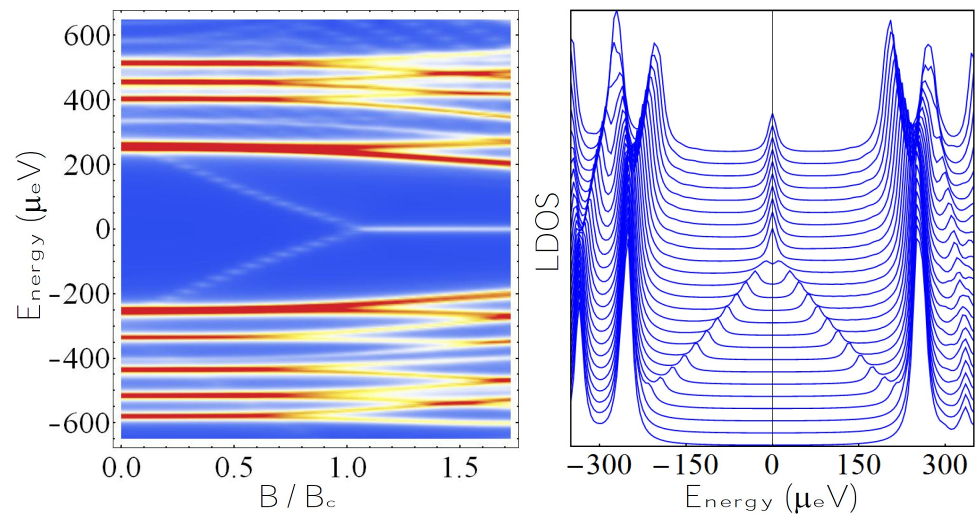
Numerical simulations



clean wire



weak disorder



Tudor D. Stanescu, Sumanta Tewari, Jay D. Sau, and S. Das Sarma, arXiv:1206.0013

Falko Pientka, Graham Kells, Alessandro Romito, Piet W. Brouwer, and Felix von Oppen, arXiv:1206.0723

Possible reasons:

(a) non-magnetic disorder in the NW;

(b) magnetic disorder in the NW;

(c) temperature;

(d) dissipative quasiparticle broadening arising;

(e) inhomogeneities at the SC-NW interface

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Theoretical model

kinetic energy

spin orbit interaction

Zeeman energy

$$\hat{H}_w = \int_0^{L_x} dx \Big\{ c_s^\dagger(x) \Big[-\frac{\partial_x^2}{2m_e^*} - \mu(x) + i\alpha_R \sigma_y \partial_x - B_Z \sigma_x - \boldsymbol{b}(x) \cdot \boldsymbol{\sigma} \Big]_{ss'} c_{s'}(x) + \Delta(x) \left(c_\uparrow^\dagger(x) c_\downarrow^\dagger(x) + \mathrm{H.c.} \right) \Big\}.$$

fluctuating proximity-induced superconductivity

Theoretical model

kinetic energy

spin orbit interaction

Zeeman energy

$$\hat{H}_w = \int_0^{L_x} dx \Big\{ c_s^\dagger(x) \Big[\boxed{-\frac{\partial_x^2}{2m_e^*}} - \mu(x) + \boxed{i\alpha_R \sigma_y \partial_x} - \boxed{B_Z \sigma_x}$$

$$- \boldsymbol{b}(x) \cdot \boldsymbol{\sigma} \Big]_{ss'} c_{s'}(x) + \boxed{\Delta(x) \left(c_\uparrow^\dagger(x) c_\downarrow^\dagger(x) + \mathrm{H.c.} \right)} \Big\}.$$

fluctuating proximity-induced superconductivity

Disorder

$$\mu(x) = \mu_0 + \delta\mu(x)$$

static non-magnetic disorder

$$\boldsymbol{b}(x)$$

static magnetic disorder

Theoretical model

kinetic energy

spin orbit interaction

Zeeman energy

$$\hat{H}_w = \int_0^{L_x} dx \Big\{ c_s^\dagger(x) \Big[\boxed{-\frac{\partial_x^2}{2m_e^*}} - \mu(x) + \boxed{i\alpha_R \sigma_y \partial_x} - \boxed{B_Z \sigma_x} - \boldsymbol{b}(x)\cdot\boldsymbol{\sigma} \Big]_{ss'} c_{s'}(x) + \boxed{\Delta(x)\left(c_\uparrow^\dagger(x) c_\downarrow^\dagger(x) + \text{H.c.}\right)} \Big\}.$$

fluctuating proximity-induced superconductivity

Disorder

$$\mu(x) = \mu_0 + \delta\mu(x)$$ static non-magnetic disorder

$$\boldsymbol{b}(x)$$ static magnetic disorder

spatial (and associated potential) fluctuations in the barrier separating the NW and the SC

spatial (and associated potential) fluctuations in the barrier separating the NW and the SC

$$\Delta(x) = \gamma(x)\Delta_{SC} / (\gamma(x) + \Delta_{SC})$$

transparency of the NW-SC interface

uniform parent pairing in the bulk SC

$$\gamma(x) = \rho_0 t_\perp^2(x)$$

LDOS in the NW at the Fermi energy in the normal phase

the tunneling matrix element connecting the SC and NW

$$t_\perp(x) = t_\perp^0 e^{-\kappa\delta d(x)}$$

$$\gamma(x) \ll \Delta_{SC}$$

$$\Delta(x) \approx \gamma(x)$$

$$\Delta(x) = \Delta_0 e^{-2\delta\beta(x)}$$

$$\delta\beta(x) = \kappa\delta d(x)$$

Numerical model - nanowire

$$\hat{H}_w = -t\sum_{\langle ij\rangle} c^\dagger_{is} c_{js} + i\alpha \sum_i \left[c^\dagger_{is}\sigma^y_{ss'}(c_{i+1s'} - c_{i-1s'})\right]$$
$$- \sum_i c^\dagger_{is}\left[\mu_0 + \delta\mu_i - B_Z\sigma^x - \boldsymbol{b}_i \cdot \boldsymbol{\sigma}\right] c_{is'}$$
$$+ \left[\Delta_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} + \text{H.c.}\right]$$

$$\left\langle \delta\mu_i \delta\mu_j \right\rangle = W_\mu^2 \delta_{ij}$$

$$\left\langle b_i^p b_j^q \right\rangle = W_b^2 \delta_{ij}\delta_{pq}$$

$$\left\langle \delta\beta_i \delta\beta_j \right\rangle = W_\beta^2 \delta_{ij}$$

Gaussian-distributed random variables

$$\Delta(x) = \Delta_0 e^{-2\delta\beta(x)}$$

$$P(\Delta_i) = \frac{1}{2\Delta_i\sqrt{2\pi}W_\beta}\exp\left[-\frac{1}{8W_\beta^2}\ln^2\left(\frac{\Delta_i}{\Delta_0}\right)\right]$$

$$\langle \Delta_i \rangle = \int_0^\infty d\Delta_i \; P(\Delta_i)\,\Delta_i$$

$$W_\Delta^2 = \int_0^\infty d\Delta_i \; P(\Delta_i)\left(\Delta_i - \langle \Delta_i \rangle\right)^2$$

Tunneling experiment

$$\hat{H} = \hat{H}_w + \hat{H}_L + \hat{H}_t$$

$$\hat{H}_L = \sum_{ks} \varepsilon_k d^\dagger_{ks} d_{ks}$$

lead

$$\hat{H}_t = t_L \sum_{ks} d^\dagger_{ks} c_{1s} + \text{H.c.}$$

tunneling

$$G(V,T) = -2\pi e^2 t_L^2 \rho_L \int_{-\infty}^{\infty} d\omega \rho_1^w(\omega) f'(\omega - eV)$$

local density of states

$$f(x) = (e^{x/T} + 1)^{-1}$$

$$\rho_i^w(\omega) = -\frac{1}{\pi} \text{Im}\, g_{ii}^w(\omega)$$

Local density of states

$$\{u^{(0)}_{is,n}, v^{(0)}_{is,n}\}$$

eigenstates

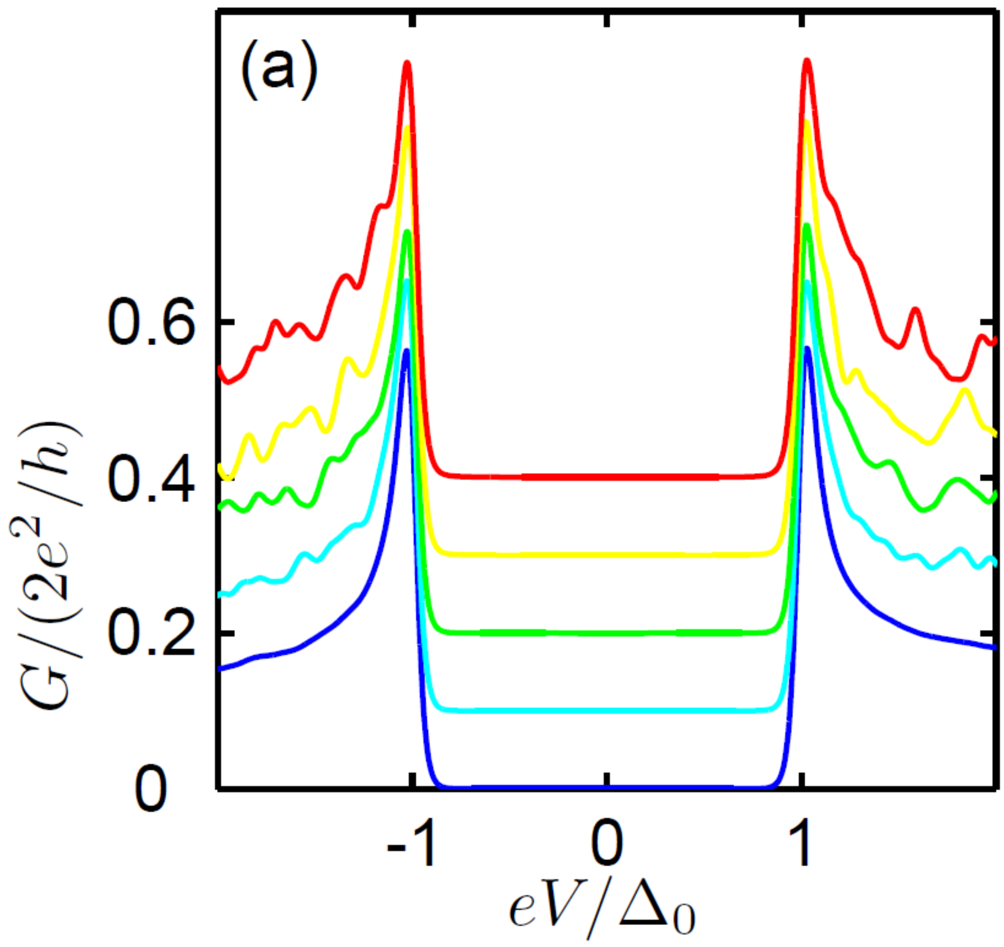
$$\rho^w_i(\omega) = -\frac{1}{\pi}\mathrm{Im}\, g^w_{ii}(\omega)$$

$t_L \to 0$

$$g^w_{ij}(\omega) = \sum_{ns} \frac{u^{(0)*}_{is,n} u^{(0)}_{js,n}}{\omega - E^{(0)}_n + i\gamma_{L,n}} + \frac{v^{(0)*}_{is,n} v^{(0)}_{js,n}}{\omega + E^{(0)}_n + i\gamma_{L,n}}$$

$$\gamma_{L,n} = -i\pi\rho_L t_L^2 \sum_s \left(|u^{(0)}_{1s,n}|^2 + |v^{(0)}_{1s,n}|^2\right)$$

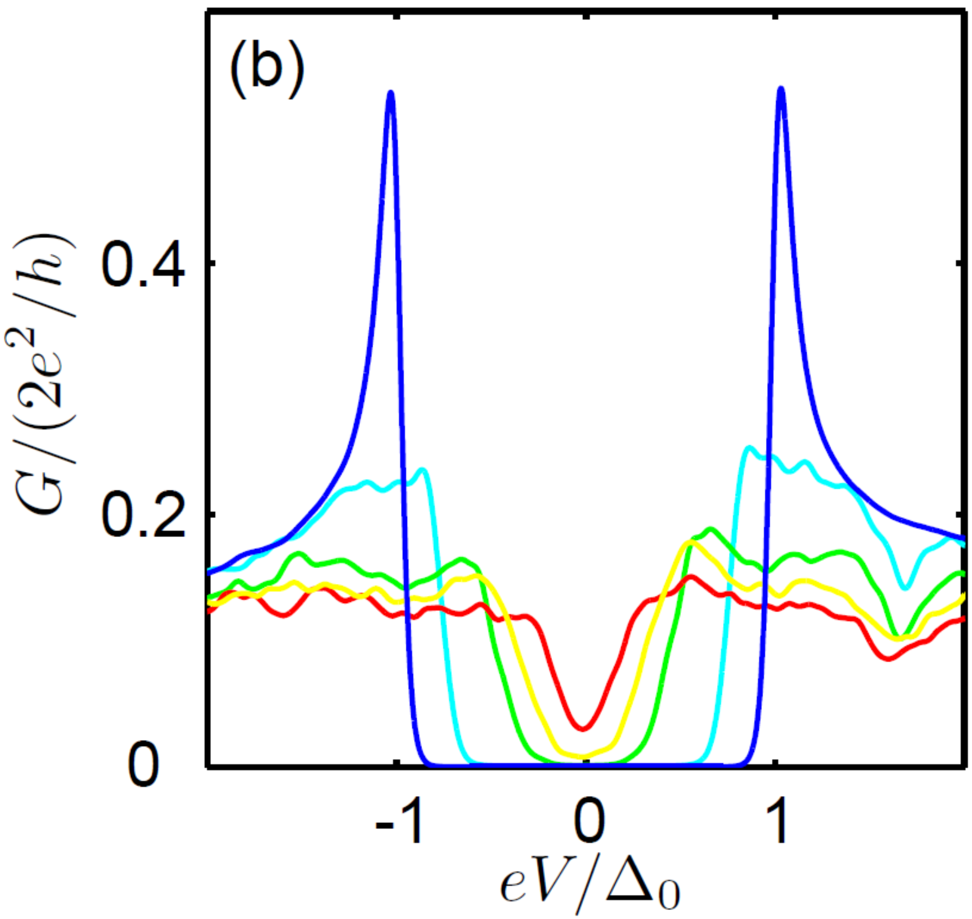
Results – static non-magnetic impurities



$T = 70\text{mK}$

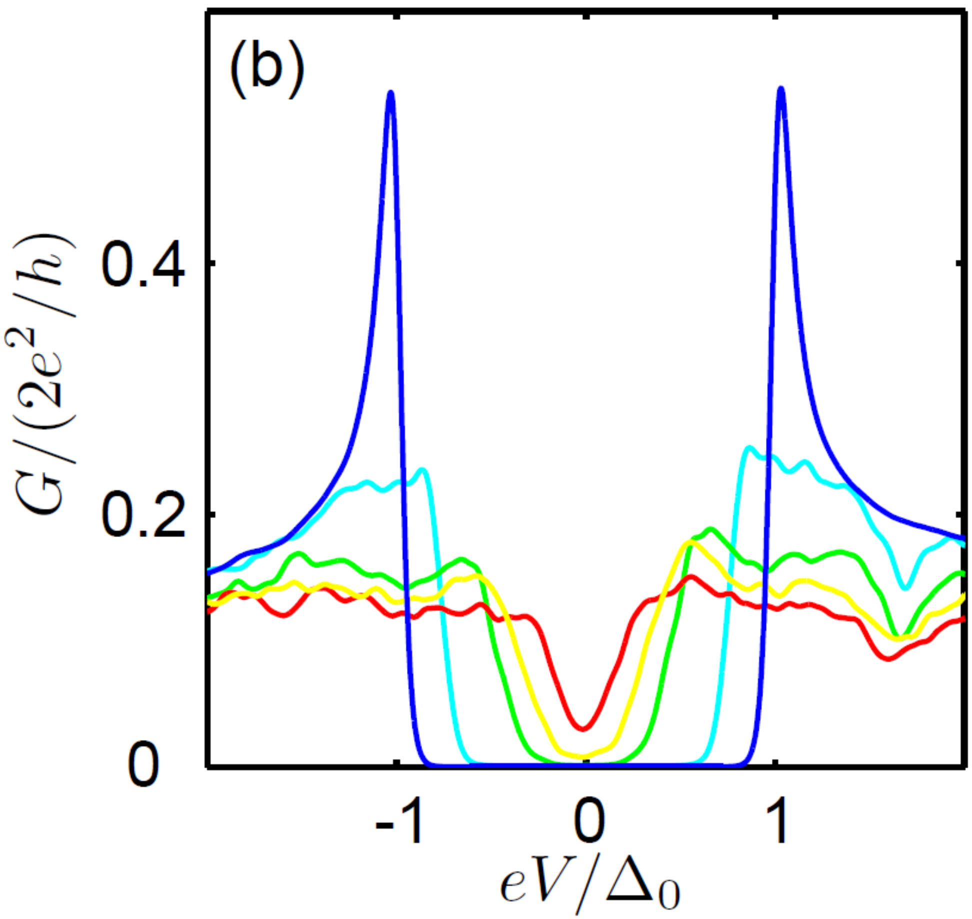
$W_\mu = 0$ $W_\mu = 0.8\Delta_0$

Results – static magnetic impurities



$W_b = 0, 0.27\Delta_0, 0.54\Delta_0, 0.68\Delta_0$ and $0.81\Delta_0$

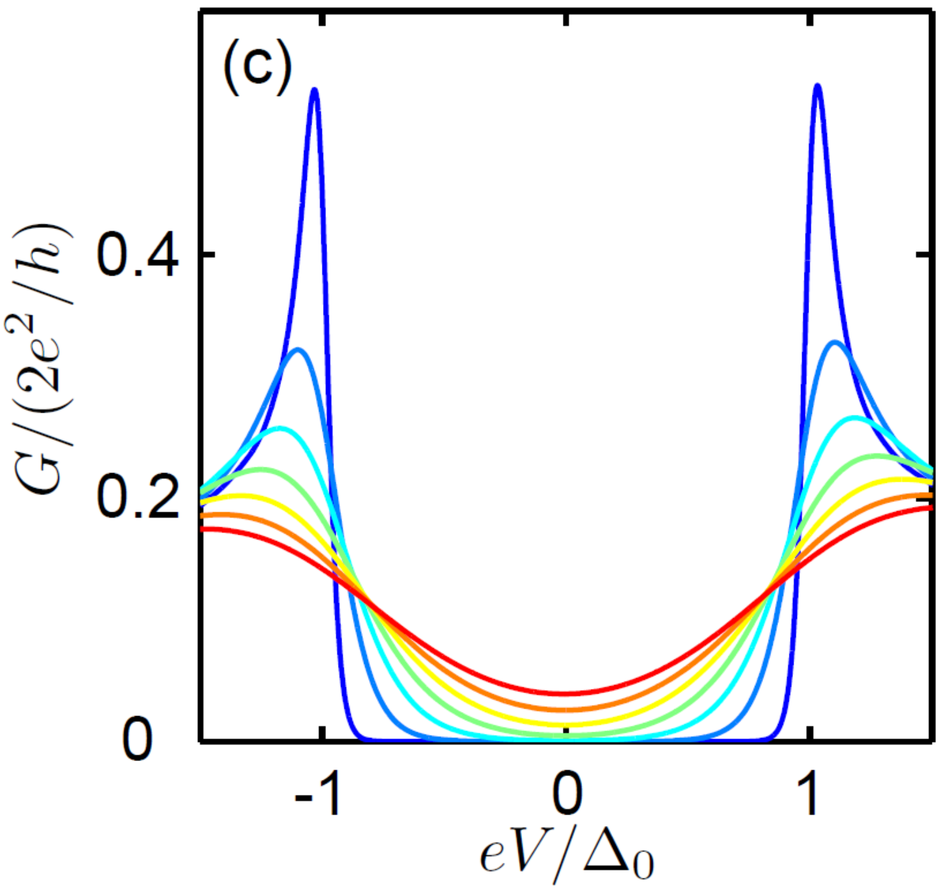
Results – static magnetic impurities



$W_b = 0,\ 0.27\Delta_0,\ 0.54\Delta_0,\ 0.68\Delta_0$ and $0.81\Delta_0$

Such a large amount of magnetic disorder is unlikely to be present in the NW used in the experiments.

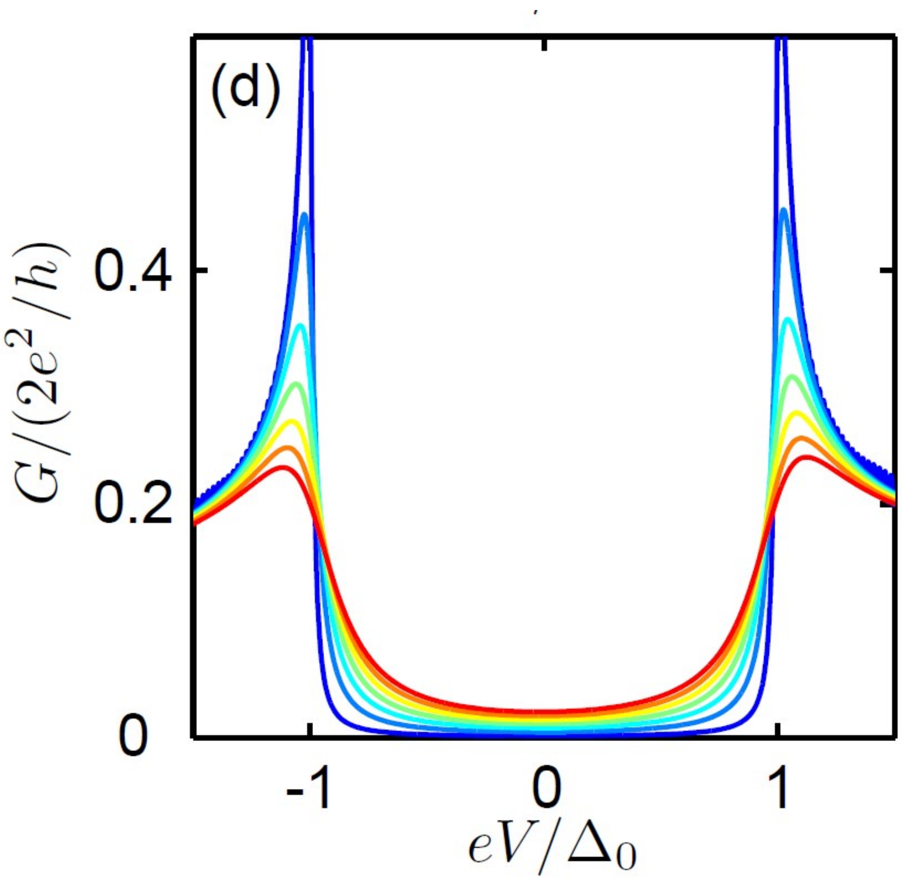
Results – thermal effects



$T = 0.027\Delta_0$ (blue curve) to $T = 0.35\Delta_0$ (red curve)

$0.027\Delta_0 \approx 78\text{mK}$

Results – quasiparticle broadening

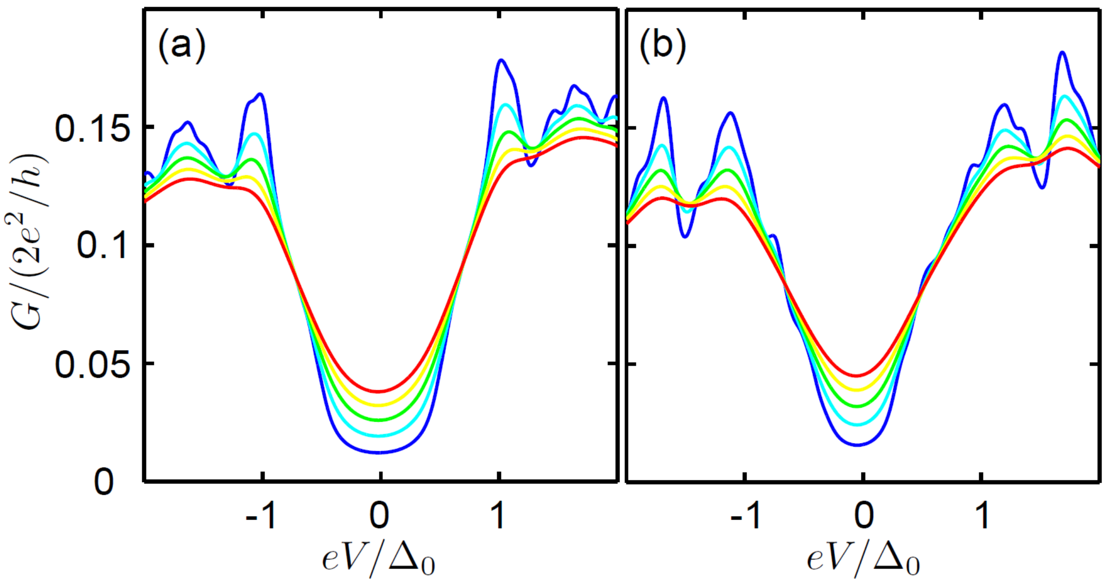


$$\gamma_N = 0.027\Delta_0 \text{ (blue curve)}$$

$$\gamma_N = 0.35\Delta_0 \text{ (red curve)}$$

$$\omega \rightarrow \omega + i\gamma_N$$

Results – interface inhomogeneity + quasiparticle broadening



$W_\beta = 0.4$ $\qquad$ $W_\beta = 0.8$

$\gamma_N = 0.11\Delta_0$ (blue curve)

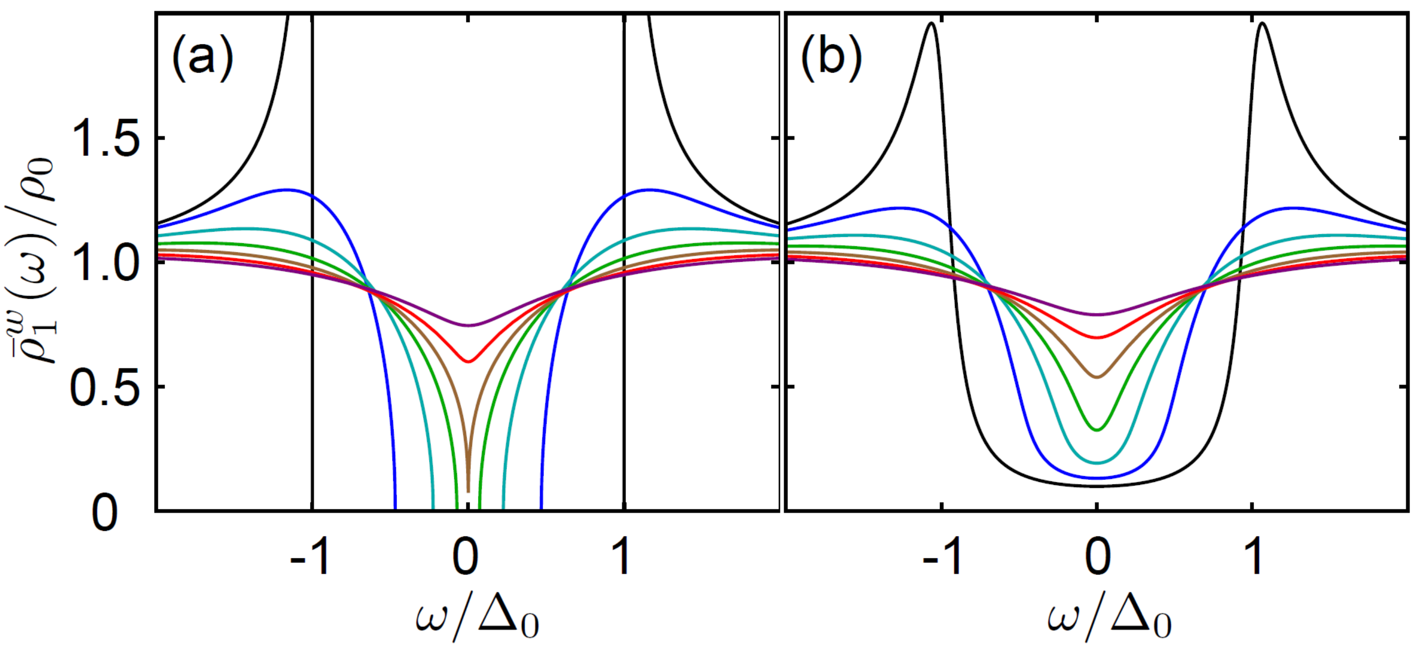
$\gamma_N = 0.32\Delta_0$ (red curve)

Minimal analytical model

$$\Delta(x) = \Delta_0 + \delta\Delta(x)$$

$$\langle \delta\Delta(x)\delta\Delta(x')\rangle = W_\Delta^2 \delta(x - x')$$

$\gamma_N = 0$



$\gamma_N = 0.11\Delta_0$

$(\Delta_0\tau_\Delta)^{-1} = 0$ to 1.5

$\tau_\Delta^{-1} \equiv \pi W_\Delta^2 \rho_0$

$$E_{\text{gap}} = \Delta_0[1 - (\Delta_0\tau_\Delta)^{-2/3}]^{3/2}$$

$$\Delta_0\tau_\Delta \le 1$$

Details of calculations

Dyson's equation

$$
\begin{aligned}
G_{kk'} &= G_k^{(0)}\delta_{kk'} + \sum_p G_k^{(0)} V_{kk'} G_{k'}^{(0)} \\
&+ \sum_p G_k^{(0)} V_{kp} G_p^{(0)} V_{pk'} G_{k'}^{(0)} \\
&+ \sum_{pp'} G_k^{(0)} V_{kp} G_p^{(0)} V_{pp'} G_{p'}^{(0)} V_{p'k'} G_{k'}^{(0)} + \ldots
\end{aligned}
$$

$$G_k^{(0)} = (z\tau_0 - \xi_k\tau_3 - \Delta_0\tau_1)^{-1}$$

$$V_{kp} = (\delta\Delta)_{k+p}\,\tau_1$$

$$
\begin{aligned}
\langle \delta\Delta_k \rangle &= 0 \\
\langle \delta\Delta_k \delta\Delta_{k'} \rangle &= W_\Delta^2 \delta_{k,-k'}
\end{aligned}
$$

Dyson's equation

$$\bar{G}_k\delta_{kk'} = G_k^{(0)}\delta_{kk'} + \sum_p G_k^{(0)}\tau_1 G_p^{(0)}\tau_1 G_{k'}^{(0)} \left\langle \delta\Delta_{kp}\delta\Delta_{pk'} \right\rangle$$
$$+ \sum_{pp'} G_k^{(0)}\tau_1 G_p^{(0)}\tau_1 G_{p'}^{(0)}\tau_1 G_{p'}^{(0)}\tau_1 G_{k'}^{(0)}$$
$$\times \left\langle \delta\Delta_{kp}\delta\Delta_{pp'}\delta\Delta_{p'p''}\delta\Delta_{p''k'} \right\rangle + \dots$$

$$\bar{G}_k = G_k^{(0)} + W_\Delta^2 G_k^{(0)}\tau_1 \left(\sum_p G_p^{(0)}\right)\tau_1 G_k^{(0)}$$
$$+W_\Delta^4 G_k^{(0)}\tau_1 \left(\sum_p G_p^{(0)}\right)\tau_1 G_k^{(0)}\tau_1 \left(\sum_p G_p^{(0)}\right)\tau_1 G_k^{(0)}$$
$$+W_\Delta^4 G_k^{(0)}\tau_1 \left(\sum_{pp'} G_p^{(0)}\tau_1 G_{p'}^{(0)}\tau_1 G_p^{(0)}\right)\tau_1 G_k^{(0)}$$
$$+W_\Delta^4 G_k^{(0)}\tau_1 \left(\sum_{pp'} G_p^{(0)}\tau_1 G_{p'}^{(0)}\tau_1 G_{k+p'-p}^{(0)}\right)\tau_1 G_k^{(0)}$$
$$+ \dots$$
$$\equiv G_k^{(0)} + G_k^{(0)}\Sigma_k\bar{G}_k$$

Non-crossing approximation

$$\bar{\sum}_{pp'} G_p^{(0)} \tau_1 G_{p'}^{(0)} \tau_1 G_{k+p'-p}^{(0)}$$

such terms are ignored

$$k_F v_F \tau_\Delta \gg 1$$

$$\Sigma_k = W_\Delta^2 \int \frac{dp}{2\pi} \tau_1 \bar{G}_p \tau_1$$

$$\tau_\Delta^{-1} = \pi W_\Delta^2 \rho_0$$

$$\begin{aligned} \bar{G}_k &= (z\tau_0 - \xi_k\tau_3 - \Delta_0\tau_1 - \Sigma)^{-1} \\ &= \left(\tilde{z}\tau_0 - \tilde{\xi}_k\tau_3 - \tilde{\Delta}\tau_1\right)^{-1} \end{aligned}$$

$$\tilde{z} = z - \Sigma_0$$

$$\tilde{\xi}_k = \xi_k + \Sigma_3$$

$$\Sigma = \Sigma_0\tau_0 + \Sigma_3\tau_3 + \Sigma_1\tau_1$$

$$\tilde{\Delta} = \Delta_0 + \Sigma_1$$

$$\Sigma_k = W_\Delta^2 \int \frac{dp}{2\pi} \tau_1 \bar{G}_p \tau_1$$

$$\begin{aligned} \bar{G}_k &= \left(z\tau_0 - \xi_k\tau_3 - \Delta_0\tau_1 - \Sigma\right)^{-1} \\ &= \left(\tilde{z}\tau_0 - \tilde{\xi}_k\tau_3 - \tilde{\Delta}\tau_1\right)^{-1} \end{aligned}$$

$$\begin{aligned} \Sigma_k &= W_\Delta^2 \int \frac{dp}{2\pi} \frac{\tilde{z} + \tilde{\xi}_k\tau_3 + \tilde{\Delta}\tau_1}{\tilde{z}^2 - \xi_p^2 - \tilde{\Delta}^2} \\ &= \frac{-1}{2\tau_\Delta} \frac{\tilde{z} + \tilde{\Delta}\tau_1}{\sqrt{\tilde{\Delta}^2 - \tilde{z}^2}} \end{aligned}$$

$$\tilde{z} = z - \Sigma_0$$

$$\tilde{\xi}_k = \xi_k + \Sigma_3$$

$$\tilde{\Delta} = \Delta_0 + \Sigma_1$$

$$\tilde{z} = z + \frac{1}{2\tau_\Delta} \frac{\tilde{z}}{\sqrt{\tilde{\Delta}^2 - \tilde{z}^2}}$$

$$\tilde{\Delta} = \Delta_0 - \frac{1}{2\tau_\Delta} \frac{\tilde{\Delta}}{\sqrt{\tilde{\Delta}^2 - \tilde{z}^2}}$$

Density of states:

$$\rho(\epsilon) = \frac{-1}{\pi}\mathrm{Im}\int\frac{dk}{2\pi}\mathrm{Tr}\bar{G}_k(z\to\epsilon+i0)$$

$$\approx \rho_0 \mathrm{Im}\frac{u}{\sqrt{1-u^2}}$$

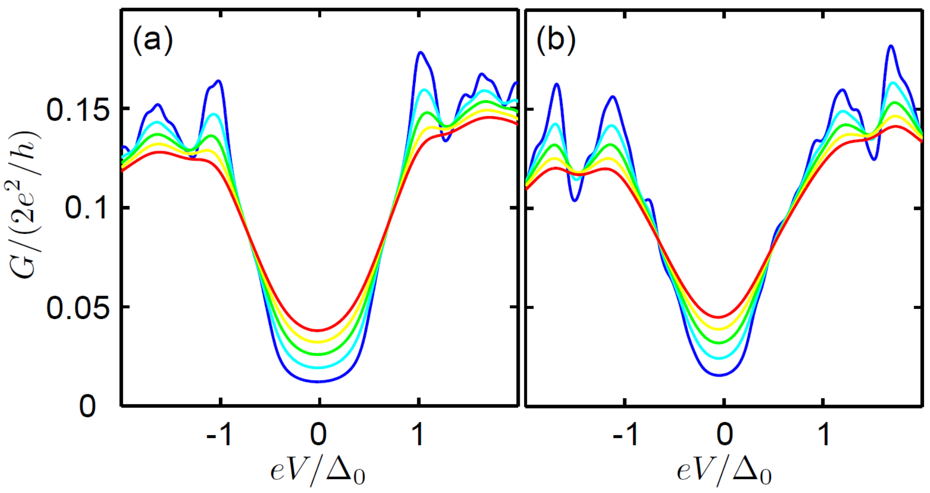
$$= \rho_0\left(\Delta_0\tau_\Delta\right)\mathrm{Im}u$$

$$u \equiv \tilde{z}/\tilde{\Delta}$$

$$E_{\mathrm{gap}} = \theta\left(\Delta_0\tau_\Delta - 1\right)\Delta_0\left(1-\left(\Delta_0\tau_\Delta\right)^{-2/3}\right)^{3/2}$$

Conclusions

“Quantitative considerations point to the interface fluctuations at the NW-SC contact leading to inhomogeneous pairing amplitude along the wire as the primary physical mechanism causing the ubiquitous soft gap behavior.”



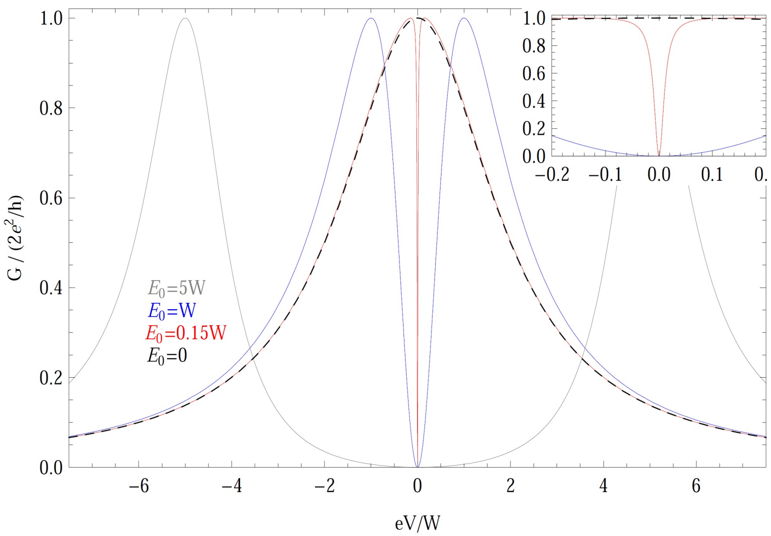
Tunneling conductance due to discrete spectrum of Andreev states

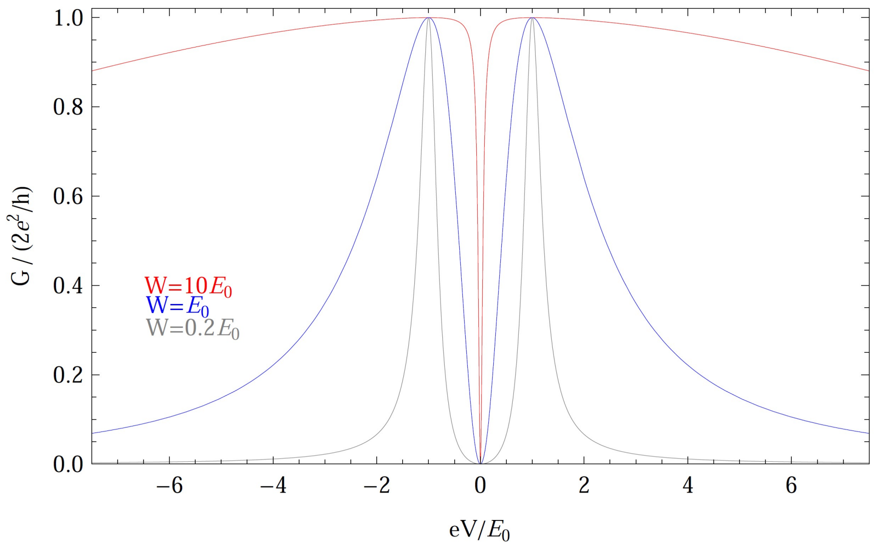
P. A. Ioselevich and M. V. Feigel’man

arXiv:1211.2722

Single channel conductance

$T = 0$





E_0^2/W