Inertia and Chiral Edge Modes of a Skyrmion Magnetic Bubble

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The dynamics of a vortex in a thin-film ferromagnet resembles the motion of a charged massless particle in a uniform magnetic field. Similar dynamics is expected for other magnetic textures with a nonzero Skyrmion number. However, recent numerical simulations reveal that Skyrmion magnetic bubbles show significant deviations from this model. We show that a Skyrmion bubble possesses inertia and derive its mass from the standard theory of a thin-film ferromagnet. In addition to center-of-mass motion, other low energy modes are waves on the edge of the bubble traveling with different speeds in opposite directions.

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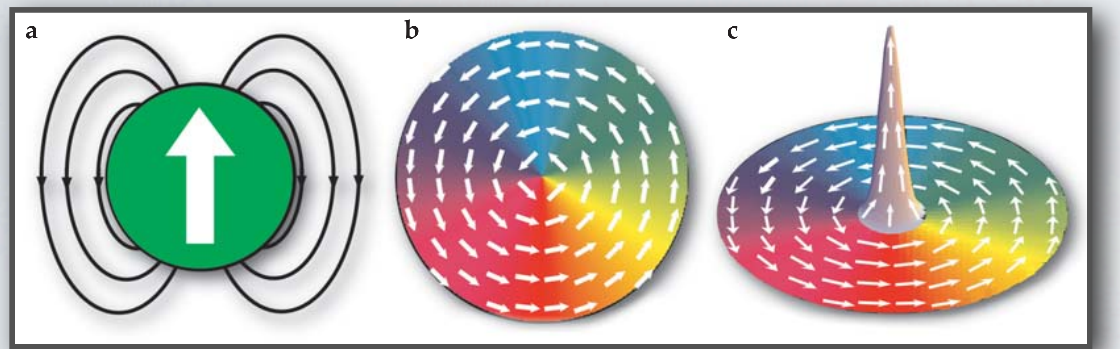
TOPOLOGICAL STATES IN FM NANOMAGNETS



Nanomagnets

Disk of 15 nm thickness and D=100 nm diameter

Competition between

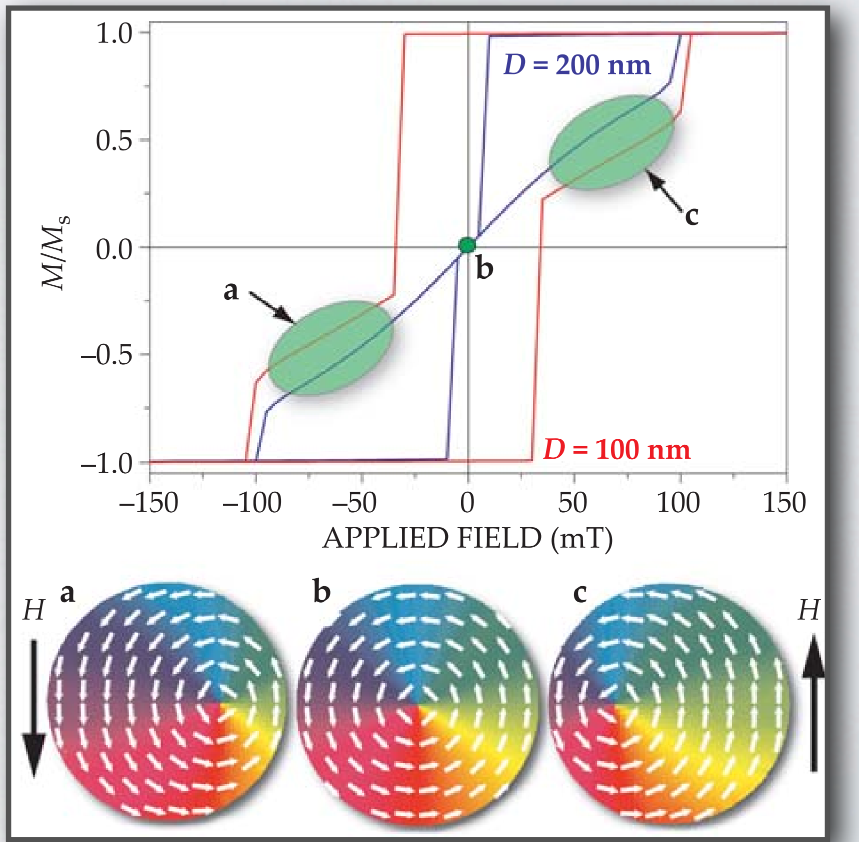
- Exchange interaction (wants to align all spins, density decreases with D)
- Magnetic stray field (wants to create vortex-like structure, increases with D)

Magnetic field

Due to Zeeman interaction spins want to align with B

For intermediate magnetic fields (up to ~50-300 mT) vortex structure stays in tact

Allows us to move vortex around in a controlled way



C. L. Chien *et al.*, Physics Today **60**(6), 40 (2007)

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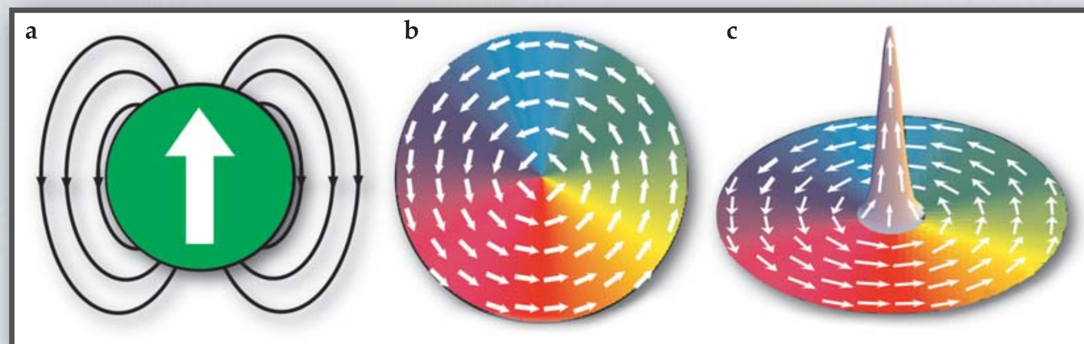

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Interest: classical magnetic memories (MRAM)

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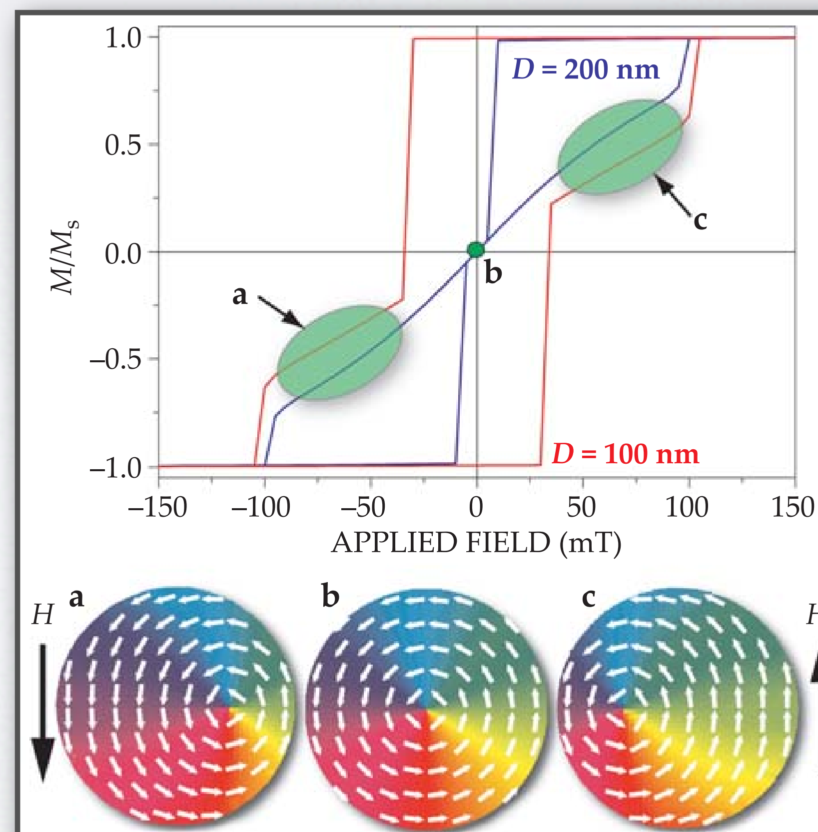

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THIELE'S EQUATION (1)

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How can we describe the motion of such **rigid** magnetization structures?

Start from the Landau-Lifshitz equation:

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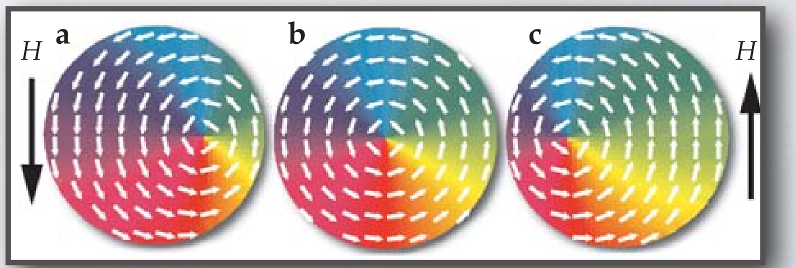
Precession $(\mathbf{B} = \delta U / \delta \mathbf{m})$

Damping



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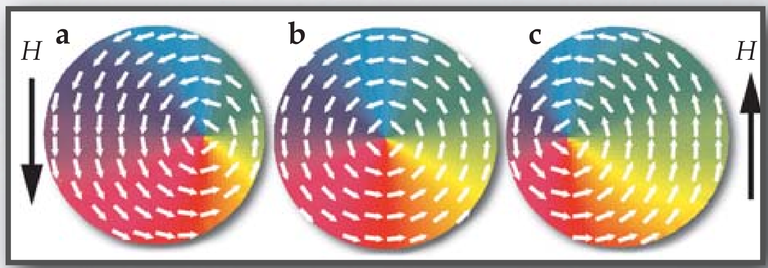

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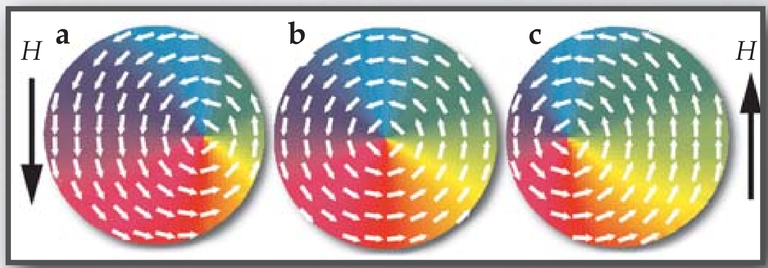

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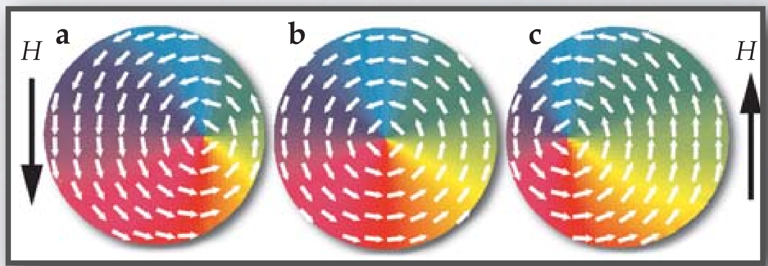

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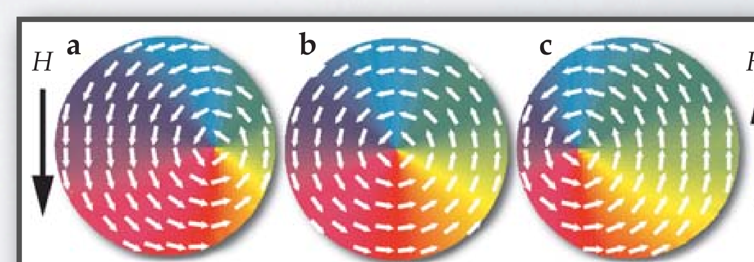
The parameters **G** and *D* depend on the magnetization profile, for vortex

$\mathbf{G} = (0, 0, \mathcal{G})$ where $\mathcal{G} \propto (1/4\pi) \int \mathrm{d}^2\mathbf{r}\mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m})$

Topological invariant, Skyrmion number q=1/2



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Corresponds to the EOM of a massless particle in a magnetic field **G**/e and force **F**=-∇*U*

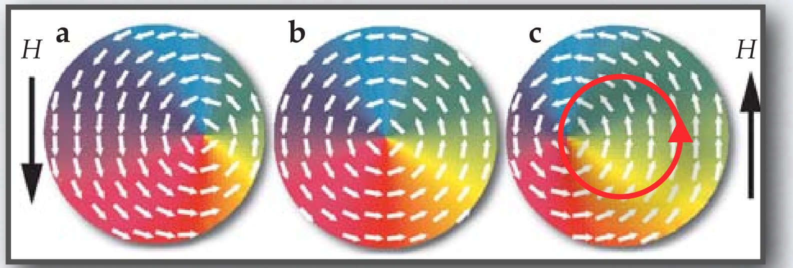
$$-M\ddot{\mathbf{R}} + e\mathbf{B} \times \dot{\mathbf{R}} + \mathbf{F} - D\dot{\mathbf{R}} = 0$$

Mass **(=0)** | Lorentz Force | External Force | Damping (=0)

Example: Motion of a magnetic vortex in a parabolic potential well (ignore damping)

$$U(X,Y) = \mathcal{K}(X^2 + Y^2)/2$$

Due to 'magnetostatic interaction between vortex and edge of disc'



Leads to circular motion with frequency: $\omega = \mathcal{K}/\mathcal{G}$

MOTION OF SKYRMIONS (MAGNETIC BUBBLES)

Skyrmions are similar to vortices, but with skyrmion number $q=1$

↓

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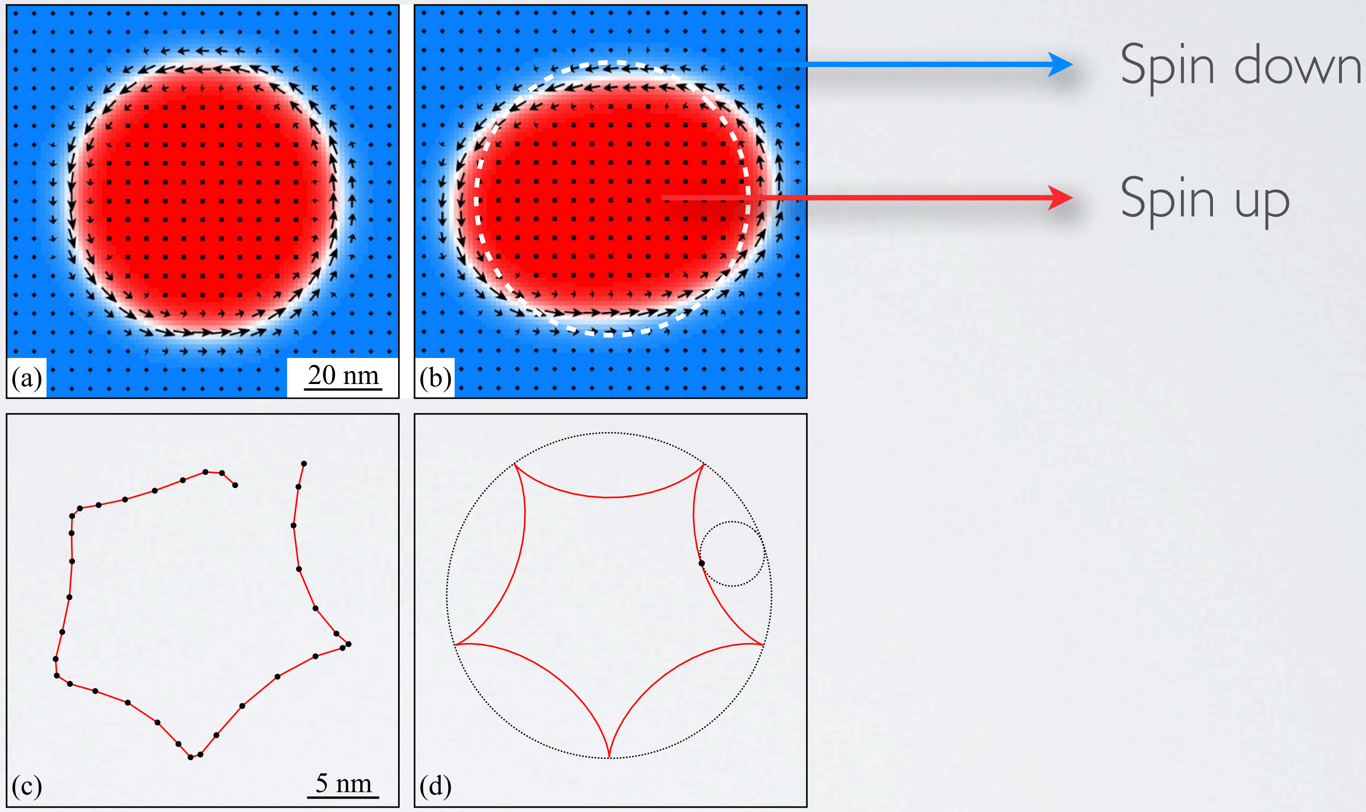

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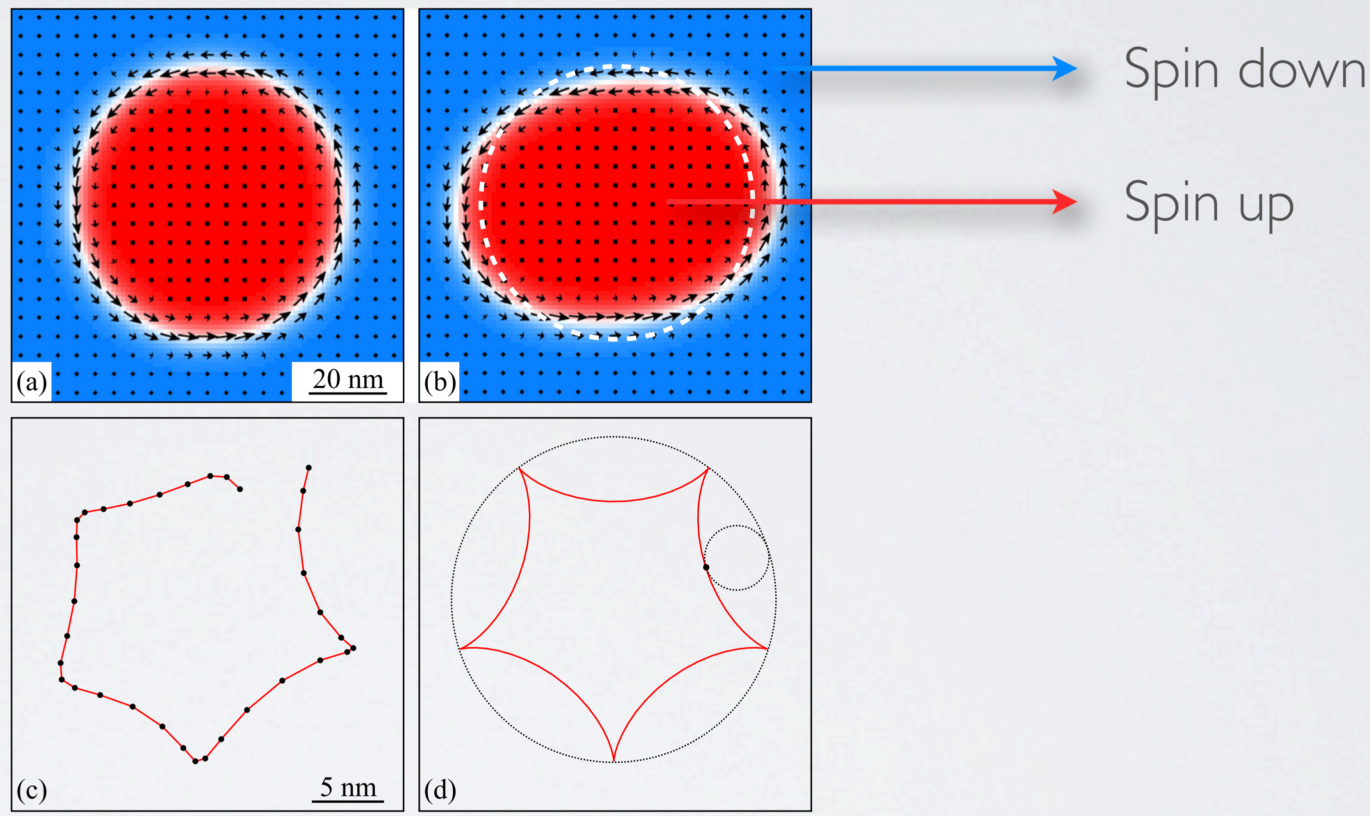

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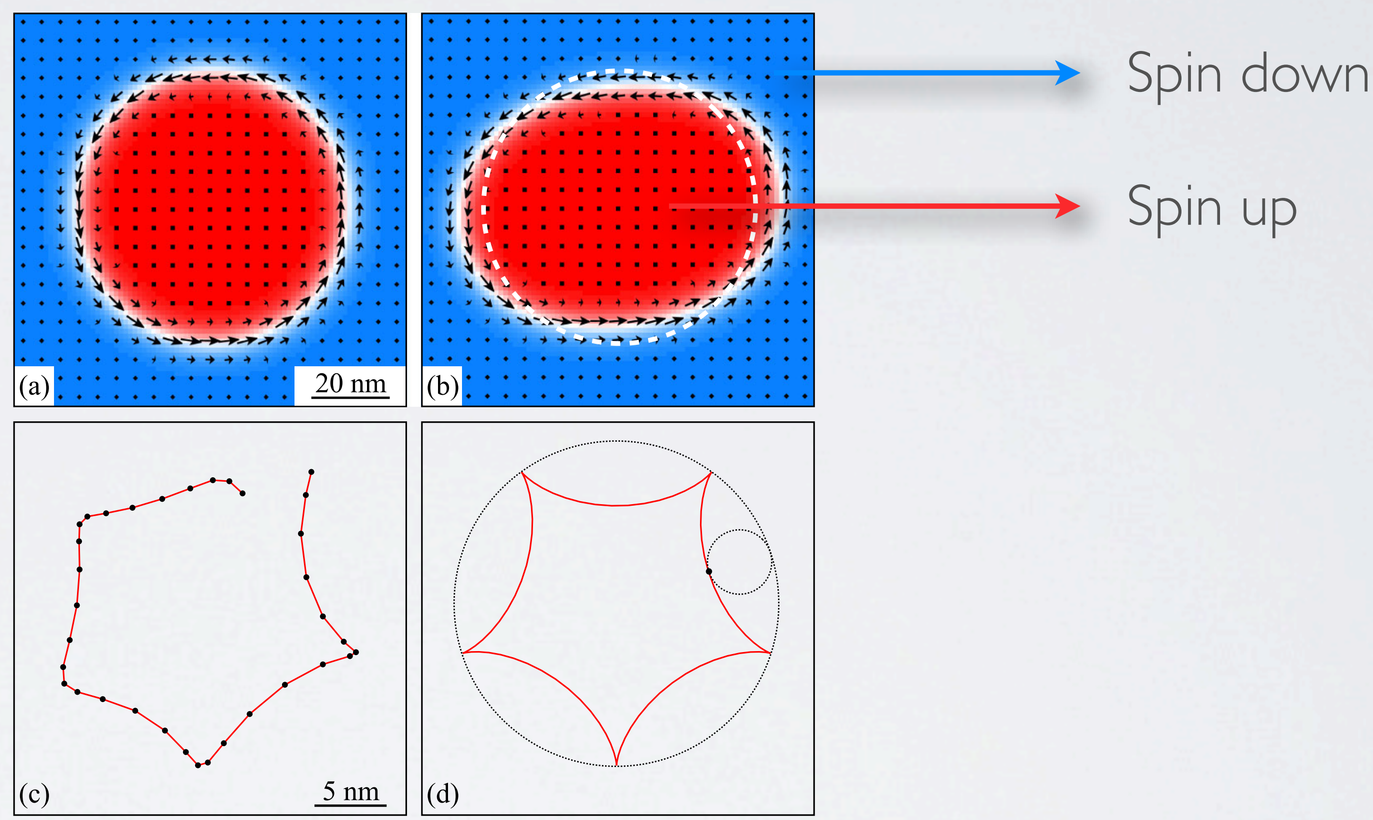

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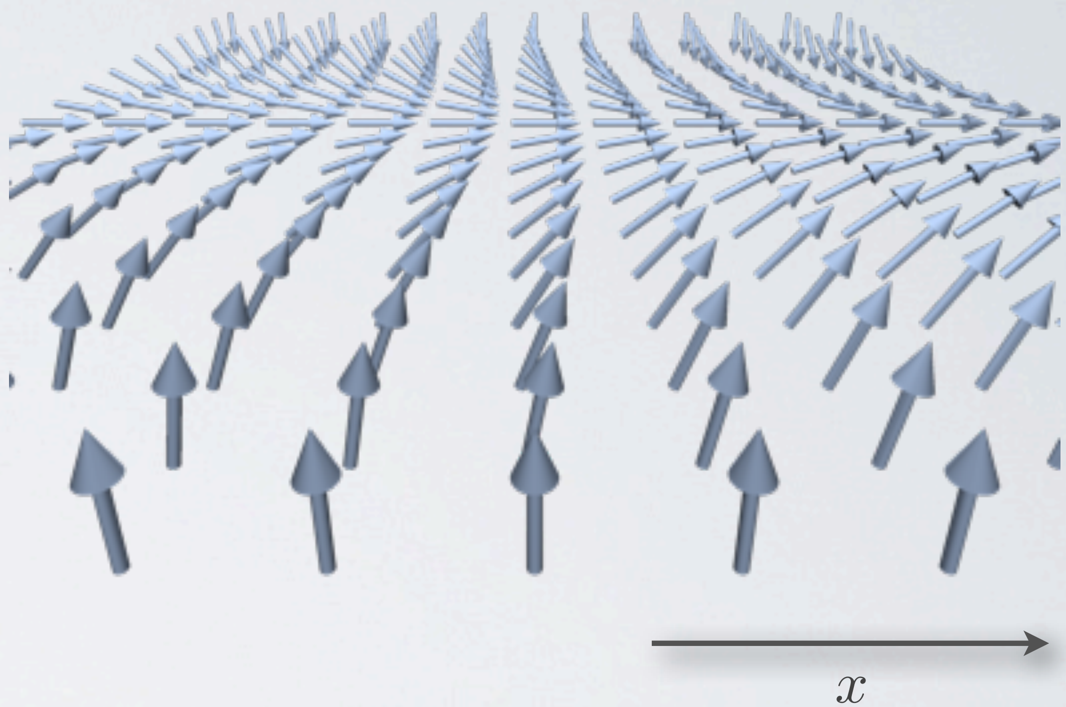
What is the physical origin of this mass?

ORIGIN OF MASS TERM: STRAIGHT DOMAIN WALL (1)

Certain small **deformations of the magnetic texture** turn out to generate the mass-term



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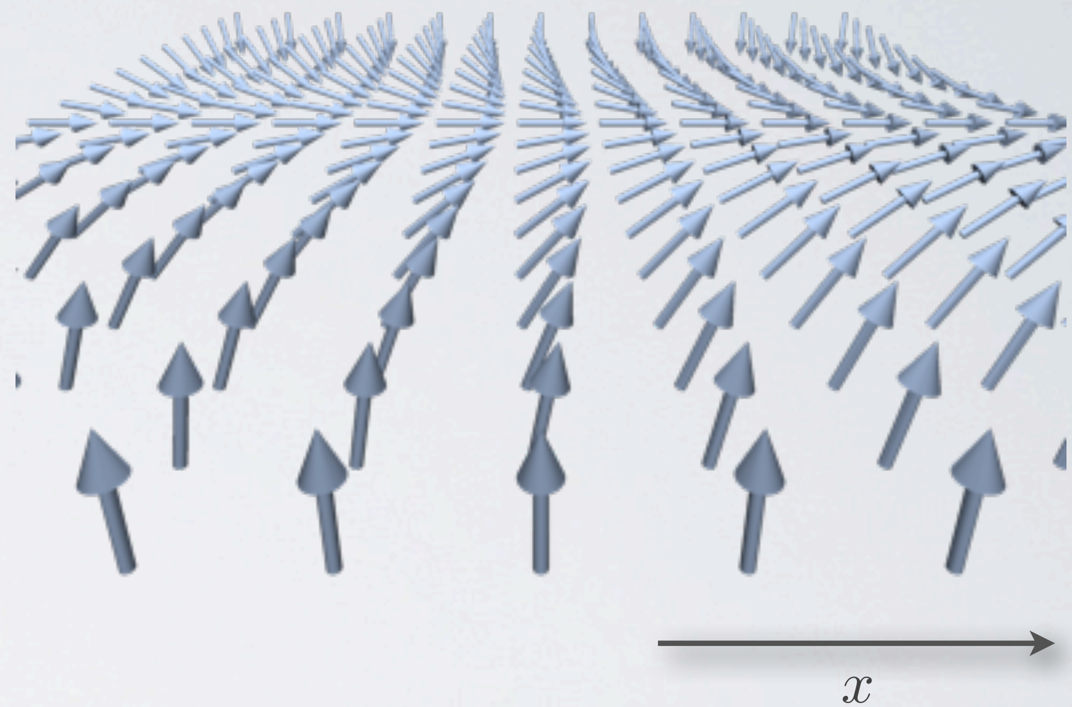
In the Landau-Lifshitz formalism:

$$\mathbf{m} = (\sin\theta\cos\psi, \sin\theta\sin\psi, \cos\theta)$$

$$L[\theta,\psi] = \int \mathrm{d}^2\mathbf{r} \left\{(1-\cos\theta)\dot{\psi} - U[\theta,\psi]\right\}$$



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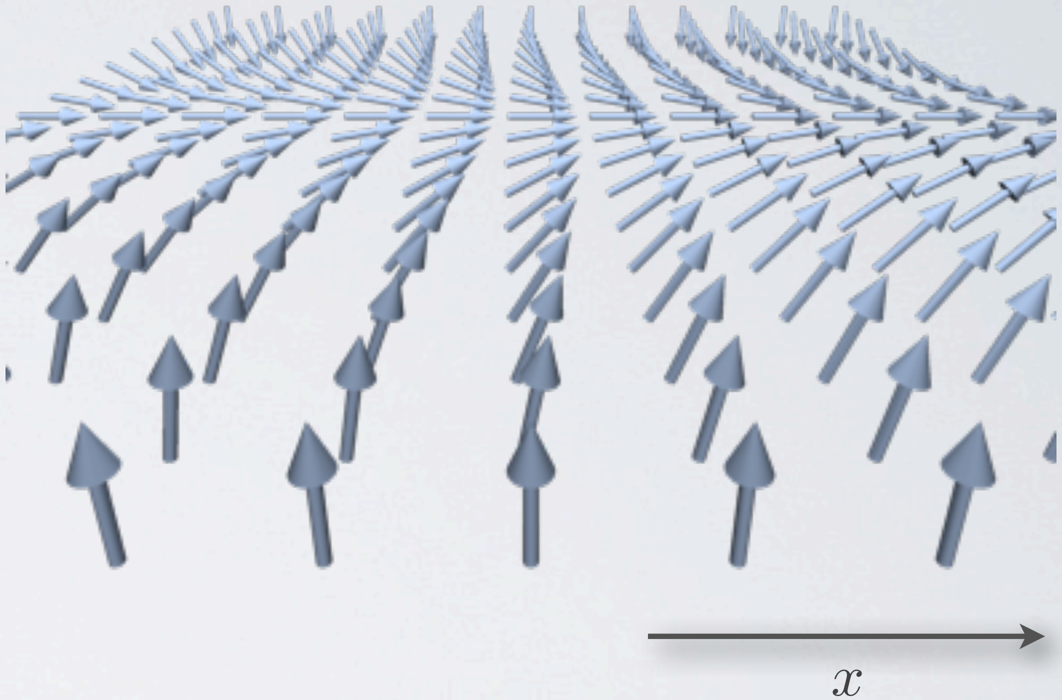
$$\psi(x,t) \text{ and } Y(x,t)$$



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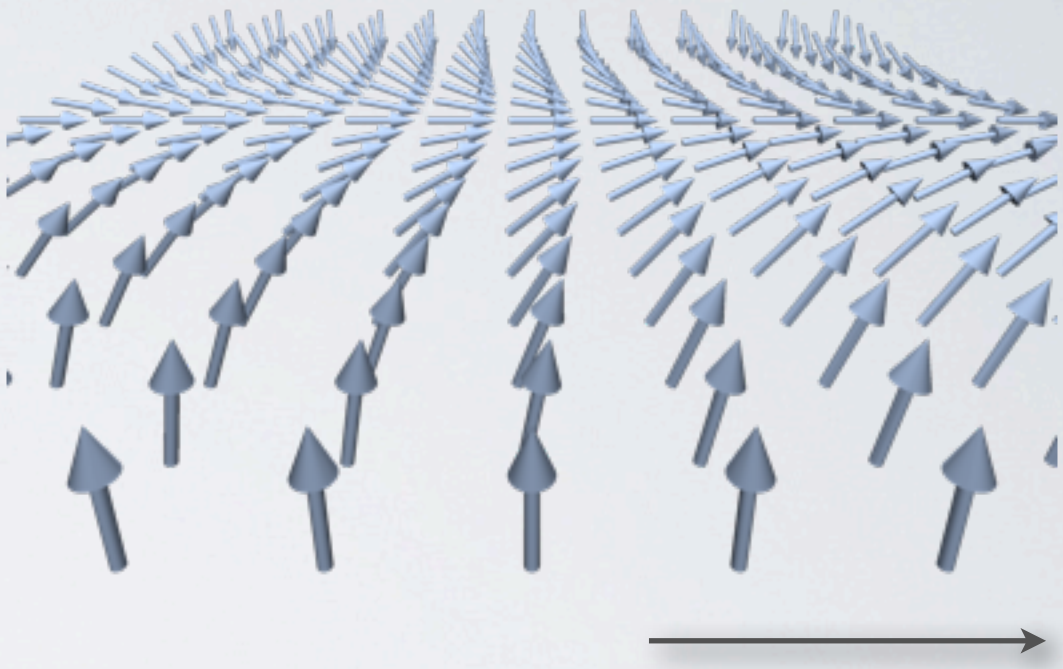

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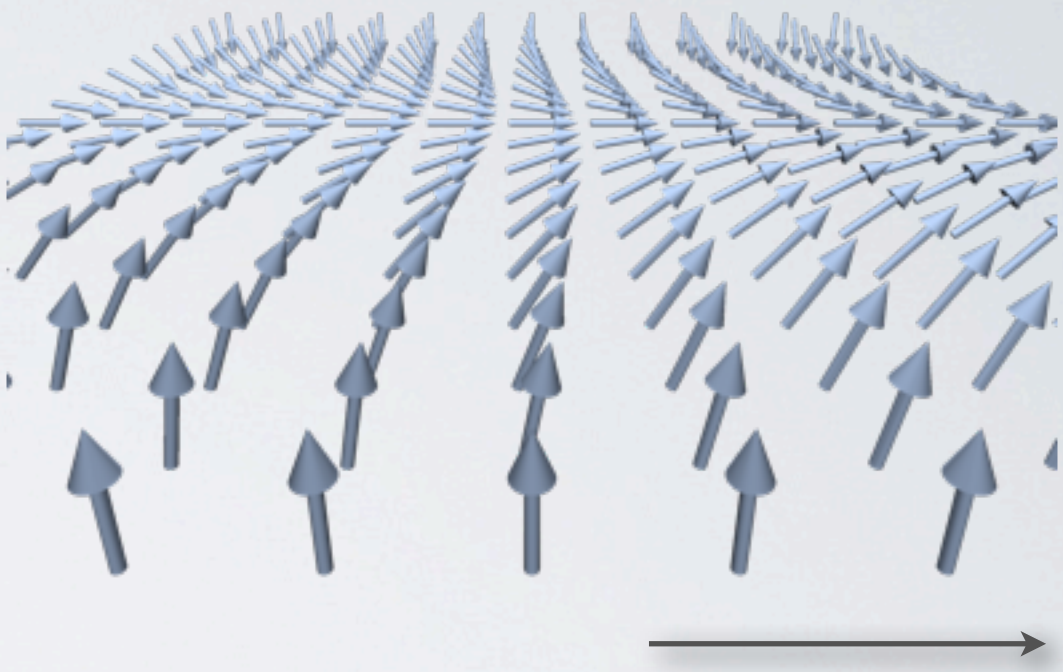

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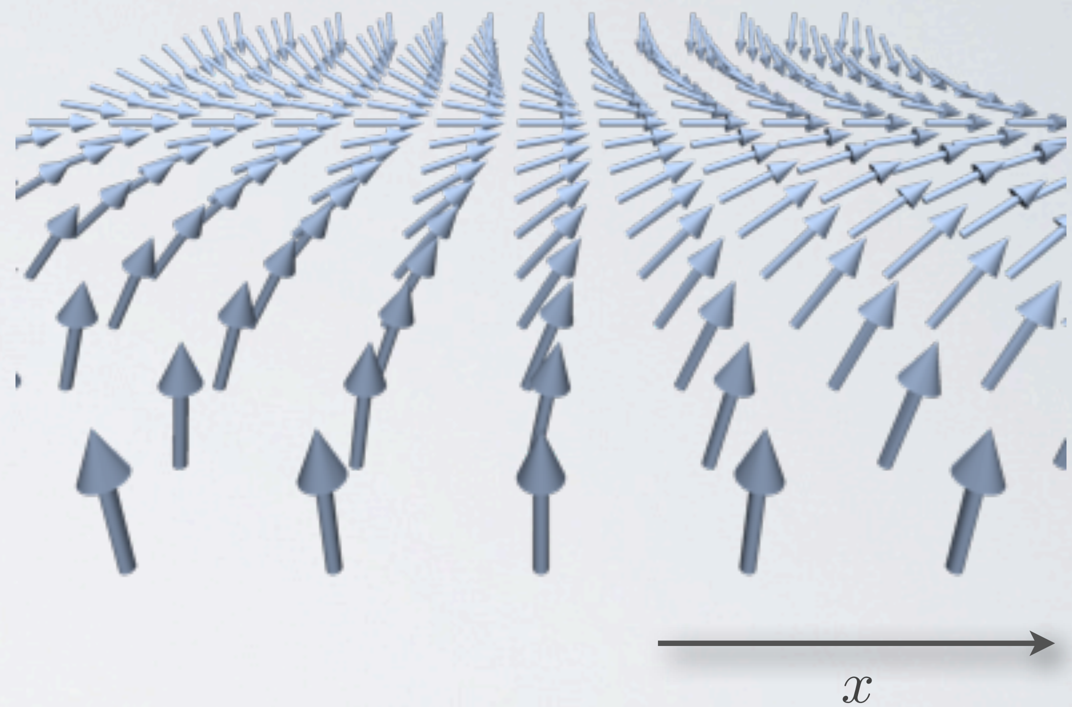

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In equilibrium: in plane magnetization is **aligned with the wall**

$$\psi = Y' \equiv \partial Y/\partial x$$

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Magnetization can be **misaligned** with the wall. Costs an energy:

$$L[Y, \psi] = \int_0^l \mathrm{d}x g \dot{Y} \psi - \mathcal{K}(\psi - Y')^2 - U[Y]$$

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- Local
- Long-range

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$$\rho\ddot{Y} + 2g\dot{Y}' - \sigma Y'' = 0$$

Waves traveling left and right with velocity

$$\omega_k = \rho^{-1}\left(-gk \pm \sqrt{g^2k^2 + \sigma\rho k^2}\right)$$

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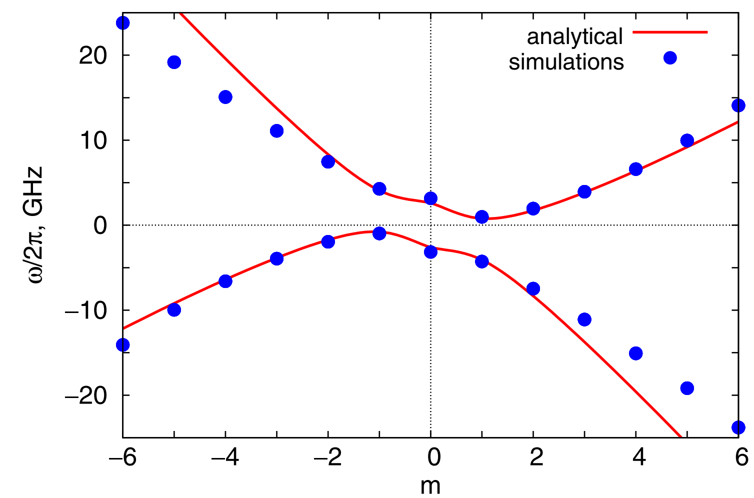

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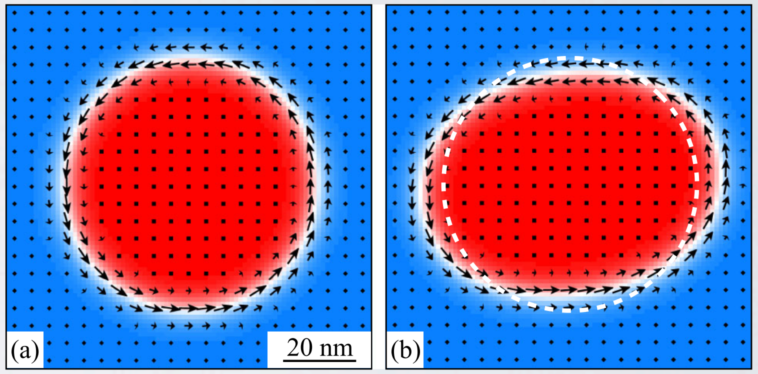
ORIGIN OF MASS TERM: MAGNETIC BUBBLE

Works pretty much the same as for domain wall.

Parametrization domain wall and magnetization:

$$r(\phi) = \bar{r} + \sum_m r_m e^{im\phi} \qquad \psi(\phi) = \phi \pm \pi/2 + \sum_m \psi_m e^{im\phi}$$

This gives the **Lagrangian**



$$L[r,\psi] = 2\pi\bar{r}\sum_m \left(g\dot{r}_m\psi_m^* - \frac{\mathcal{K}|\psi_m + imr_m/\bar{r}|^2}{2} \right) - U[r]$$

After removing magnetization angle

$$L[r] = \sum_m \left(\pi\bar{r}\rho|\dot{r}_m|^2 - 4\pi mig r_m^* \dot{r}_m\right) - U[r]$$

Assuming only the lowest mode $r_1 = (X - iY)/2$ is excited

$$L[X,Y] = \mathcal{M}(\dot{X}^2 + \dot{Y}^2)/2 + \mathcal{G}\dot{X}Y - \mathcal{K}(X^2 + Y^2)/2$$

CONCLUSIONS

Thiele's equation predicts that rigid magnetic textures behave as massless particles moving in a magnetic field determined by the texture

This approach works well for magnetic bubbles, but fails for skyrmions

The authors show that the correct motion is predicted when a mass term is added to Thiele's equation

The authors find the origin of this mass term

THANK YOU FOR YOUR ATTENTION