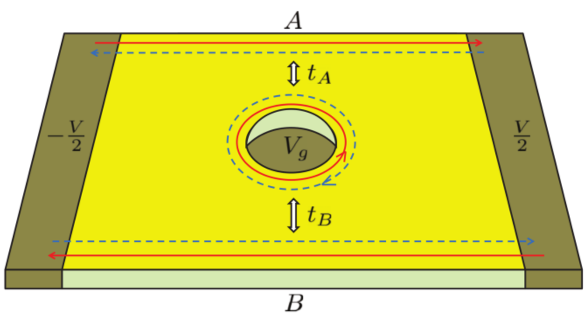
PHYSICAL REVIEW B **87**, 085425 (2013)

Generating and controlling spin-polarized currents induced by a quantum spin Hall antidot

G. Dolcetto,[1,2,3] F. Cavaliere,[1,2] D. Ferraro,[2,3,4] and M. Sassetti[1,2]

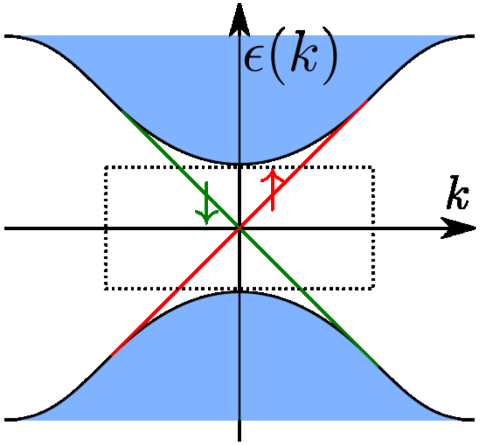
We study an electrically controlled quantum spin Hall antidot embedded in a two-dimensional topological insulating bar. Helical edge states around the antidot and along the edges of the bar are tunnel coupled. The close connection between spin and chirality, typical of helical systems, allows to generate a *spin-polarized* current flowing across the bar. This current is studied as a function of the external voltages, by varying the asymmetry between the barriers. For asymmetric setups, a switching behavior of the spin current is observed as the bias is increased, both in the absence and in the presence of electron interactions. This device allows to generate and control the spin-polarized current by simple electrical means.



2D topological insulators

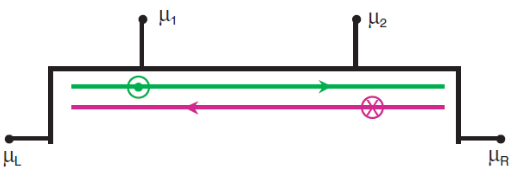
2D topological insulators:

- Bulk band gap
- Gapless 1D edge states



Single edge state of a 2D topological insulator:

- Energies below the bulk gap: helical 1D liquid
- Degrees of freedom: $\psi_\uparrow(x)$, $\psi_\downarrow(x)$



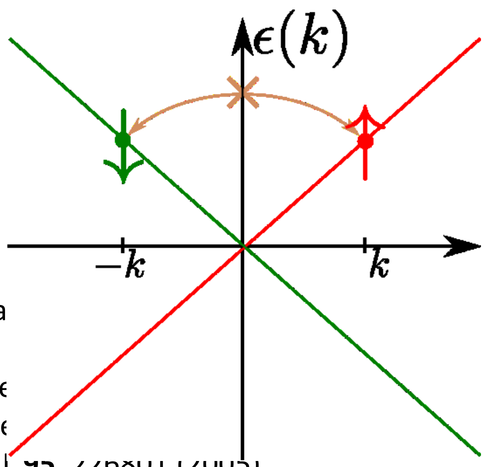
Time-reversal (TR) operator for spin-1/2 particles:

- Anti-unitary TR operator: $\Theta^2 = -1$
- Action on fermionic fields:

Kramers doublet

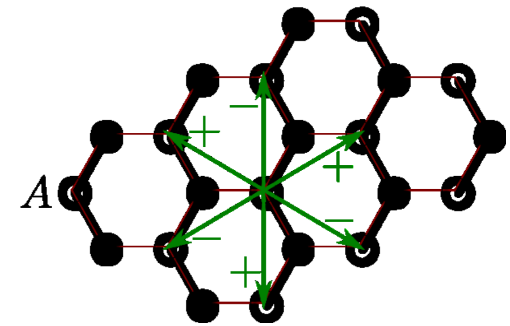
$$\begin{cases} \Theta\psi_{\uparrow,k}\Theta^{-1} = \psi_{\downarrow,-k} \\ \Theta\psi_{\downarrow,k}\Theta^{-1} = -\psi_{\uparrow,-k} \end{cases}$$

$$G_0 = \frac{e^2}{h}$$



rnevig, Hughes a
and Zhang, Rev.
[3] Hasan and Kane, Re
[4] König et al., Science
[5] Kane and Mele, PRL 95, 226801 (2005)

Graphene with spin-orbit coupling



Kane-Mele (KM) model: [1]

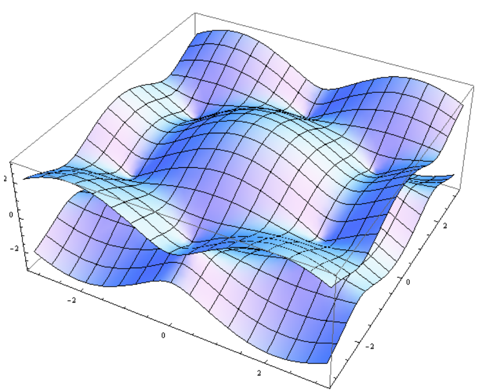
- **Bad**: not experimentally realizable (yet)
- **Good**: simple, microscopic paradigm

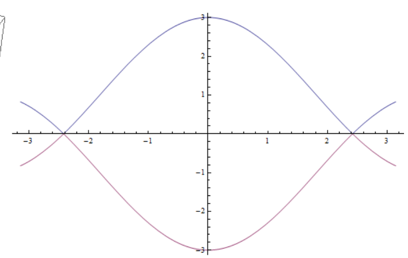
Hopping Hamiltonian on honeycomb lattice:

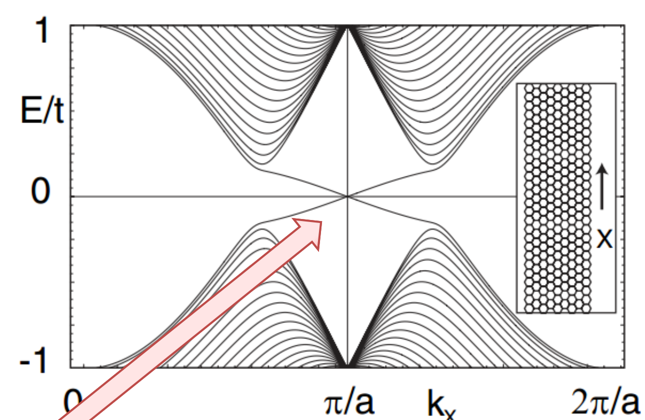
$$H_{KM} = \sum_{\langle ij \rangle \alpha} t c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{\langle\langle ij \rangle\rangle \alpha\beta} i t_2 \nu_{ij} c_{i\alpha}^{\dagger} c_{j\beta}$$

Intrinsic spin-orbit coupling

Topological property of the band structure!







Helical edge states

[1] Kane and Mele, PRL **95**, 226801 (2005)

Silicene

"Silicene":
- Honeycomb lattice with **Si** instead of **C**
- Larger interatomic distance than graphene
- Less $\pi - \pi$ overlap: buckling
- Stable configuration: $\theta = 101.73°$
- Enhanced spin-orbit coupling

PRL **107,** 076802 (2011) PHYSICAL REVIEW LETTERS week ending 12 AUGUST 2011

Quantum Spin Hall Effect in Silicene and Two-Dimensional Germanium

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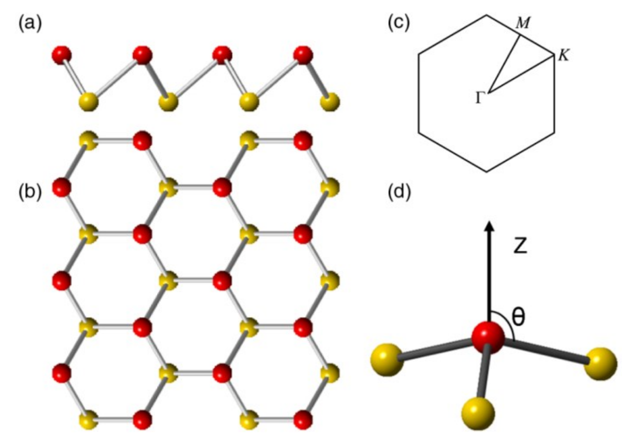
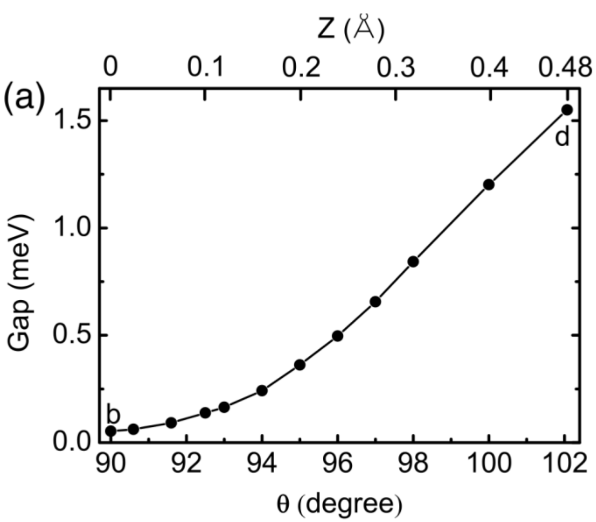
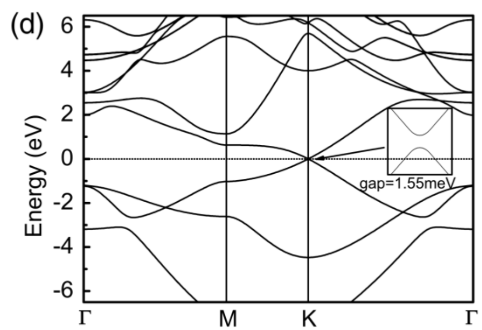
(Received 18 April 2011; published 9 August 2011)

We investigate the spin-orbit opened energy gap and the band topology in recently synthesized silicene as well as two-dimensional low-buckled honeycomb structures of germanium using first-principles calculations. We demonstrate that silicene with topologically nontrivial electronic structures can realize the quantum spin Hall effect (QSHE) by exploiting adiabatic continuity and the direct calculation of the Z_2 topological invariant. We predict that the QSHE can be observed in an experimentally accessible low temperature regime in silicene with the spin-orbit band gap of 1.55 meV, much higher than that of graphene. Furthermore, we find that the gap will increase to 2.9 meV under certain pressure strain. Finally, we also study germanium with a similar low-buckled stable structure, and predict that spin-orbit coupling opens a band gap of 23.9 meV, much higher than the liquid nitrogen temperature.



Spin-orbit gap: $\Delta \approx 1.55$ meV ≈ 18 K

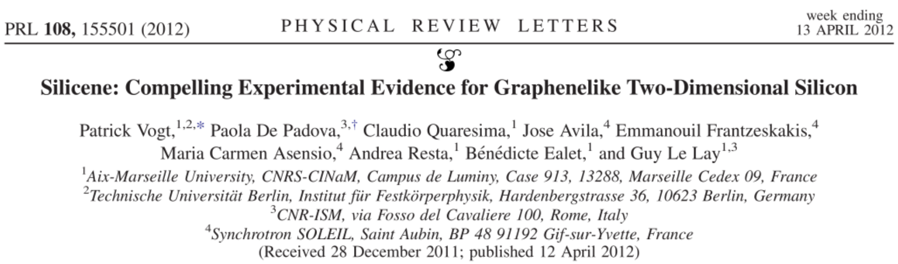




[1] Liu *et al.*, PRL **107**, 076802 (2011)

Silicene experiments

- **Si** atoms forming honeycomb lattice grown on Ag(110)
- STM reveals hexagonal structure: $a \approx 0.22\text{nm}$
- ARPES reveals Dirac spectrum: $v_F \approx 1.3 \times 10^6 \text{m/s}$


PRL 108, 155501 (2012) PHYSICAL REVIEW LETTERS week ending 13 APRIL 2012

Silicene: Compelling Experimental Evidence for Graphenelike Two-Dimensional Silicon

Patrick Vogt,[1,2,*] Paola De Padova,[3,†] Claudio Quaresima,[1] Jose Avila,[4] Emmanouil Frantzeskakis,[4] Maria Carmen Asensio,[4] Andrea Resta,[1] Bénédicte Ealet,[1] and Guy Le Lay[1,3]
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[2]Technische Universität Berlin, Institut für Festkörperphysik, Hardenbergstrasse 36, 10623 Berlin, Germany
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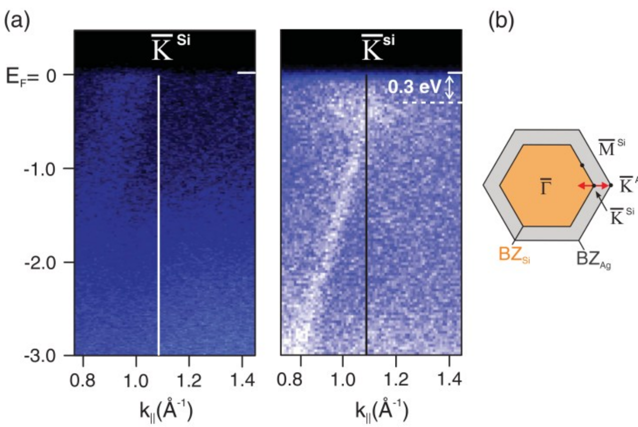
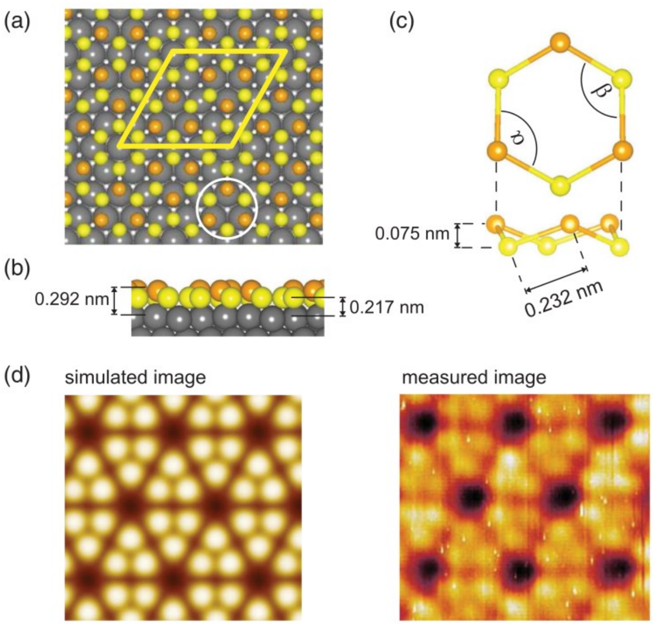
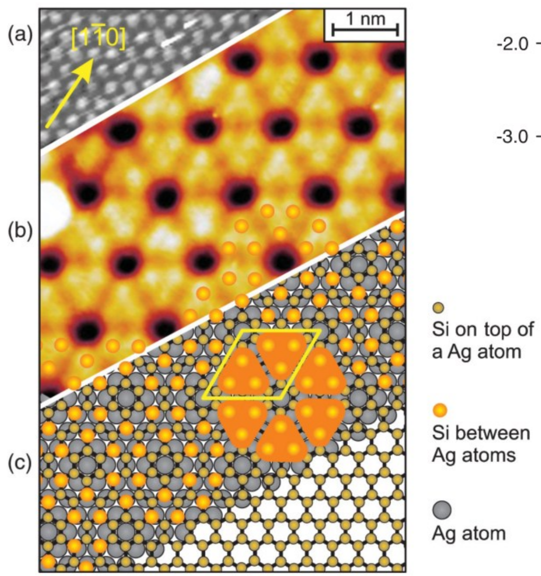
[4]Synchrotron SOLEIL, Saint Aubin, BP 48 91192 Gif-sur-Yvette, France
(Received 28 December 2011; published 12 April 2012)







[1] Vogt *et al.*, PRL **108**, 155501 (2012)

Type-II semiconductor quantum wells

InAs/GaSb heterostructure:

- Loss toxic than HgTe!
- Off-the-shelf materials: Easier to manufacture
- Tunable by application of gate voltage

PRL **100**, 236601 (2008) PHYSICAL REVIEW LETTERS week ending 13 JUNE 2008

Quantum Spin Hall Effect in Inverted Type-II Semiconductors

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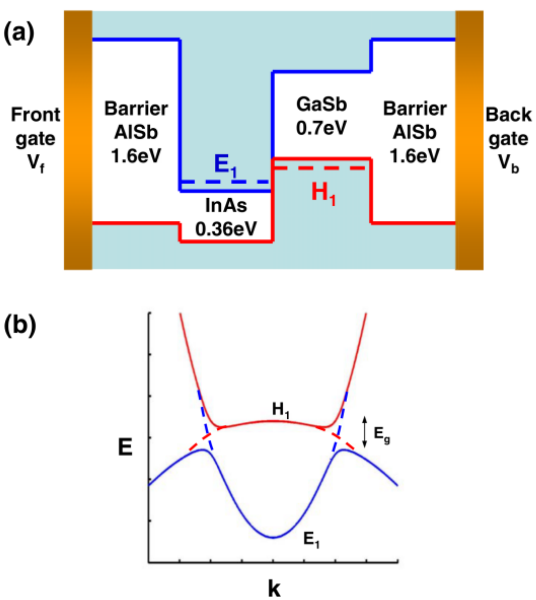
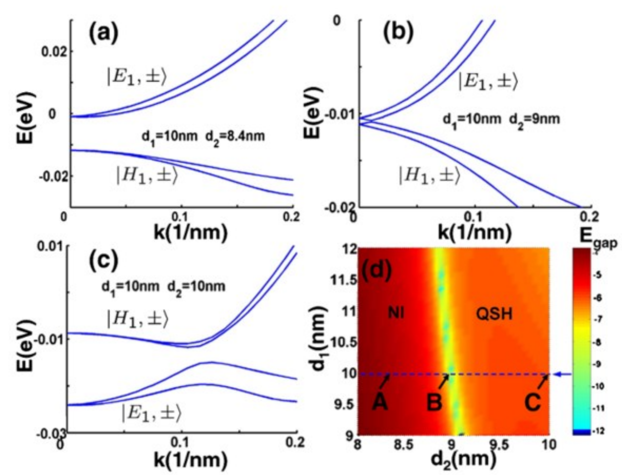
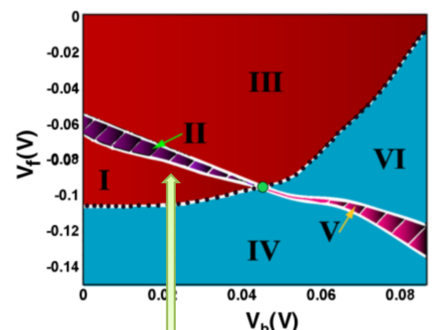
(Received 24 January 2008; published 11 June 2008)

The quantum spin Hall (QSH) state is a topologically nontrivial state of quantum matter which preserves time-reversal symmetry; it has an energy gap in the bulk, but topologically robust gapless states at the edge. Recently, this novel effect has been predicted and observed in HgTe quantum wells and in this Letter we predict a similar effect arising in Type-II semiconductor quantum wells made from InAs/GaSb/AlSb. The quantum well exhibits an "inverted" phase similar to HgTe/CdTe quantum wells, which is a QSH state when the Fermi level lies inside the gap. Due to the asymmetric structure of this quantum well, the effects of inversion symmetry breaking are essential. Remarkably, the topological quantum phase transition between the conventional insulating state and the quantum spin Hall state can be continuously tuned by the gate voltage, enabling quantitative investigation of this novel phase transition.







II: Topological phase

[1] Liu *et al.*, PRL **100**, 236601 (2008)

Type-II semiconductor quantum wells

Experimental results:

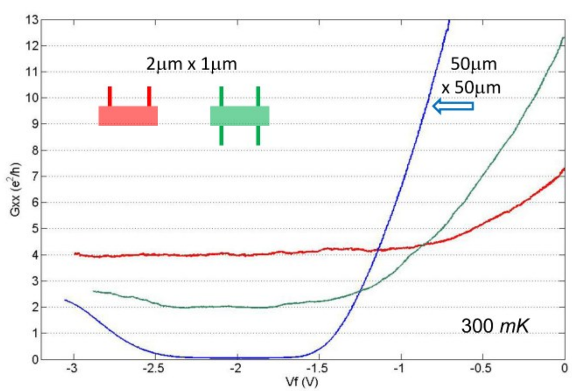
- Four-terminal conductance

$$G \approx 4\frac{e^2}{h}$$

- Problem: Large bulk conductance

- Latest update:

News Flash: Conductance Quantization in InAs/GaSb



Raw data:
Lingjie Du, Ivan Knez, Rui-Rui Du, Gerry Sullivan

I. Knez (unpublished, presented at KITP)

PRL **107,** 136603 (2011) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2011

Evidence for Helical Edge Modes in Inverted InAs/GaSb Quantum Wells

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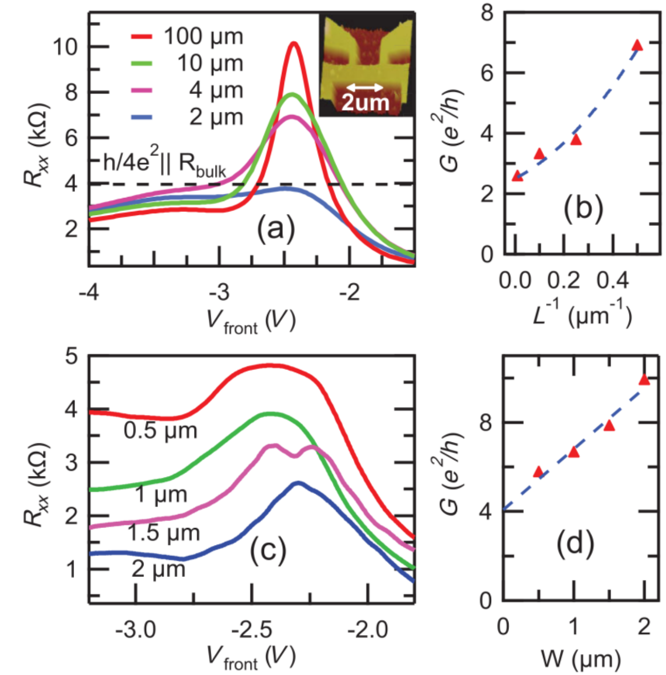
(Received 29 April 2011; published 19 September 2011)

We present an experimental study of low temperature electronic transport in the hybridization gap of inverted InAs/GaSb composite quantum wells. An electrostatic gate is used to push the Fermi level into the gap regime, where the conductance as a function of sample length and width is measured. Our analysis shows strong evidence for the existence of helical edge modes proposed by Liu *et al* [Phys. Rev. Lett. **100**, 236601 (2008)]. Edge modes persist in spite of sizable bulk conduction and show only a weak magnetic field dependence—a direct consequence of a gap opening away from the zone center.

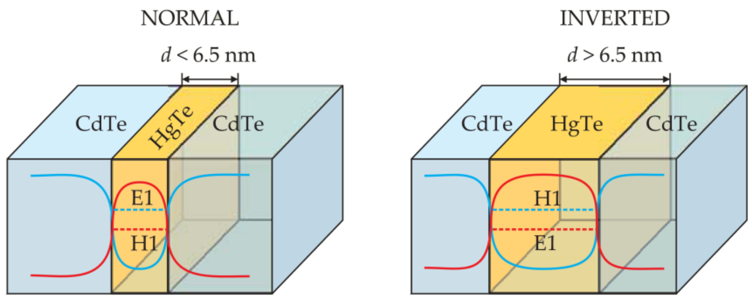


[1] Knez *et al.*, PRL **107**, 136603 (2011)

HgTe/CdTe quantum wells

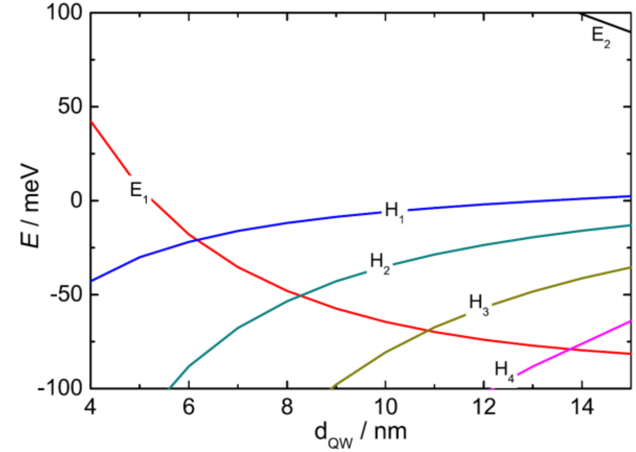
HgTe/CdTe quantum wells [1,2]

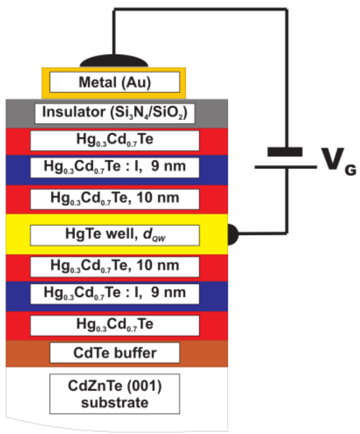
- Band inversion above critical quantum well thickness $d > d_c \approx 6.3\text{nm}$

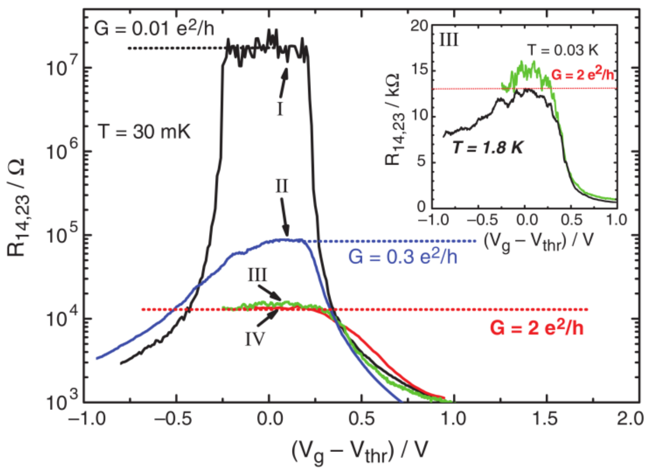


Evidence for quantum spin Hall state:

- Strong increase in conductance for $d > d_c$
- Conductance G_0 independent of sample size







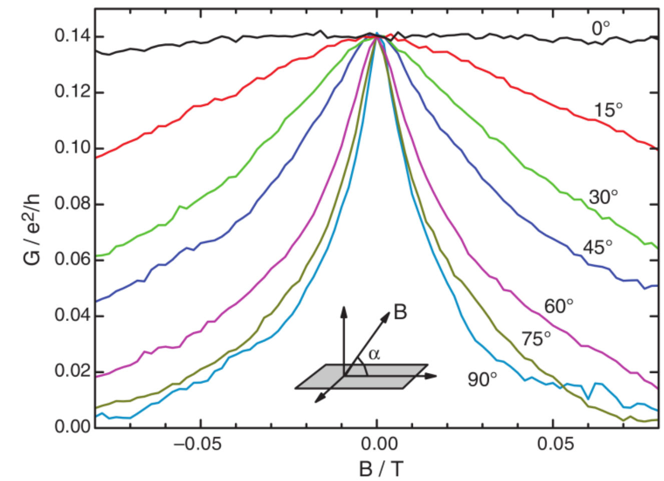
[1] König *et al.*, Science **318**, 766 (2007)

[3] Qi and Zhang, Physics Today **63**, 33 (2010)

HgTe/CdTe quantum wells

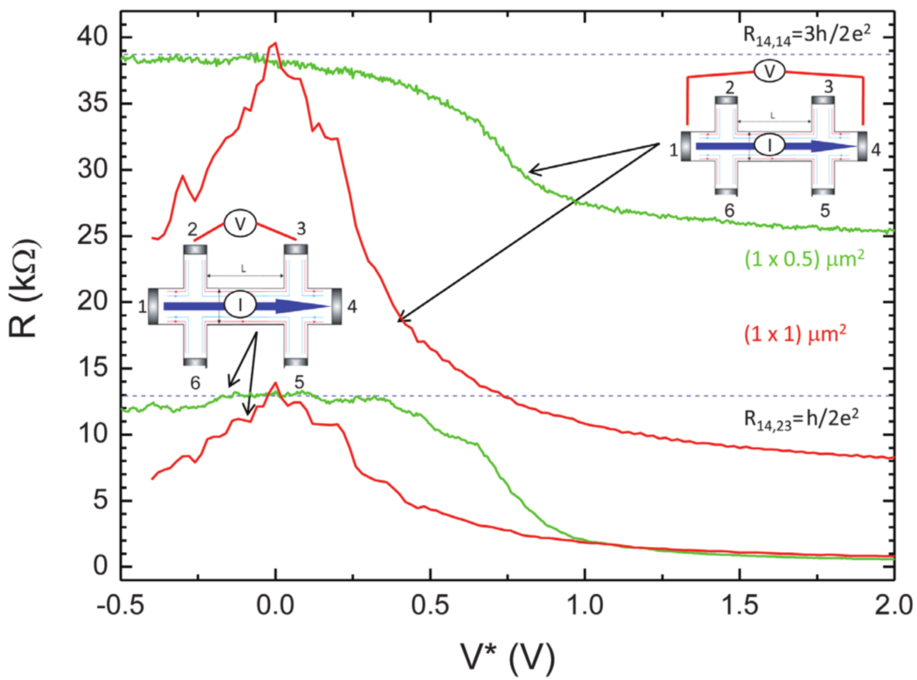
Some more evidence:

- Magnetic field dependence: breaking of TR invariance destroys quantized conductance
- Multi-terminal measurements consistent with Landauer-Büttiker prediction



$$I_j = \frac{e^2}{h}\sum_j (T_{ji}V_i - T_{ij}V_j)$$

$$T_{i+1,i} = T_{i,i+1} = 1$$

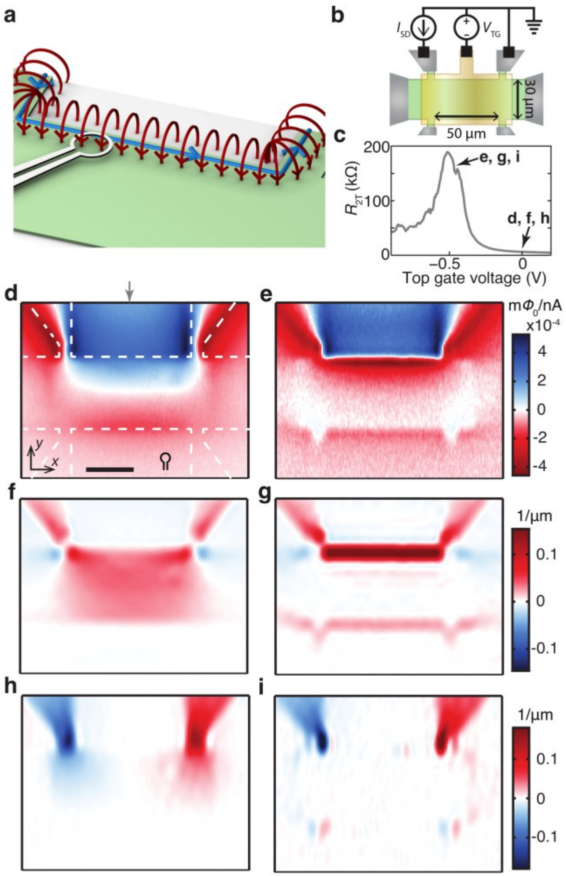


Some other experiments:

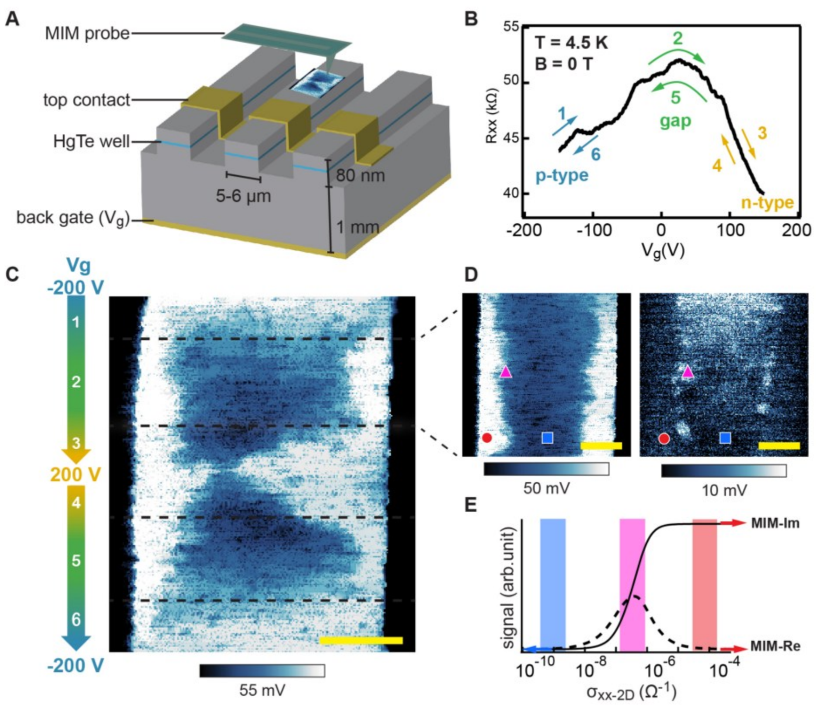
[1] G.M. Minkov, arXiv:1211.2563

[2] Gusev et al., PRB **84**, 121302 (2011)

More recent experiments



K. Nowack *et al.*, arXiv:1212.2203 (2012)

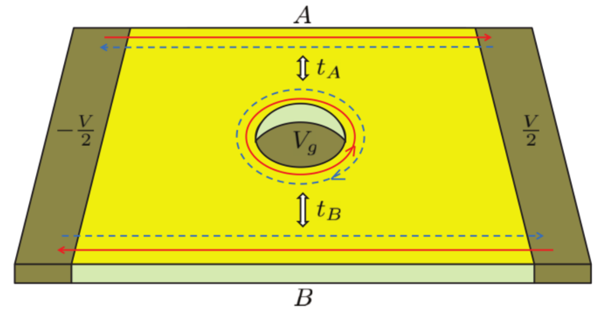


Yue Ma *et al.*, arXiv:1212.6441 (2012)

Model

*...let's return to PRB **87**, 085425 (2013)*

Setup:
- 2D topological insulator bar
- Hole in the middle ("antidot")
- (Local) tunneling between edges and antidot



Hamiltonian:

[redacted]

- Edges: Helical Luttinger liquids $\psi_{AB,\uparrow\downarrow}(x)$
- Antidot: Helical LL $\psi_{ad,\uparrow\downarrow}(x)$
- Bosonization $\psi_{ad} \rightarrow d_{q,\nu}$
- Symmetrically applied voltage
- Single-electron tunneling

$$H_{\rm ad} = -iv_F \int_0^L dx(\psi^\dagger_{\rm ad,\uparrow}\partial_x\psi_{\rm ad,\uparrow} - \psi^\dagger_{\rm ad,\downarrow}\partial_x\psi_{\rm ad,\downarrow}) + \frac{g}{2}\int_0^L dx\left(\sum_{\sigma=\uparrow,\downarrow}\psi^\dagger_{\rm ad,\sigma}\psi_{\rm ad,\sigma}\right)^2,$$

Energy scales:
- Charging energy E_n
- Spin addition energy $E_s \ll E_n$

$$\epsilon_0 = \frac{2\pi v_F}{L}, \quad \epsilon = \frac{\epsilon_0}{K}, \quad E_n = \frac{\epsilon_0}{2K^2}, \quad E_s = \frac{\epsilon_0}{2}$$

Transport through the antidot

Transport theory:

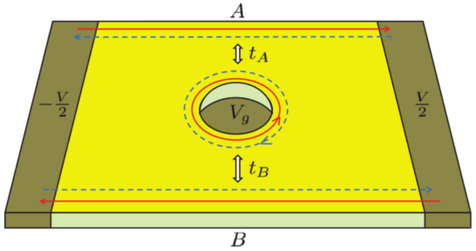
- Antidot Hilbert space: $|\text{charge } n, \text{spin } s, \text{plasmons } \{m_\nu\}\rangle$
- Sequential tunneling:

$$\Delta_n = \pm 1, \Delta_s = \pm 1$$

- Transition rates:

$$\Gamma^{\lambda}_{i\to f} = |t_\lambda|^2 \int dt e^{-i\Delta U^{\lambda}_{i\to f} t} G_{ad}(t) G_\lambda(t)$$

$$\Delta U^{\lambda}_{i\to f} = E_n\left[\tfrac{1}{2} + \Delta_n(n - n_g)\right] + E_s\left[\tfrac{1}{2} + \Delta_s s\right] \pm \tfrac{1}{2} eV\Delta_s$$



- Green's function from LL theory: (thermal average)

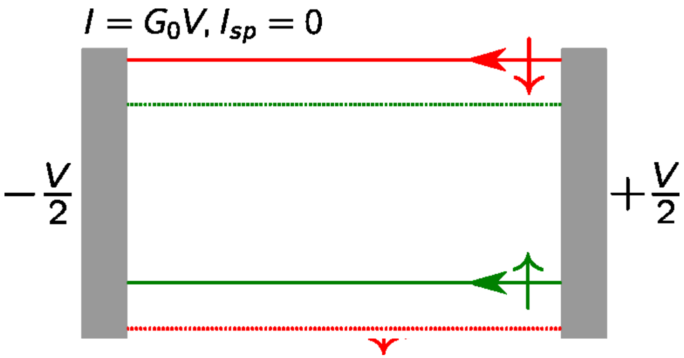
$$G_{\lambda,ad}(t) = \left\langle \psi_{\lambda,ad}(0,t)\psi^{\dagger}_{\lambda,ad}(0,0)\right\rangle$$

Total current:

$$I = \frac{2e^2}{h}V - I_{BS}$$
$$= G_0 V - \frac{2e}{\hbar} I^{(tun)}_{\sigma}$$

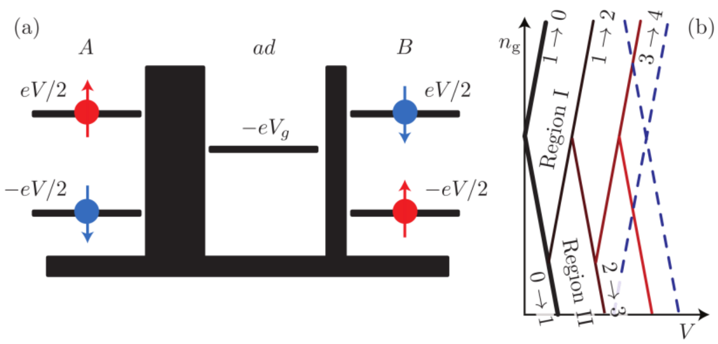
Spin-polarized current:

$$I_{sp} = \frac{\hbar}{2}\sum_{\sigma}(\dot{n}_{A\sigma} - \dot{n}_{B\sigma})$$
$$= \frac{2e}{\hbar} I^{(tun)}_{\rho}$$

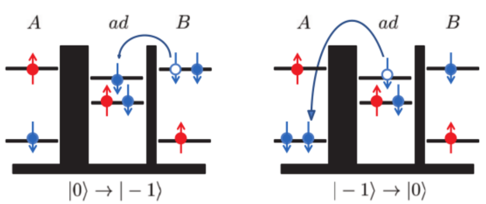


Asymmetric junctions

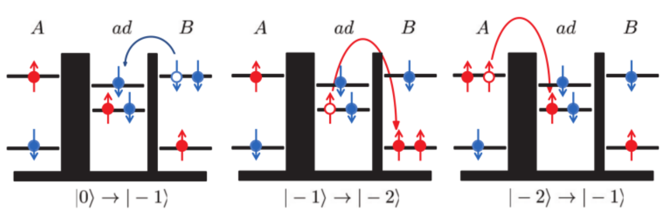
Asymmetric junctions: $\Gamma_B = \eta\Gamma_A, \quad \eta > 1$



Region 1:



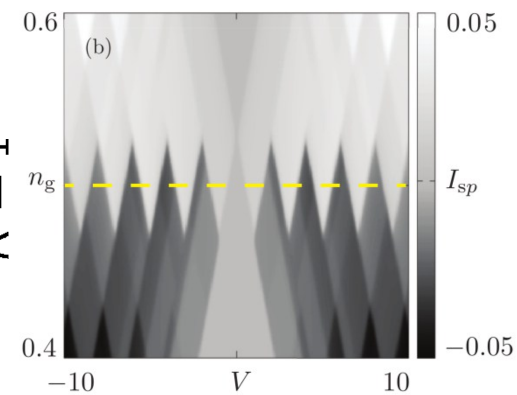
Region 2:



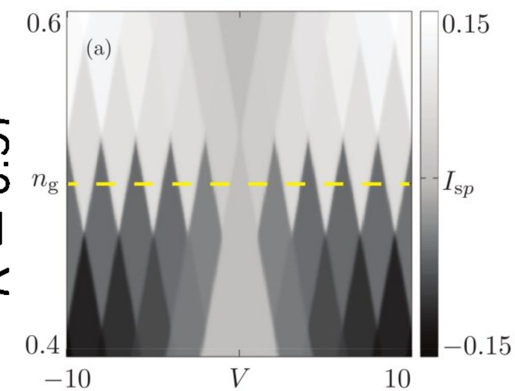
Spin-polarized current I_{sp}:

$(\eta = 10, E_n/\epsilon_0 = 20)$

$K = 1$

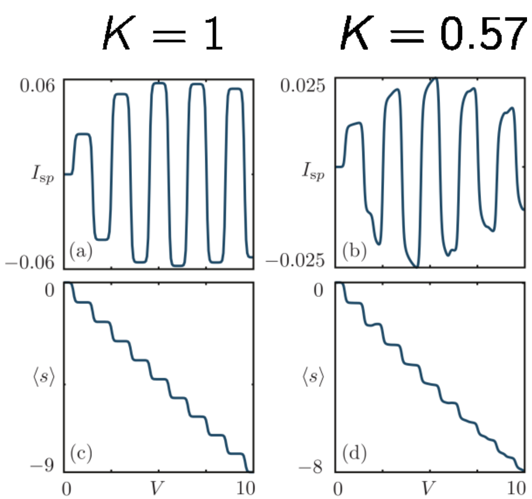


$K = 0.57$

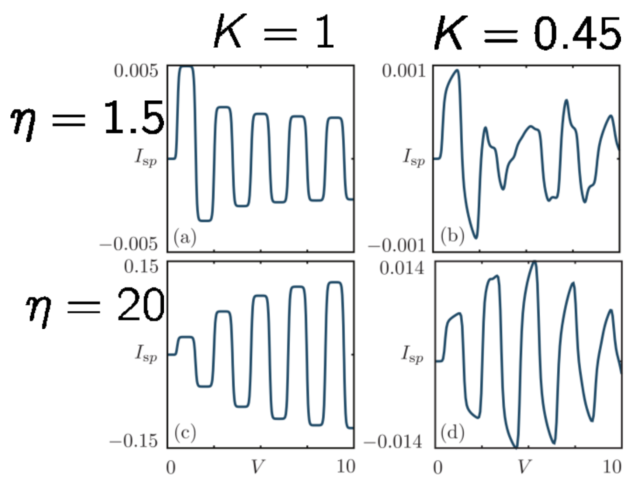


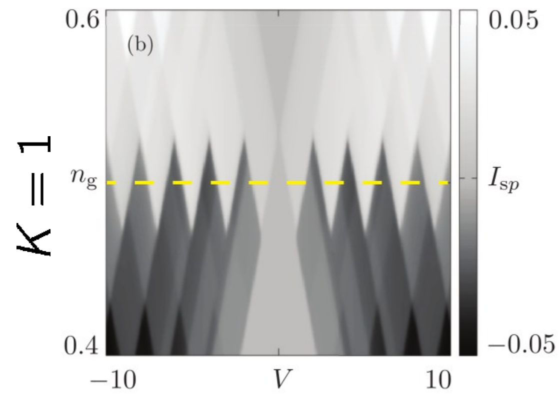
$$\Gamma_A = \frac{|t_A|^2}{(2\pi a)^2}\frac{1}{\omega_c}$$

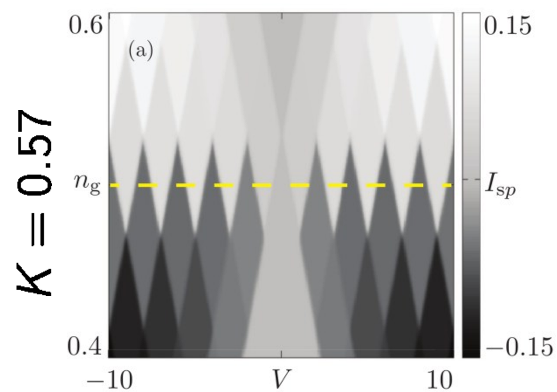
Current and dot spin:
$(\eta = 10, E_n/\epsilon_0 = 20)$



Current for different asymmetries:







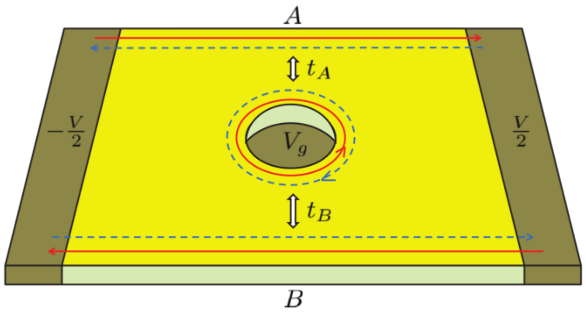
Analytics for few dot levels:

$$\frac{I_{\rm sp}}{\hbar\Gamma_0} = \begin{cases} \frac{\eta(\eta-1)}{\eta^2+\eta+1} & \text{region } I, \\ -\frac{2\eta(\eta-1)(\eta^2-\eta+1)}{(\eta^2+1)^2+2\eta(\eta^2+\eta+1)} & \text{region } II. \end{cases}$$

…no comparison to numerics… ☹

Conclusions

- 2D topological insulator antidot configuration with helical states at edges and around the antidot
- Charging energy on the antidot is large
- Asymmetric coupling the edge states produces spin-polarized current
- Electrically controllable spin-filter



Dolcetto *et al.*,
Phys. Rev. B **87**, 085425 (2013)