

Photonic Floquet topological insulators

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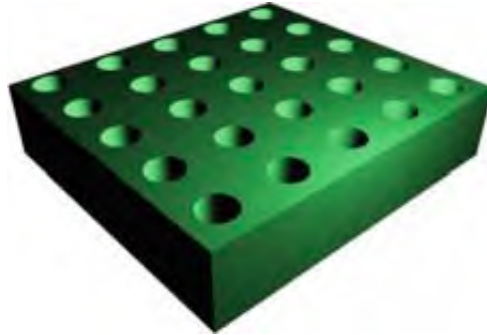
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$$\dot{\mathbf{X}} = \mathbf{F}(t)\mathbf{X}$$

Lyapunov–Floquet

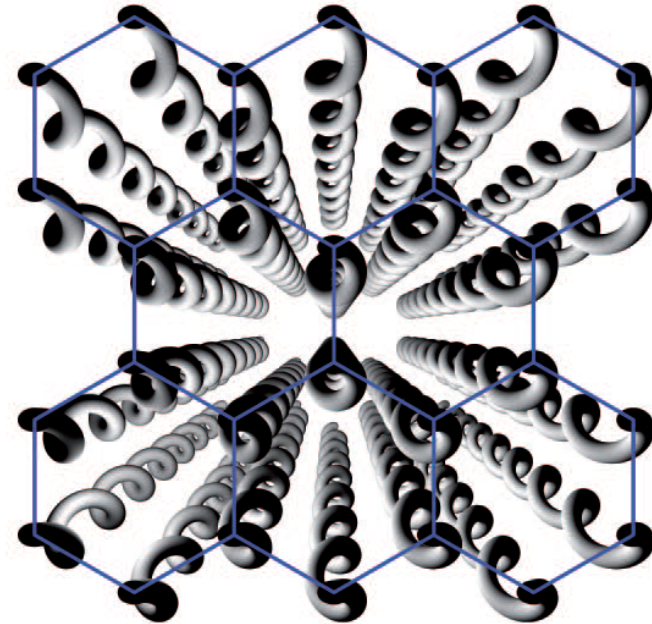
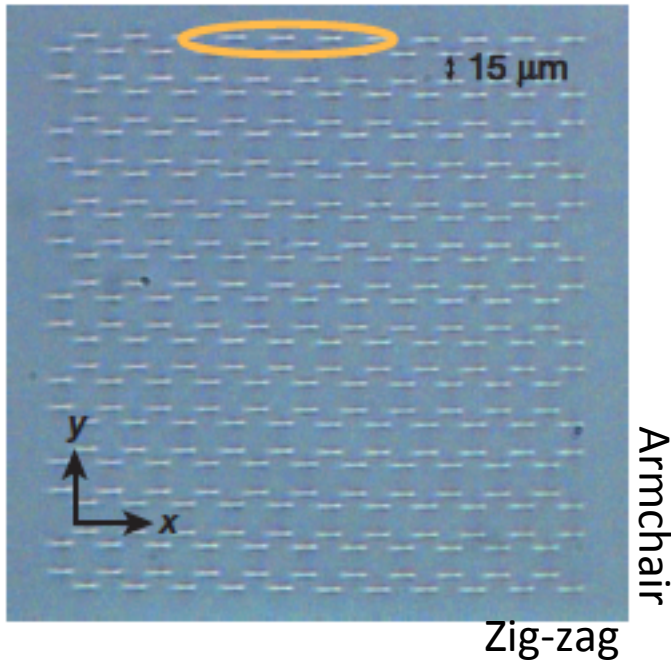
$$\mathbf{F}(t + T) = \mathbf{F}(t)$$

Optical media with periodic modulation of the permittivity.



Examples of 1D, 2D and 3D photonic crystals:
Bragg grating, porous silicon, opal.

- Periodic modulation of the permittivity forms a lattice similar to atomic lattice of solid-state.
- Behavior of photons in a photonic crystal is similar to electron and hole behavior in an atomic lattice.
- Due to the lattice periodicity photonic crystal provide band gap.
- Difference is the particle energy distribution.



The photonic lattice is an array of evanescently - coupled waveguides arranged in a honeycomb structure with nearest-neighbor spacing of 15 μm

Medium - fused silica. Refractive index $n_0 = 1.45$

Each waveguide has a cross-section with major and minor axis diameters of 11 μm and 4 μm. The total propagation length is 10 cm.

$$\lambda = 633 \text{ nm}$$

HeNe, gas, red

Paraxial propagation of light in photonic lattice.

$$\nabla^2 \mathbf{E}(r) + \frac{\omega^2}{c^2} \varepsilon(r) \mathbf{E}(r) = 0$$

$$\omega = \frac{2\pi c}{\lambda}$$

$$\mathbf{E}(r) = \psi(r) e^{ik_0 z} \mathbf{x}$$

$$k_0 = \frac{2\pi n_0}{\lambda}$$

$$2ik_0 \partial_z \psi(r) + \nabla_{\perp}^2 \psi(r) + \left(\frac{\omega^2}{c^2} \varepsilon(r) - k_0^2 \right) \psi(r) = 0$$

Assumption $\left| \frac{\partial^2 \psi(r)}{\partial z^2} \right| < k_0 \left| \frac{\partial \psi(r)}{\partial z} \right|, \left| \frac{\partial^2 \psi(r)}{\partial x^2} \right|, \left| \frac{\partial^2 \psi(r)}{\partial y^2} \right|$

Effective potential $k_0^2 \left(\frac{\varepsilon(r)}{n_0^2} - 1 \right) \sim 2k_0^2 \frac{\sqrt{\varepsilon(r)} - n_0}{n_0} \equiv 2k_0^2 \frac{\Delta n(r)}{n_0}$

$$\Delta n = 7 \times 10^{-4}$$

Schrodinger-type equation

$$i\partial_z\psi(x,y,z) = -\frac{1}{2k_0}\nabla^2\psi(x,y,z) - \frac{k_0\Delta n(x,y,z)}{n_0}\psi(x,y,z) \quad (1)$$

Wave guides are invariant in the z direction

$$\begin{aligned}x' &= x + R\cos(\Omega z) & \Omega &= 2\pi / Z_p \\y' &= y + R\cos(\Omega z) & Z_p &= 1\text{cm} \\z' &= z\end{aligned}$$

In transformed coordinates

$$i\partial_{z'}\psi(\mathbf{r}') = -\frac{1}{2k_0}(\nabla + i\mathbf{A}(z'))^2\psi(\mathbf{r}') - \frac{k_0 R^2 \Omega^2}{2}\psi(\mathbf{r}') - \frac{k_0 \Delta n(x', y')}{n_0}\psi(\mathbf{r}')$$

$$\mathbf{A}(z') = k_0 R \Omega [\sin(\Omega z'), -\cos(\Omega z'), 0]$$

$$i\partial_{z'}\psi_n(z') = \sum_{\langle m \rangle} c e^{i\mathbf{A}(z')\mathbf{r}_{mn}} \psi_m(z')$$

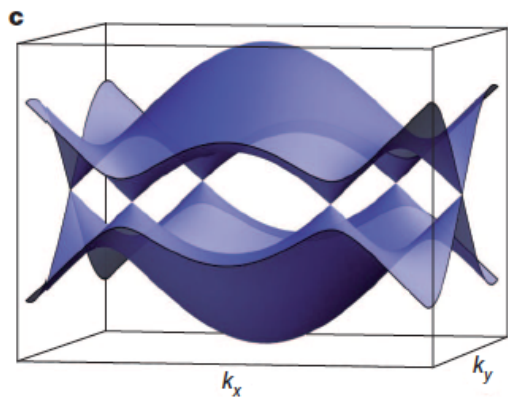
Summation over neighborhood waveguides

$$\text{Solution: } \psi_n(z') = e^{i\beta z'} \varphi_n(z')$$

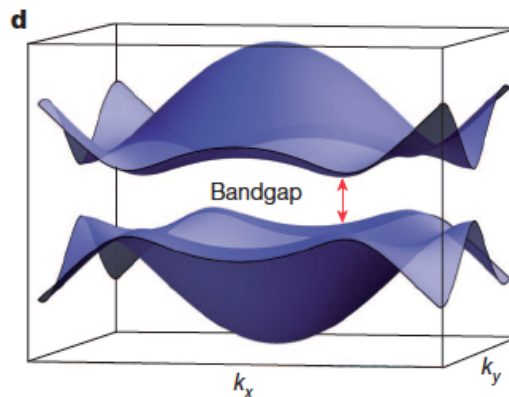
$\varphi_n(z')$ is Z_p periodic function.

Spectrum of β

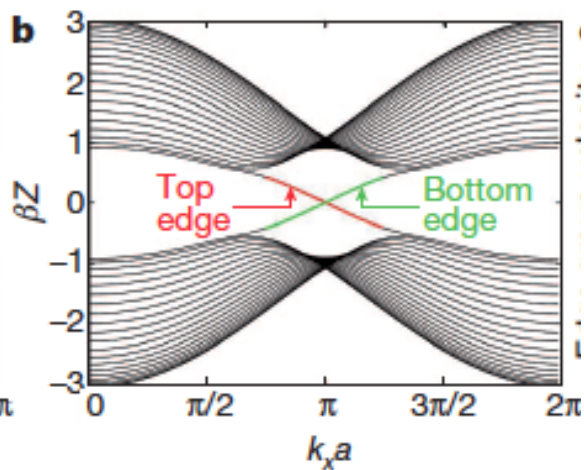
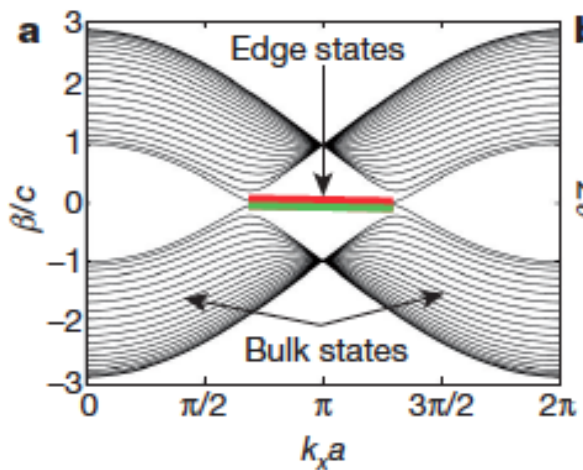
$$\beta(k_x, k_y)/c$$



$$R = 0$$

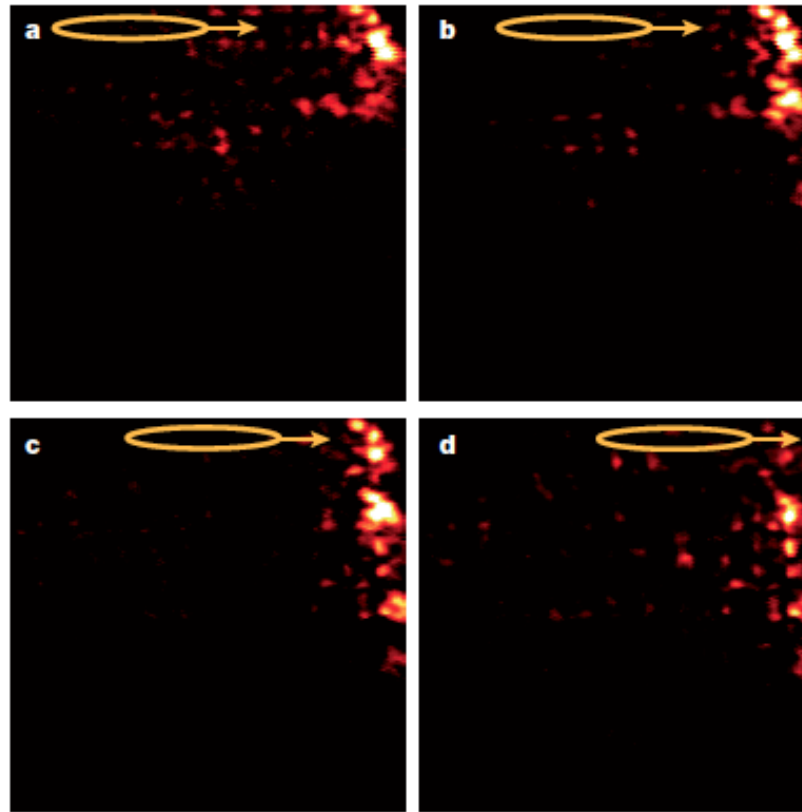


$$R = 8\mu\text{m}$$



Group velocity in the x-y plane

$$R = 8\mu\text{m}$$



to the right in a–d. The beam propagates along the top edge of the array (which is in the zig-zag configuration), hits the corner, and clearly moves down the vertical edge (which is in the armchair configuration). Note that the wavepacket shows no evidence of backscattering or bulk scattering due to its impact with the corner of the lattice. This scattering of the edge state is prevented by topological protection.

