Measures of quantum synchronization in continuous variable systems

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Motivation

Why study synchronization?

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Why study synchronization?

Happens all over in nature:

- neuronal networks
- chemical reactions
- pacemaker heart cells
- fireflies
- ...

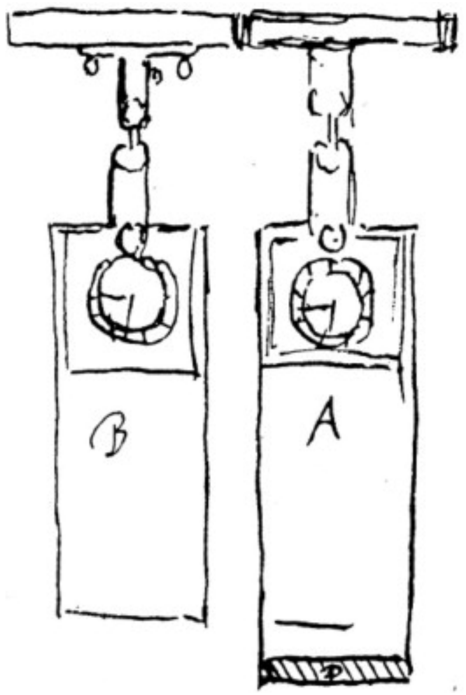
Outline

1. Synchronization
2. Measures for *classical* Synchronization
3. Measures for *quantum* Synchronization
4. Synchronization in opto-mechanical systems

Synchronization

Ingredients to synchronization:

Two or more mutually coupled dynamical systems



C. Huygens, Œuvres Completes de Christiaan Huygens, Vol.15 (1893), Vol. 17 (1932)

- Classical non-linear dynamical systems
- Self-sustained oscillators

(each oscillators stable limit cycle)

Synchronization

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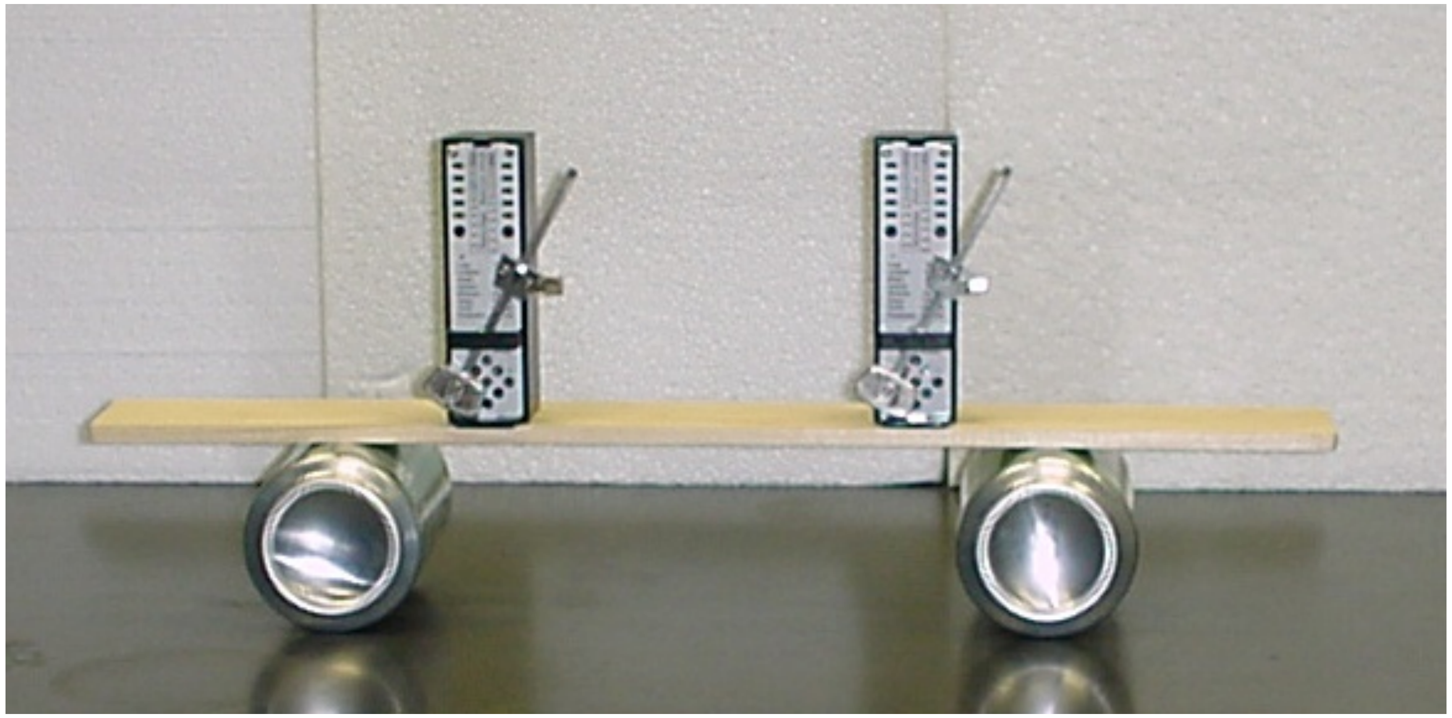


- Classical non-linear dynamical systems
- Self-sustained oscillators

(each oscillators stable limit cycle)

Classical Synchronization

$S_1 : q_1(t),\ p_1(t)$ different initial states $S_1^{(0)} \neq S_2^{(0)}$

$S_2 : q_2(t),\ p_2(t)$ mutual interaction $\lambda S_1 S_2$

Complete synchronization

$$p_-(t) := [p_1(t) - p_2(t)]/\sqrt{2} \rightarrow 0 \text{ for t} \rightarrow \infty$$

$$q_-(t) := [q_1(t) - q_2(t)]/\sqrt{2} \rightarrow 0 \text{ for t} \rightarrow \infty$$

Phase synchronization

$$\varphi_-(t) := [\varphi_1(t) - \varphi_2(t)] \rightarrow \varphi_0 \in [0, 2\pi] \text{ for t} \rightarrow \infty$$

$$\varphi_j(t) = \arctan[p_j(t)/q_j(t)]$$

Quantum Synchronization

Extension to quantum systems not straight forward

$[q_j(t), p_{j'}(t)] = i\delta_{jj'}$

$\Rightarrow \quad q_-(t), p_-(t)$ gen. coordinates of same mode

$\Rightarrow$ Heisenberg!

Introduce a measure of *qm complete* synchronization

$$\mathcal{S}_c(t) := \langle q_-(t)^2 + p_-(t)^2 \rangle^{-1}$$

Heisenberg principle $\langle q_-(t)^2 \rangle \langle p_-(t)^2 \rangle \geq 1/4$

$$\mathcal{S}_c(t) \leq \frac{1}{2\sqrt{\langle q_-(t)^2 \rangle \langle p_-(t)^2 \rangle}} \leq 1$$

Quantum Synchronization

Introduce a measure of *qm phase* synchronization

$$a_j(t) := [q_j(t) + ip_j(t)]/\sqrt{2} = [r_j(t) + a'_j(t)]e^{i\varphi_j(t)}$$

$$\langle a_j(t) \rangle = r_j(t) e^{i\varphi_j(t)}$$

$$\Rightarrow a'_j(t) = [q'_j(t) + ip'_j(t)]/\sqrt{2}$$

$q'_j(t)$ fluctuations of amplitude

$p'_j(t)$ fluctuations of phase

Quantum Synchronization

Introduce a measure of *qm phase* synchronization

$$a_j(t) := [q_j(t) + ip_j(t)]/\sqrt{2} = [r_j(t) + a'_j(t)]e^{i\varphi_j(t)}$$

$$\langle a_j(t) \rangle = r_j(t)e^{i\varphi_j(t)}$$

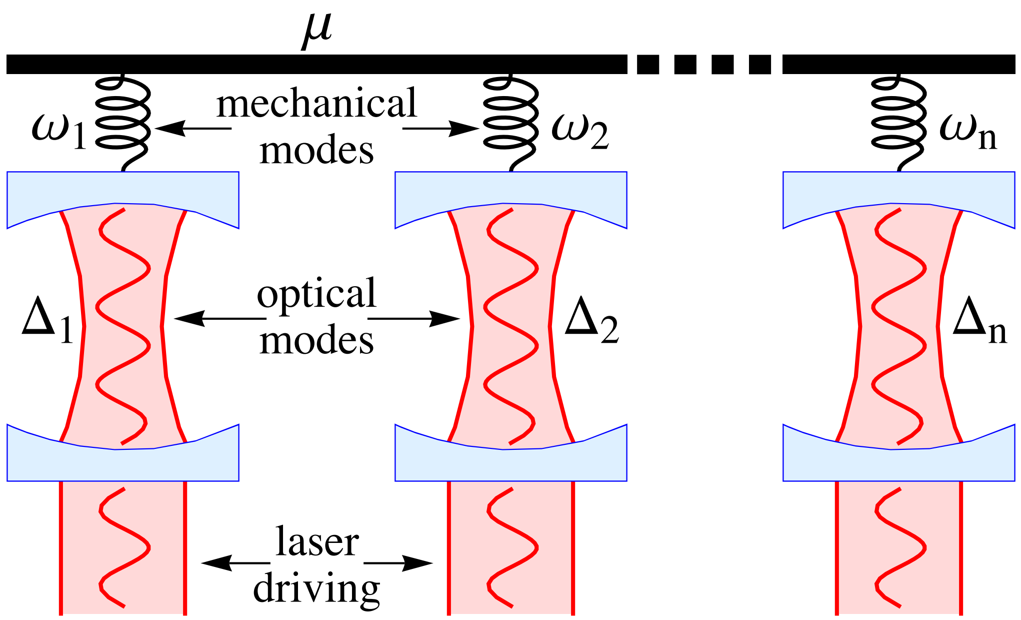
$$\Rightarrow a'_j(t) = [q'_j(t) + ip'_j(t)]/\sqrt{2}$$

if the phases of $\langle a_1(t) \rangle$ and $\langle a_2(t) \rangle$ are locked, then

$$p'_-(t) = [p'_1(t) - p'_2(t)]/\sqrt{2}$$

$$\mathcal{S}_p(t) := \frac{1}{2}\langle p'_-(t)^2 \rangle^{-1} \leq 1 \qquad \text{(pos. P-function)}$$

Synchronization in Optomechanics



$$H = \sum_{j=1,2} [\Delta_j a_j^\dagger a_j + \omega_j b_j^\dagger b_j - g a_j^\dagger a_j (b_j + b_j^\dagger) + iE(a_j - a_j^\dagger)]$$
$$- \mu(b_1 b_2^\dagger + b_2^\dagger b_1)$$

$$\dot{a}_j = [-\kappa - i\Delta_j + ig(b_j + b_j^\dagger)]a_j + E + \sqrt{2\kappa} a_j^{in}$$
$$\dot{b}_j = [-\gamma - i\omega_j]b_j + iga_j^\dagger a_j + i\mu b_{3-j} + \sqrt{2\gamma} b_j^{in}$$

Synchronization in Optomechanics

Semiclassical treatment: $O(t) = \langle O(t) \rangle + O'(t)$

mean value describing classical trajectories

small qm fluctuations

qm Langevin equations:

set of classical non-linear DEQ

qm fluctuations

$$C_{i,\ell}(t) = \langle \{ R_i(t) R_\ell(t)^\dagger \} \rangle / 2$$
$$R = (a_1', a_1'^\dagger, b_1', b_1'^\dagger, a_2', a_2'^\dagger, b_2', b_2'^\dagger)$$
$$\Rightarrow \quad \langle q_-(t)^2 \rangle, \; \langle p_-(t)^2 \rangle \quad \Rightarrow \quad \mathcal{S}_c$$

Synchronization in Optomechanics

Semiclassical treatment: $O(t) = \langle O(t) \rangle + O'(t)$

mean value describing classical trajectories → $\langle O(t) \rangle$

small qm fluctuations → $O'(t)$

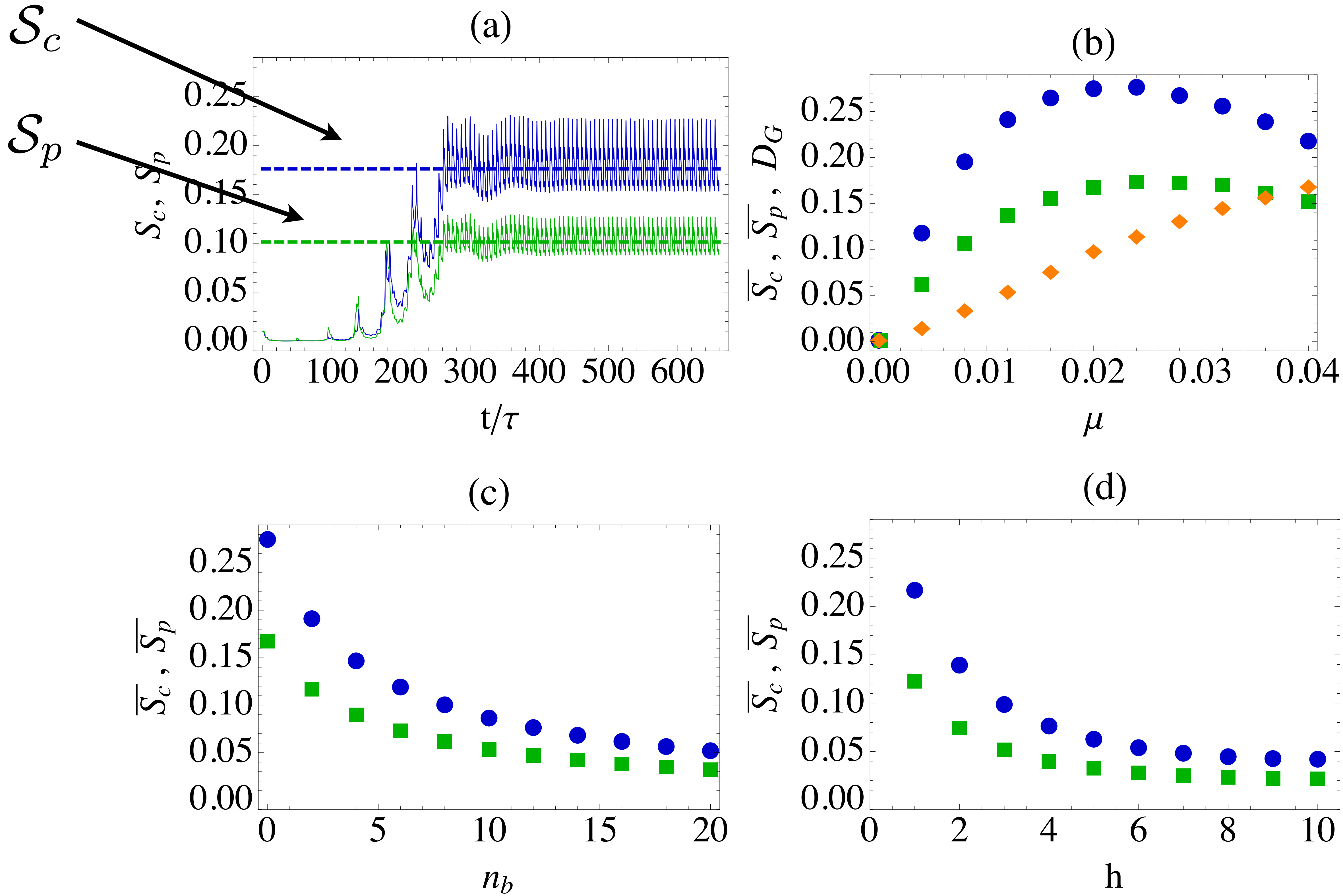
qm Langevin equations:

→ set of classical non-linear DEQ

→ qm fluctuations

$$C'(t) = U(t)C(t)U^{\dagger}(t)$$
$$U(t) = \mathrm{diag}[e^{-i\varphi_{a_1}(t)}, e^{i\varphi_{a_1}(t)}, \ldots]$$
$$\Rightarrow \langle p'_{-}(t)^2 \rangle \Rightarrow \mathcal{S}_p$$

Synchronization in Optomechanics



Conclusion

- Proposal of a measure for quantum synchronization of coupled continuous variable systems inspired by classical synchronization

- Universal bounds by quantum mechanics to level of synchronization

- Application to an opto-mechanical system