

# Antiferromagnetic states and phase separation in doped AA-stacked graphene bilayers

A.O. Sboychakov,<sup>1,2</sup> A.V. Rozhkov,<sup>1,2</sup> A.L. Rakhmanov,<sup>1,2,3</sup> and Franco Nori<sup>1,4</sup>

<sup>1</sup>*CEMS, RIKEN, Saitama, 351-0198, Japan*

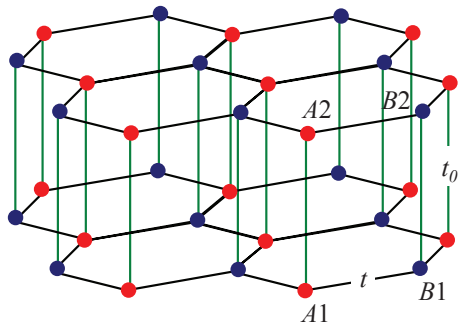
<sup>2</sup>*Institute for Theoretical and Applied Electrodynamics,  
Russian Academy of Sciences, 125412 Moscow, Russia*

<sup>3</sup>*Moscow Institute of Physics and Technology, Dolgoprudnyi, Moscow Region, 141700 Russia*

<sup>4</sup>*Department of Physics, University of Michigan, Ann Arbor, MI 48109-1040, USA*

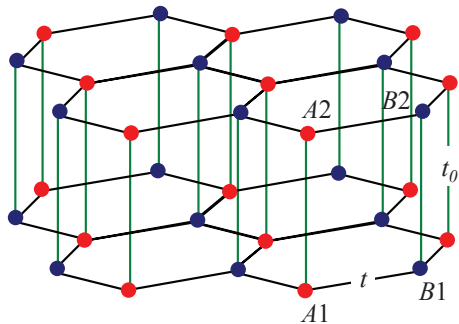
We study electronic properties of AA-stacked graphene bilayers. In the single-particle approximation such a system has one electron band and one hole band crossing the Fermi level. If the bilayer is undoped, the Fermi surfaces of these bands coincide. Such a band structure is unstable with respect to a set of spontaneous symmetry violations. Specifically, strong on-site Coulomb repulsion stabilizes antiferromagnetic order. At small doping and low temperatures, the homogeneous phase is unstable, and experiences phase separation into an undoped antiferromagnetic insulator and a metal. The metallic phase can be either antiferromagnetic (commensurate or incommensurate) or paramagnetic depending on the system parameters. We derive the phase diagram of the system on the doping-temperature plane and find that, under certain conditions, the transition from paramagnetic to antiferromagnetic phase may demonstrate re-entrance. When disorder is present, phase separation could manifest itself as a percolative insulator-metal transition driven by doping.

# AA-stacked graphene bilayer

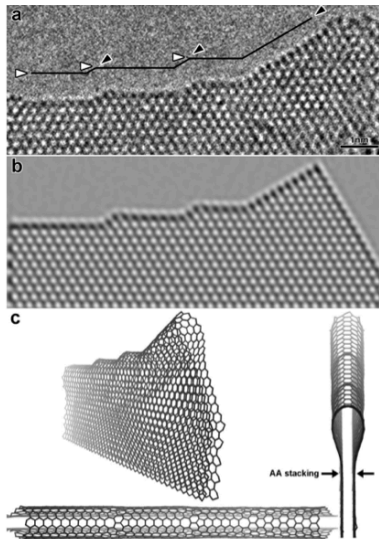


- Not energetically favourable over AB stacking  
⇒ need a pinning mechanism

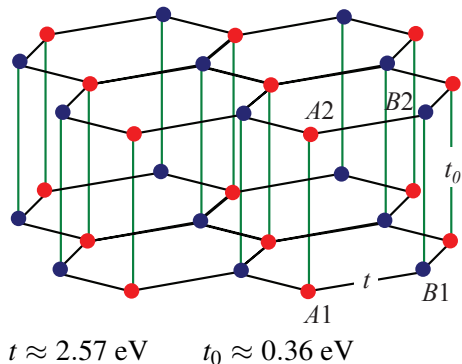
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# Single-particle description



$$\begin{aligned}
 H_0 = & -t \sum_{\langle \mathbf{nm} \rangle i \sigma} d_{\mathbf{n}iA\sigma}^\dagger d_{\mathbf{m}iB\sigma} + \text{H.c.} \\
 & -t_0 \sum_{\mathbf{n}a\sigma} d_{\mathbf{n}1a\sigma}^\dagger d_{\mathbf{n}2a\sigma} + \text{H.c.} \\
 & -\mu \sum_{\mathbf{n}ia\sigma} d_{\mathbf{n}ia\sigma}^\dagger d_{\mathbf{n}ia\sigma}
 \end{aligned}$$

# Spectrum

$$\epsilon_{0\mathbf{k}}^{(1)} = -t_0 - t\zeta_{\mathbf{k}}$$

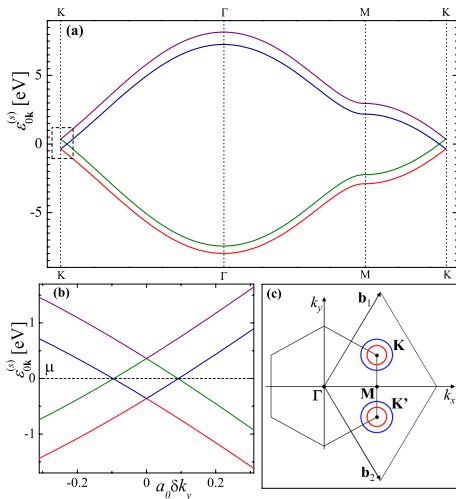
$$\epsilon_{0\mathbf{k}}^{(2)} = -t_0 + t\zeta_{\mathbf{k}}$$

$$\epsilon_{0\mathbf{k}}^{(3)} = +t_0 - t\zeta_{\mathbf{k}}$$

$$\epsilon_{0\mathbf{k}}^{(4)} = +t_0 + t\zeta_{\mathbf{k}}$$

$$\zeta_{\mathbf{k}} = \left| 1 + 2 \exp\left(\frac{3ik_x a_0}{2}\right) \cos\left(\frac{\sqrt{3}k_y a_0}{2}\right) \right|$$

⇒ Twice graphene  $\pm t_0$



Fermi surface nesting, prone to instabilities (PRL **109**, 206801 (2012))

# Antiferromagnetism with mean field

On-site repulsion  $U \sim 4 - 10$  eV

$$H_{\text{int}} = \frac{U}{2} \sum_{\mathbf{n}i\sigma} (n_{\mathbf{n}i\sigma} - \frac{1}{2})(n_{\mathbf{n}i\bar{\sigma}} - \frac{1}{2})$$

Order parameter  $\Delta_{ia} \equiv U \langle d_{\mathbf{n}i\uparrow}^\dagger d_{\mathbf{n}i\downarrow} \rangle$  (magnetization along  $x$ )

"G-type"  $\Delta_{1\mathcal{A}} = \Delta_{2\mathcal{B}} = -\Delta_{1\mathcal{B}} = -\Delta_{2\mathcal{A}} \equiv \Delta$

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$$H_{\text{int}}^{\text{MF}} = \underbrace{\mathcal{N} \left( \frac{4\Delta^2}{U} - U(n^2 - 1) \right)}_{E_0} + \underbrace{\frac{Ux}{2} \sum_{\mathbf{n}i\sigma} n_{\mathbf{n}i\sigma}}_{\mu \rightarrow \mu'} - \sum_{\mathbf{n}i} \Delta_{ia} (d_{\mathbf{n}i\uparrow}^\dagger d_{\mathbf{n}i\downarrow} + \text{H.c.})$$

$n$ : #el/site

$x \equiv n - 1$ : doping

# Self consistency

$$\epsilon_{\mathbf{k}}^{(1,4)} = \mp \sqrt{\Delta^2 + (t\zeta_{\mathbf{k}} + t_0)^2}$$
$$\epsilon_{\mathbf{k}}^{(2,3)} = \mp \sqrt{\Delta^2 + (t\zeta_{\mathbf{k}} - t_0)^2}$$

Minimization of the grand potential w.r.t.  $\Delta$

$$\Omega = E_0 - 2T \sum_{s=1}^4 \int \frac{d\mathbf{k}}{V_{\text{BZ}}} \log(1 + e^{(\mu' - \epsilon_{\mathbf{k}}^{(s)})/T})$$

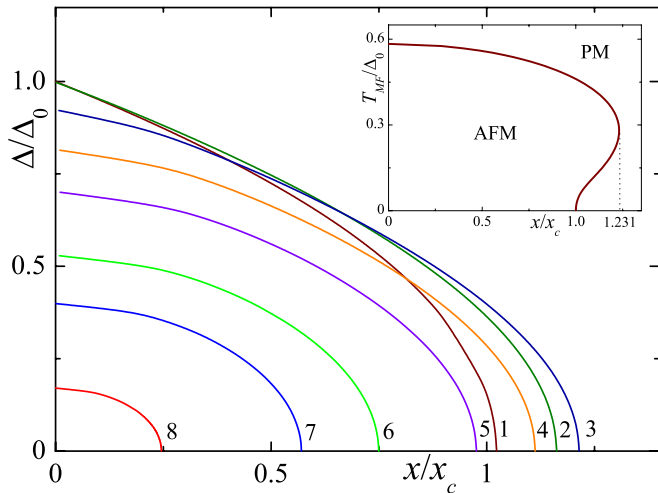
Zero temperature

$$\Delta(x, 0) = \Delta_0 \sqrt{1 - x/x_c}$$
$$\mu'(x, 0) = \Delta_0 (\text{sgn}(x) - x/2x_c)$$

Transition temperature  $T_{\text{MF}} \sim 0.567\Delta_0$  at  $x = 0$  for small  $U$



# Phase diagram



Re-entrant behavior: ordering with increasing  $T$ . Artifact of mean field?

## Validity of mean field

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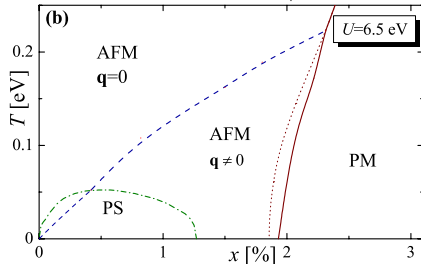
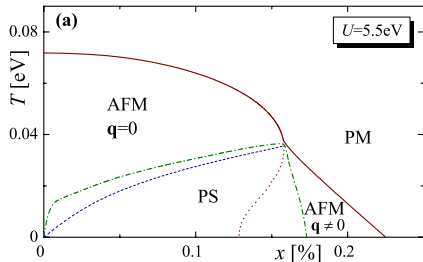
- Mean field reasonable locally if  $\xi_{\text{MF}} < \xi_{\text{sw}}$
- Breakdown of MF  $T^* \geq 0.8T_{\text{MF}}$ ;  $T_{\text{MF}}$  still a good estimate of the transition temperature
- MF not quantitatively good for large  $U$

# Incommensurate AFM

Spatial modulation of the order parameter  $\Delta_{\mathbf{n}ia} \equiv e^{i\mathbf{q}\mathbf{n}}U\langle d_{\mathbf{n}ia\uparrow}^\dagger d_{\mathbf{n}ia\downarrow} \rangle$

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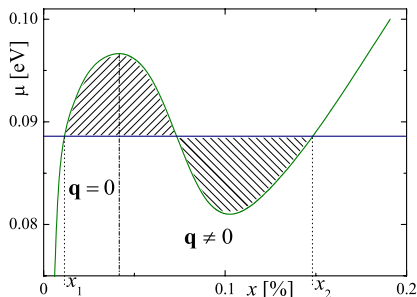
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- Incommensurate AFM wins only for low  $T$  (less symmetrical)
- Hides re-entrance for low  $U$

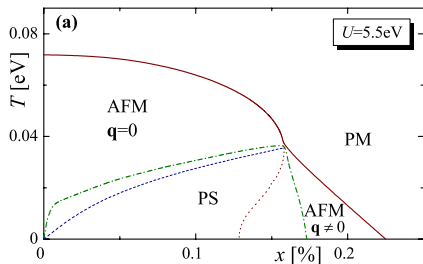
# Phase separation

In some regions of  $(x, T)$ ,  $\partial\mu/\partial x < 0 \Rightarrow$  unphysical  
Separation of two phases (comm. AFM, incomm. AFM) with locked  
electron densities  $x_1, x_2$

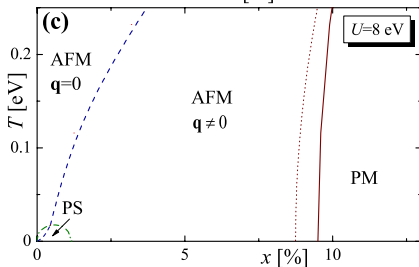
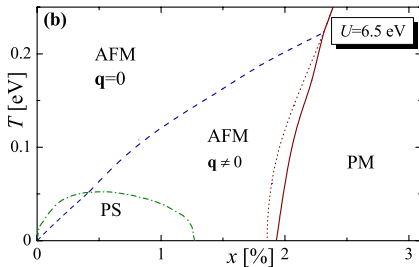


$x_2 \gg x_1 \Rightarrow$  percolative metal-‘insulator’ transition upon doping  
(arXiv:1302.1994)

# Phase diagram



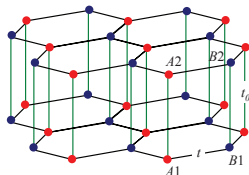
- Actual values of  $x_c, T_c$  strongly depend on  $U$   
 $T_c \sim 50\text{ K}$  for  $U = 5.5\text{ eV}$   
 $T_c \geq \text{RT}$  for  $U \geq 6.5\text{ eV}$





# Conclusions

- Magnetic properties of AA-stacked bilayer graphene



- Antiferromagnetism observable (maybe up to RT)
- Possibility of re-entrance, incommensurate AFM, phase separation and percolative metal-insulator transition; some may be artifacts

Series of papers by the same authors: PRL **109**, 206801; PRB **87**, 075128; PRB **87**, 121401; arXiv 1302.1994; arXiv 1305.0330