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# Macroscopic Quantum Mechanics in a Classical Spacetime

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# Outline

# Semiclassical gravity

Classical theory of spacetime that supports quantum matter,

$$G_{\mu\nu} = 8\pi \langle\psi|\hat{T}_{\mu\nu}|\psi\rangle \,.$$

**Flaw:**
It does not allow state reduction $\rightarrow$ violates $\nabla^\nu G_{\mu\nu} = 0$

**Consequence:**
We need an interpretation of QM that does not require state reduction yet still explains the phenomenology of quantum measurements.

# Diòsi-Penrose model[1,2]

Relates the process of decoherence and state reduction to gravity.

**Basic idea:**

Decoherence rate for a superposition state $\frac{|\psi_1\rangle + |\psi_2\rangle}{\sqrt{2}}$:

$$\Gamma \sim \frac{\Delta E_G}{\hbar}, \qquad \Delta E_G = |U_g(1,1) + U_g(2,2) - 2U_g(1,2)|,$$

$U_g(i,j) =$ Newtonian interaction between mass densities.

[1] L. Diòsi, Phys. Lett. 105A (1984)

[2] R. Penrose, Gen. Relativ. Gravit. 28 (1996)

# Many-particle Schrödinger-Newton equation

Wavefunction of $n$ nonrelativistic particles: $\varphi(t, \mathbf{X})$

$$i\hbar\partial_t\varphi = \sum_k \left[-\frac{\hbar^2\nabla_k^2}{2m_k} + \frac{m_k\,U(t,\mathbf{x}_k)}{2}\right]\varphi + V(\mathbf{X})\varphi\,,$$

| | |
|---|---|
| $3n$-$D$ coordinate vector: | $\mathbf{X} \equiv (\mathbf{x}_1, \dots, \mathbf{x}_n)$ |
| Potential energy: | $V(\mathbf{X})$ |

Newtonian potential:

$$\nabla^2 U(t,\,\mathbf{x}) = 4\pi \sum_j \int \mathrm{d}^{3n}\mathbf{X}\,|\varphi(t,\mathbf{X})|^2 m_j\,\delta(\mathbf{x}-\mathbf{x}_j)\,.$$

# Separation of scales

- We only probe the c.m. motion,

$$\tau_{meas} \gg \text{ time at which atoms oscillates internally}$$

- $1\text{ Hz} < \omega_{c.m.} < 1\text{ kHz}, \quad 1\text{ g} < M < 10\text{ kg},$

$$\Delta x_{c.m.} \sim \sqrt{\hbar/(M\omega_{c.m.})} \sim 10^{-19} - 10^{-17}\text{m}.$$

- Internal motions of atoms

$$\begin{aligned}\langle x^2 \rangle &= \frac{\hbar^2}{mk_BT} \int_0^{+\infty} \frac{g(\nu)}{h\nu/k_BT} \left( \frac{1}{2} + \frac{1}{e^{h\nu/k_BT} - 1} \right) d\nu \\ &= \Delta x_{zp} + \Delta x_{th}\end{aligned}$$

- Appropriate regime: $\Delta x_{zp} \gg \Delta x_{th} \gg \Delta x_{c.m.}$

# Center of mass SN equation

Center of mass: $\mathbf{x} = \frac{1}{n}\sum_{k=1}^{n} \mathbf{x}_k$

Motion of atom $k$: $\mathbf{y}_k \equiv \mathbf{x}_k - \mathbf{x}$

**Standard quantum mechanics:**
Interaction depending only on the separation of atoms,

$$\varphi(t, \mathbf{X}) = \Psi(t, \mathbf{x})\, \chi_{int}(t, \mathbf{Y}), \qquad \mathbf{Y} \equiv (\mathbf{y}_1, \ldots, \mathbf{y}_{n-1})$$

Independent evolution of $\Psi$ and $\chi_{int}$,

$$i\hbar\partial_t\Psi(t, \mathbf{x}) = H_{c.m.}\Psi(t, \mathbf{x})\,,$$
$$i\hbar\partial_t\chi_{int}(t, \mathbf{Y}) = H_{int}\chi_{int}(t, \mathbf{Y})\,.$$

**Classical gravity:**

Assuming separability, $\varphi = \Psi\,\chi_{int}$,

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2M} + \frac{1}{2}M\omega_{c.m.}^2 x^2 + \frac{1}{2}\mathcal{C}(x - \langle x\rangle)^2\right]\Psi.$$

Expectation of the center of mass position: $\langle x\rangle = \langle\Psi|\hat{x}|\Psi\rangle$

SN coupling constant:

$$\mathcal{C} = -\frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\int \frac{G\tilde{\rho}_{\mathrm{int}}(\mathbf{y})\tilde{\rho}_{\mathrm{int}}(\mathbf{y}')}{|\mathbf{z}+\mathbf{y}-\mathbf{y}'|}d\mathbf{y}d\mathbf{y}'\right]_{\mathbf{z}=0}$$

Gravitational frequency: $\omega_{SN} = \sqrt{\mathcal{C}/M}$

# Estimates for the gravitational frequency $\omega_{SN}$

Assuming homogeneous mass distribution,

$$\mathcal{C}^{hom} \approx GM\rho_0, \qquad \omega_{SN}^{hom} \approx \sqrt{G\rho_0}.$$

Assuming high mass concentration near lattice site,

$$\omega_{SN}^{\text{crystal}} = \sqrt{\frac{Gm}{12\sqrt{\pi}\Delta x_{\text{zp}}^3}}.$$

Silicon crystal at low temperature ($T \sim 10$ K):

$$\omega_{SN}^{\text{crystal}} \approx 0.036\ \text{s}^{-1} \approx 100 \times \omega_{SN}^{\text{hom}}$$

# Evolution of Gaussian states

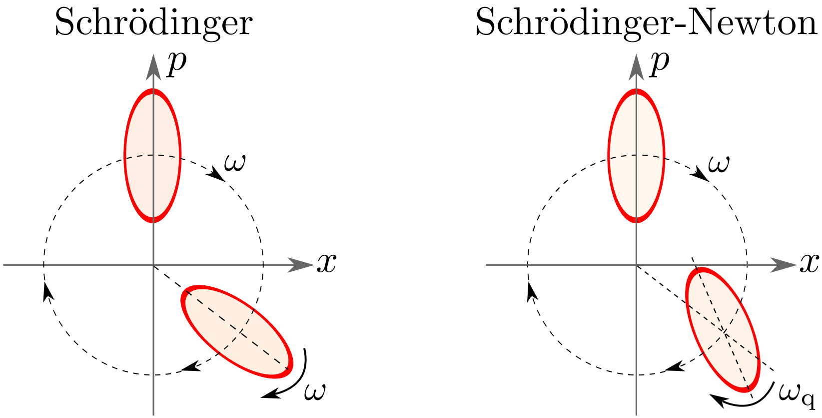
Gaussian states remain Gaussian under the SN equation for the center of mass.

Dynamics of $\langle\hat{x}\rangle$ and $\langle\hat{p}\rangle$ is unchanged,

$$\langle\dot{\hat{x}}\rangle = \langle\hat{p}\rangle/M, \qquad \langle\dot{\hat{p}}\rangle = -M\omega_{c.m.}^2\langle\hat{x}\rangle.$$

Second order moments: $V_{AB} = \langle\hat{A}\hat{B} + \hat{A}\hat{B}\rangle/2 - \langle\hat{A}\rangle\langle\hat{B}\rangle$

$$\dot{V}_{xx} = 2V_{xp}/M\,, \qquad \dot{V}_{pp} = -2M(\omega_{c.m.}^2 + \omega_{SN}^2)V_{xp}\,,$$
$$\dot{V}_{xp} = V_{pp}/M - M(\omega_{c.m.}^2 + \omega_{SN}^2)V_{xx}\,.$$




Uncertainty ellipse rotates at $\omega_q = \sqrt{\omega_{c.m.}^2 + \omega_{SN}^2}$

# Experimental test

- Prepare a mechanical resonator in a squeezed state.
- Let it evolve for a duration $\tau$.
- Carry out state tomography.

$$\Delta\theta = \omega_{c.m.}\tau \left(\frac{\omega_{SN}}{\omega_{c.m.}}\right)^2 \qquad \text{small, but reproducible}$$

Could be detected in an optomechanical setup with:

- backaction (radiation-pressure) $\approx$ thermal noise.
- $Q = \frac{\omega_{c.m}}{\gamma_m} \gtrsim \left(\frac{\omega_{c.m.}}{\omega_{SN}}\right)^2$