Quantum annealing with more than one hundred qubits

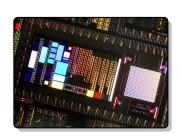
Sergio Boixo, ¹ Troels F. Rønnow, ² Sergei V. Isakov, ² Zhihui Wang, ³ David Wecker, ⁴ Daniel A. Lidar, ⁵ John M. Martinis, ⁶ and Matthias Troyer* ²

arXiv:1304.4595

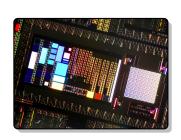
Journal Club
Daniel Becker

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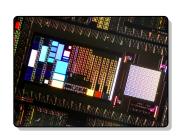
■ **D-Wave One** device successfully used to solve NP hard optimization problems



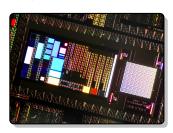
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- quantum annealing with 108 qubits on D-Wave One device performed (adiabatic quantum computation)



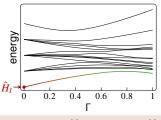
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- **D-Wave One** device successfully used to solve NP hard optimization problems
- quantum annealing with 108 qubits on D-Wave One device performed (adiabatic quantum computation)
- large-scale entanglement of annealed state after times much longer than coherence time of device
- promising prospects to see "quantum speedup" in larger systems (**D-Wave Two** with 512 qubits)



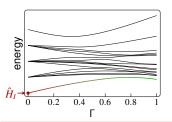
1 encode computational problem into Hamiltonian \hat{H}_P (spin glass, ground state is solution)



$$\hat{H}_P = \sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i < j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

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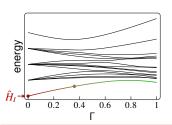
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time sweep with
$$\Gamma := t/T$$

$$\hat{H}(\Gamma) = (1 - \Gamma)\hat{H}_I + \Gamma\hat{H}_P$$



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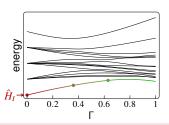
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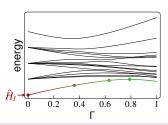
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the smaller gap to excited states the slower the sweep



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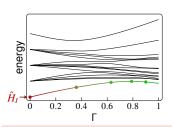
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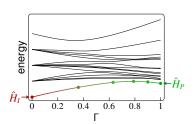
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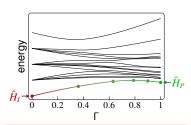
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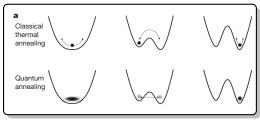
- equivalent to quantum circuit model
- D-Wave: "no errorcorrection needed"

Classical VS Quantum Annealing

general principle

slowly increase ratio of:

- 1 size of energy barrier between system states
- 2 thermal (classical) and quantum fluctuations



classical:

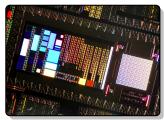
keep Hamiltonian constant and adiabatically cool the system

quantum:

keep temperature constant (at zero) and adiabatically switch Hamiltonian

D-Wave's Quantum Annealing Device

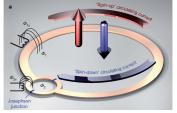
a "special purpose optimization engine"

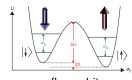


photograph of 512 qubit D-Wave chip

design parameters:

- quadratic lattice of 8 flux qubits per unit cell (temp. $\sim 20mK$)
- flux qubit pairs inductively coupled
- interaction tunable in size and sign
- coherence time: tens of nanoseconds

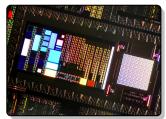




flux qubit

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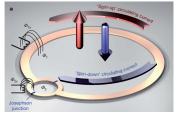
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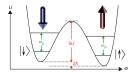


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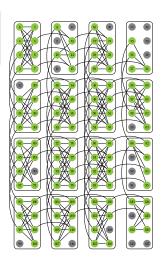




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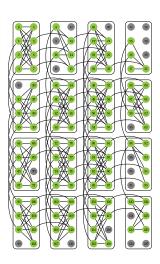
the promise



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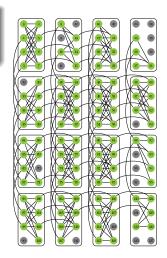
solve (NP hard?) optimization problem encoded into "Chimera" graph

■ flux qubits → vertices of graph



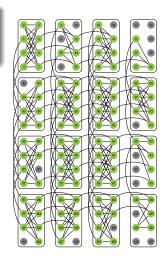
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- lacktriangle programmable qubit couplers \longrightarrow edges with interaction strength J_{ij}



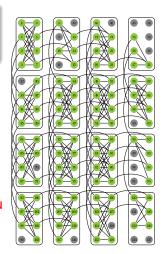
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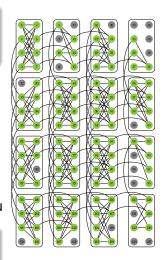
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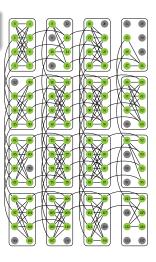
the ultimate goal

solve NP hard problems faster than classical computers (not in polynomial time)



the "D-Wave One" device

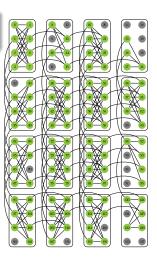
128 flux qubits in a lattice of 4×4 unit cells



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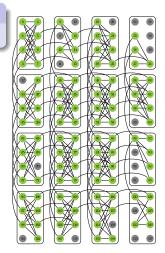
■ 108 of 128 green qubits activated and calibrated to zero on-site energy $h_i = 0$



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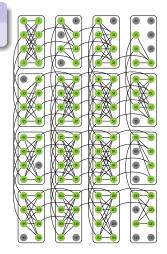
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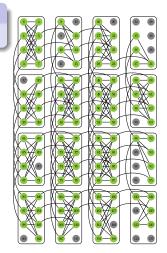
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■ 1000 D-Wave annealing runs $(5 - 20\mu s)$ total annealing time)



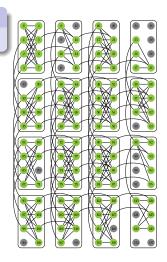
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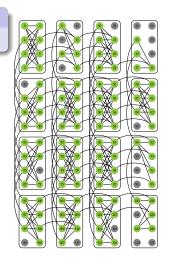
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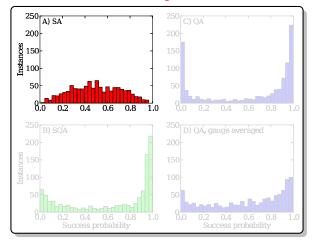
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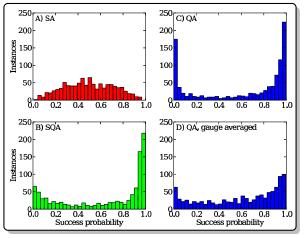
Comparing Success Probability Histograms

classical simulated annealing: monomodal distribution

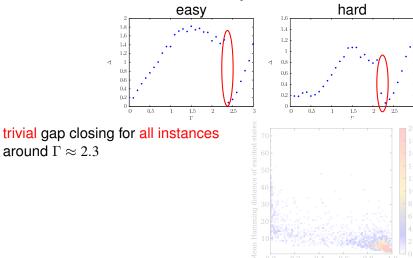


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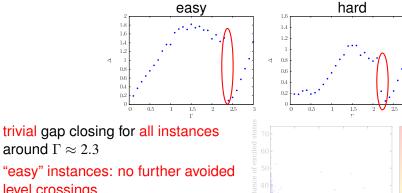
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quantum annealing: bimodal distribution

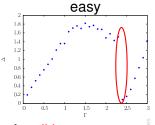


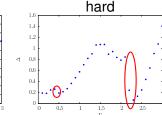
around $\Gamma \approx 2.3$



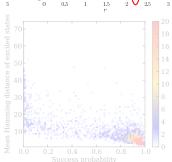
■ "easy" instances: no further avoided level crossings

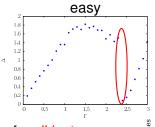
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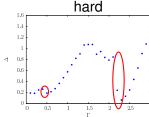




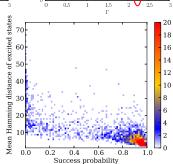
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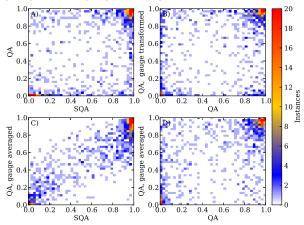


- trivial gap closing for all instances around $\Gamma \approx 2.3$
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- Hamming distance between ground and excited state larger for hard instances

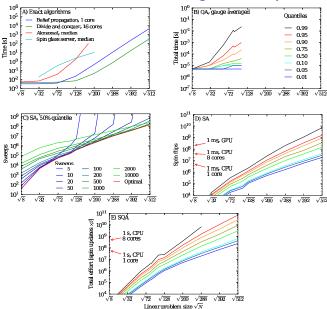


Correlations Between QA and SQA

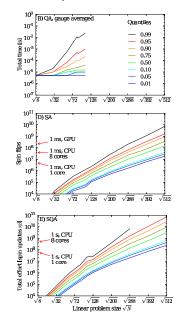
gauge averaging to compensate calibration errors



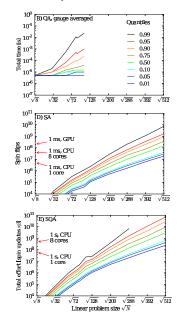
 uncorrelated measurement and simulation results explained by calibration errors



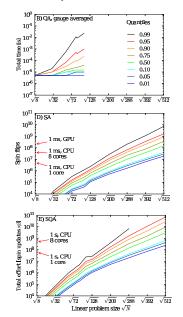
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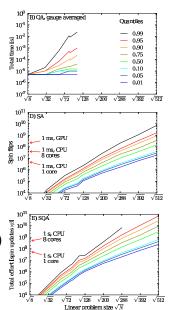
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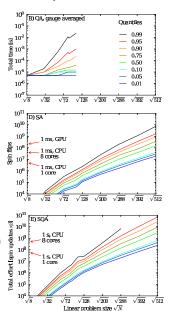
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- future work: quantum speedup of QA for larger systems?



concise summary: http://www.scottaaronson.com/blog/

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- larger systems (more) volatile against: noise, thermal fluctuations, single-qubit decoherence

Daniel Lidar:

John A. Smolin, Graeme Smith, arXiv:1305.4904

classical (thermal) annealing NOT classical analog of quantum annealing

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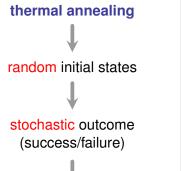
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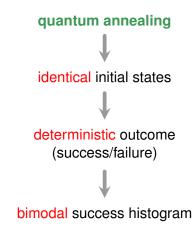
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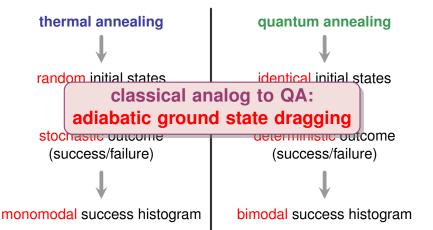


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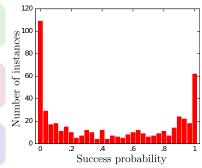
Classical Adiabatic Ground State Dragging

John A. Smolin, Graeme Smith, arXiv:1305.4904

adiabatic switching
$$V(\Gamma) = (1-\Gamma)V_{\mathsf{trans}} + \Gamma V_{\mathsf{lsing}}$$

equations of motion $rac{d}{dt} heta_i=\dot{ heta}_i$ and $rac{d}{dt}\dot{ heta}_i=rac{d}{d heta_i}V(t)$

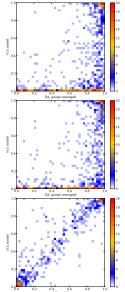
initial state $|\rightarrow\rightarrow\rightarrow\rightarrow\cdots\rightarrow\rangle$



Comment from Wang et al.

Wang et al., arXiv:1305.5837

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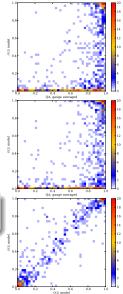


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- weak correlations between classical models and quantum system



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- quantum speedup will require quantum error correction (D. Lidar)