# Spin Backflow and ac Voltage Generation by Spin Pumping and the Inverse Spin Hall Effect

HuJun Jiao[1] and Gerrit E. W. Bauer[2,1]

[1]*Kavli Institute of NanoScience, Delft University of Technology, 2628 CJ Delft, The Netherlands*
[2]*Institute for Materials Research and WPI-AIMR, Tohoku University, Sendai 980-8577, Japan*
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The spin current pumped by a precessing ferromagnet into an adjacent normal metal has a constant polarization component parallel to the precession axis and a rotating one normal to the magnetization. The former is now routinely detected as a dc voltage induced by the inverse spin Hall effect (ISHE). Here we compute ac ISHE voltages much larger than the dc signals for various material combinations and discuss optimal conditions to observe the effect. The backflow of spin is shown to be essential to distill parameters from measured ISHE voltages for both dc and ac configurations.

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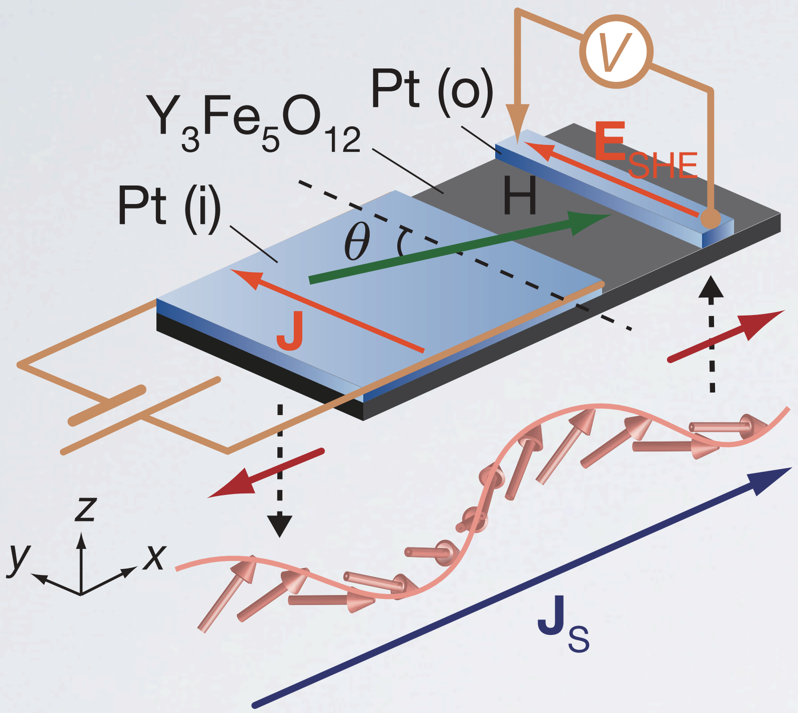
# MOTIVATION: GENERATION & READ-OUT SPIN CURRENTS

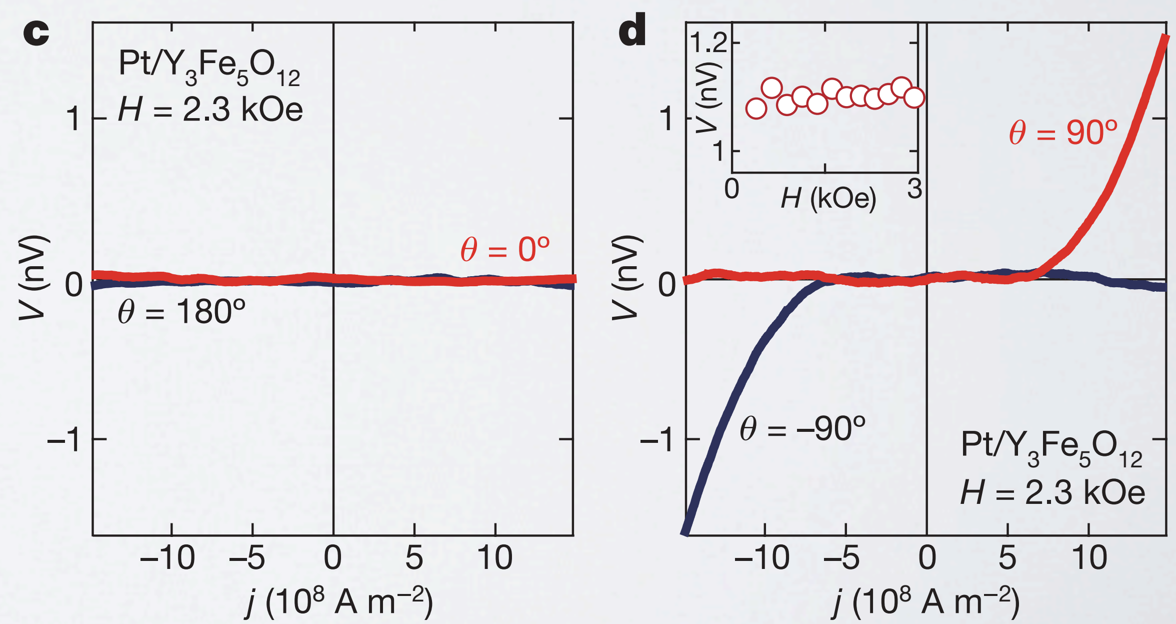



1. J induces a spin current $J_s$ in the Pt layer, gets transferred into insulator by STT

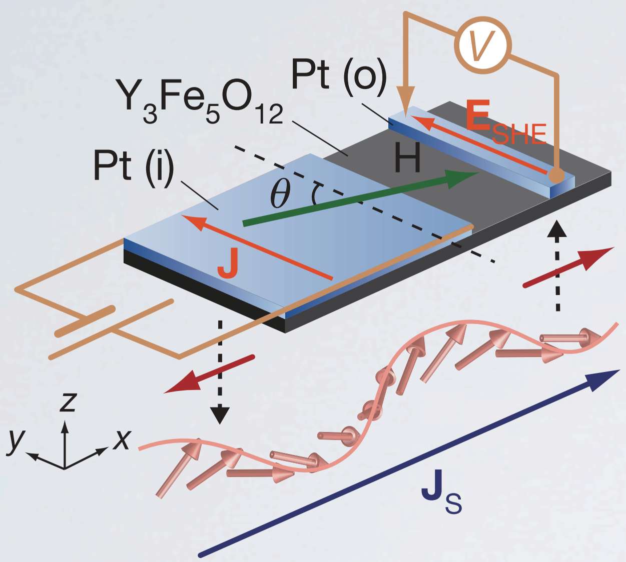
2. Spin current propagates over ~mm distance

3. Transferred into conductor by spin pumping, measured by ISHE

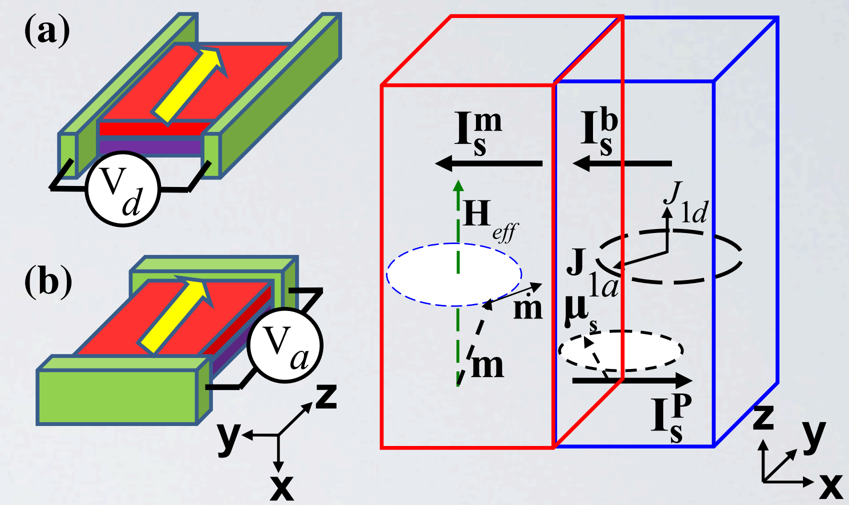
Depending on the angle between $J_S$ and **H**, the electric signal is transmitted through the insulator, carried by spin waves



# SETUP: MEASURING SPIN CURRENTS




Focus on measuring part of the setup




**Important ingredients:**

1. Spin pumping (transfer spin precession into spin accumulation/ current in metal)
2. Backflow of spins into magnet due to spin accumulation
3. Conversion spin current into voltage by inverse spin Hall effect
4. Focus on AC component of voltage

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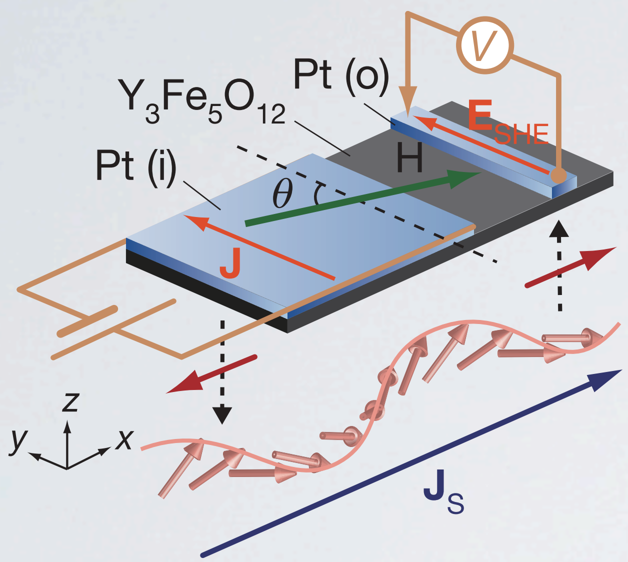


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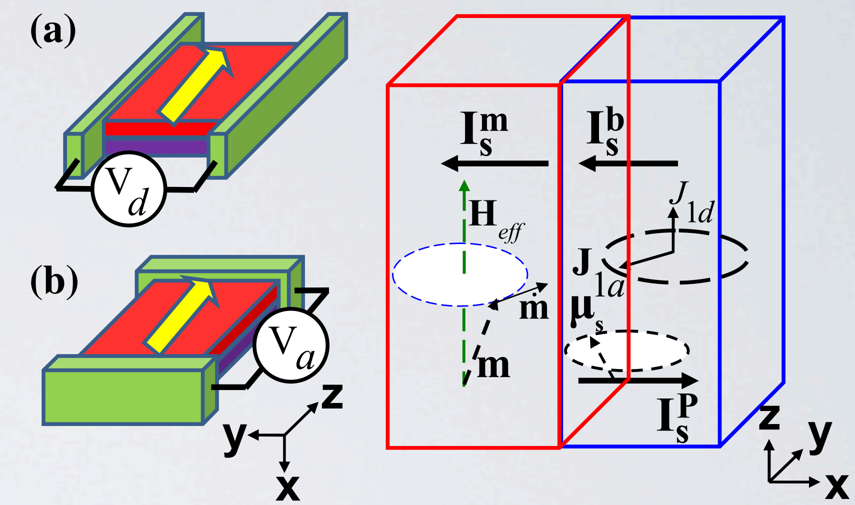


**Important ingredients:**

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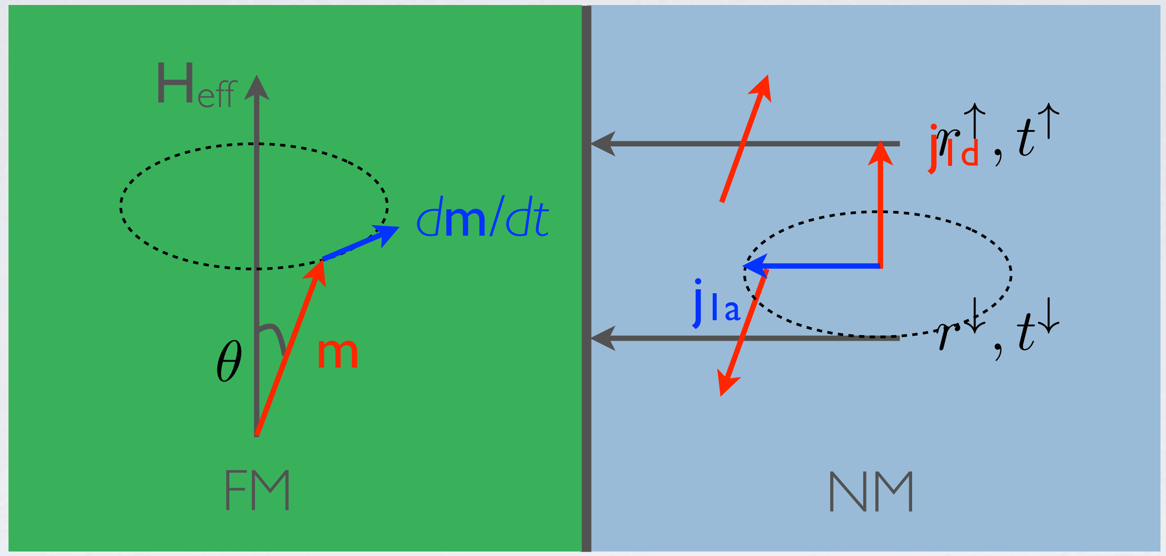
# SPIN PUMPING

Spin pumping: transfer magnetization from FM to (paramagnetic) NM



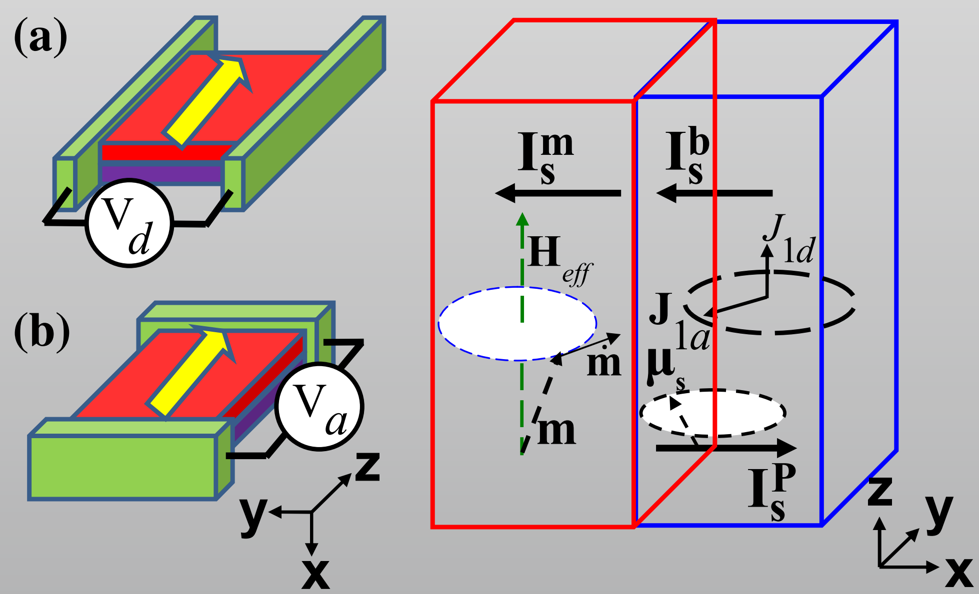

Magnetization dynamics:

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Generated spin current:

$$\mathbf{I}_s^p = \frac{\hbar}{4\pi}\left(\text{Re}g^{\uparrow\downarrow}\mathbf{m} \times \frac{d\mathbf{m}}{dt} + \text{Im}g^{\uparrow\downarrow}\frac{d\mathbf{m}}{dt}\right)$$

# SPIN BACKFLOW




Generated spin current:

$$\mathbf{I}_s^p = \frac{\hbar}{4\pi}\left(\mathrm{Re}g^{\uparrow\downarrow}\mathbf{m}\times\frac{d\mathbf{m}}{dt} + \mathrm{Im}g^{\uparrow\downarrow}\frac{d\mathbf{m}}{dt}\right)$$

Spin pumping leads to spin accumulation $\boldsymbol{\mu}_s^N$ in NM

$$\frac{\partial \boldsymbol{\mu}_s^N(\mathbf{r},t)}{\partial t} = \gamma_N \mathbf{H}_{\mathrm{ex}} \times \boldsymbol{\mu}_s^N + D_N \frac{\partial^2 \boldsymbol{\mu}_s^N}{\partial x^2} - \frac{\boldsymbol{\mu}_s^N}{\tau_{\mathrm{sf}}^N}$$

This accumulation can lead to backflow of spins

$$\mathbf{I}_s^b = \frac{g}{8\pi}[2p(\mu_0^F - \mu_0^N) + \mu_s^F - \mathbf{m}\cdot\boldsymbol{\mu}_s^N]\mathbf{m} - \frac{\mathrm{Re}g^{\uparrow\downarrow}}{4\pi}\mathbf{m}\times(\boldsymbol{\mu}_s^N\times\mathbf{m}) + \frac{\mathrm{Im}g^{\uparrow\downarrow}}{4\pi}\mathbf{m}\times\boldsymbol{\mu}_s^N,$$

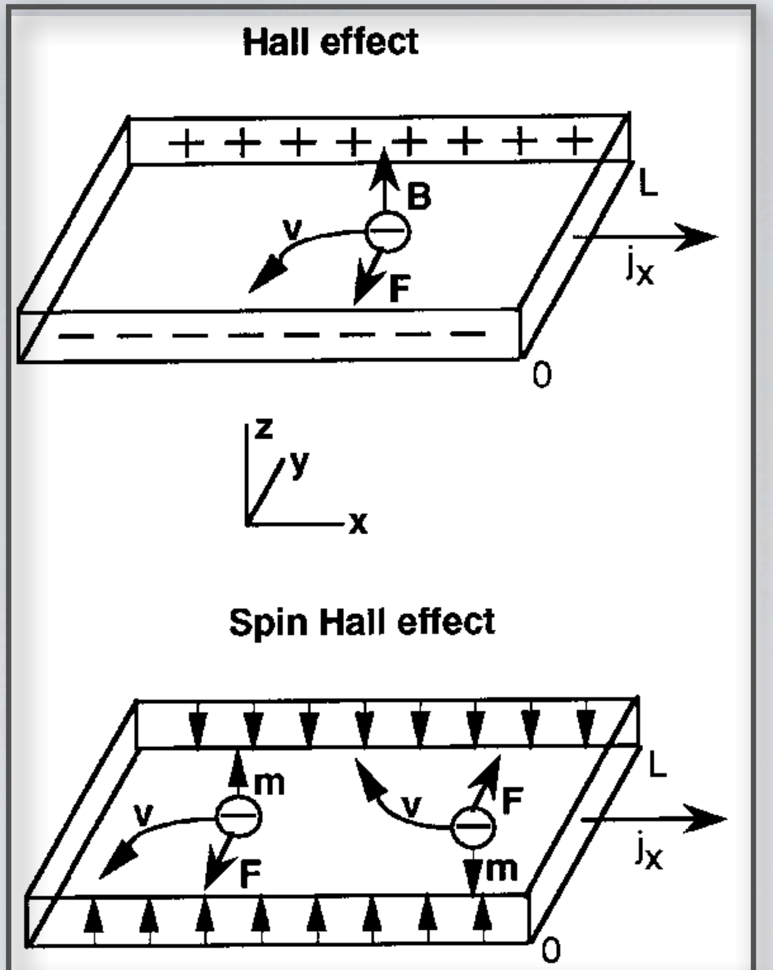
$\boldsymbol{\mu}_s^F$ is the spin accumulation in the FM

$$\frac{\partial^2 \mu_s^F(x)}{\partial x^2} = \frac{\mu_s^F(x)}{(\lambda_{\mathrm{sd}}^F)^2}$$

Can solve these equations self-consistently, using boundary conditions:

1. Spin current is continuous at interface
2. Spin current $\mathbf{I}_s^b + \mathbf{I}_s^P$ vanishes at boundary of material

# SPIN HALL EFFECT




Pic: J. E. Hirsch, PRL **83**, 1834 (1999)

**Hall effect** of spinless particle in constant magnetic field **B** = (0, 0, B):

$$H_{\mathrm{EM}} = -\mu_B \mathbf{B} \cdot \mathbf{L}$$

Gives rise to charge accumulation (voltage/current) in a finite-sized sample

**spin Hall effect** of spinfull particle with spin-orbit interaction:

$$H_{\mathrm{SO}} = V_{\mathrm{SO}}(r)\sigma \cdot \mathbf{L}$$

Gives rise to spin accumulation (spin voltage/spin current) in a finite-sized sample

**Resulting electric field** inverse spin hall effect:

$$\mathbf{E} \parallel \mathbf{n} \times \mathbf{j}_S$$

**spin Hall effect:** Creation of spin current by applying voltage difference

**inverse spin Hall effect:** Creation of voltage difference due to spin current



M. I. D'yakonov and V. I. Perel, JETP Lett. **13**, 467 (1971) and Y. K. Kato *et al.*, Science **306**, 1910 (2004)

# THE EXPRESSIONS (I)

Have seen the equations that determine spin accumulation and currents. Resulting expressions:

## In the metal

$$\boldsymbol{\mu}_s^N(x,\omega) = \sum_{i=1}^{3} \vec{v}_i \frac{\cosh[\kappa_i(x-d_N)]}{\sinh[\kappa_i d_N]} \frac{2j_{is}(x=0,\omega)}{\hbar\nu D_N \kappa_i}. \quad (4)$$

$\kappa_1^2(\omega) = (1+i\omega\tau_{\rm sf}^N)/(\lambda_{\rm sd}^N)^2$, $\kappa_{2,3}^2(\omega) = \kappa_1^2(\omega) \pm iC$, $C = -\gamma_N H_{\rm ex}/D_N$, and $\lambda_{\rm sd}^N = \sqrt{D_N \tau_{\rm sf}^N}$. $j_{1s} = I_s^z/A$ and $j_{(2,3)s} = (I_s^x \pm iI_s^y)/(\sqrt{2}A)$ are spin current densities, where $\nu$ is the one-spin density of state and $A$ is the interface area. The eigenvectors associated with $\kappa_i^2(\omega)$ $(i=1,2,3)$ are, respectively, $\vec{v}_1 = (0\,0\,1)$, $\vec{v}_2 = (1\ -i\ 0)/\sqrt{2}$, $\vec{v}_3 = (1\,i\,0)/\sqrt{2}$. In the position-time domain

$$\mathbf{j}_{1s}(x,t) = -\frac{\hbar\nu D_N}{2}\frac{\partial \boldsymbol{\mu}_s^N(x,t)}{\partial x} = j_{1s}^z(x)\mathbf{e}_z + \mathbf{j}_{1s}^a(x,t), \quad (5)$$

with

$$j_{1s}^z(x)\mathbf{e}_z = \frac{\sinh[\kappa_1(0)(d_N-x)]}{\sinh[\kappa_1(0)d_N]} j_{1s}^z(0)\mathbf{e}_z, \quad (6)$$

$$\mathbf{j}_{1s}^a(x) = 2\,\mathrm{Re}\left\{\frac{\sinh[\kappa_2(\omega)(d_N-x)]}{\sinh[\kappa_2(\omega)d_N]}\mathbf{j}_{1s}^a(0)e^{i\omega t}\right\}. \quad (7)$$

The analytic expressions for $j_{1s}^z(0)$ and $\mathbf{j}_{1s}^a(0)$, the dc and ac components of the spin current at the $N$ side of the interface, respectively, are given in the Supplemental Material [29].

AC contribution

## In the ferromagnet

$$\mu_s^F(x) = \frac{\cosh[(d_F+x)/\lambda_{\rm sd}^F]\tilde{g}}{[g_F\tanh[d_F/\lambda_{\rm sd}^F]+\tilde{g}]\cosh(d_F/\lambda_{\rm sd}^F)}\mathbf{m}\cdot\boldsymbol{\mu}_s^N, \quad (9)$$

where $g_F = 4hA\sigma_\uparrow\sigma_\downarrow/[e^2\lambda_{\rm sd}^F(\sigma_\uparrow+\sigma_\downarrow)]$ and $\tilde{g} = (1-p^2)g$. Here, $\sigma_{\uparrow(\downarrow)}$ is the conductivity of spin-up (spin-down) electrons in $F$. The spin current density in $F$ reads

$$\mathbf{j}_{2s}(x) = \frac{\sinh[(d_F+x)/\lambda_{\rm sd}^F]}{\sinh(d_F/\lambda_{\rm sd}^F)}\mathbf{j}_{2s}(0), \quad (10)$$

with

$$\mathbf{j}_{2s}(0) = -\frac{1}{8\pi}\frac{\tilde{g}g_F\tanh[d_F/\lambda_{\rm sd}^F]}{\tilde{g}+g_F\tanh[d_F/\lambda_{\rm sd}^F]}(\mathbf{m}\cdot\boldsymbol{\mu}_s^N)\mathbf{m} \quad (11)$$

# THE EXPRESSIONS (2)

### Electric fields due to ISHE

The ISHE generates a charge current $\mathbf{j}_c$ transverse to an applied spin current due to the spin-orbit interaction. With the spin current direction along $\mathbf{e}_x$ [8,16,18–21],

$$\mathbf{j}_c(x) = \alpha_{N/F}(2e/\hbar)\mathbf{e}_x \times \mathbf{j}_s(x), \quad (12)$$

where $\alpha_N$ is the spin Hall angle in $N$ and $\alpha_F = (\alpha_{F\uparrow} + \alpha_{F\downarrow})/2$ is that in $F$, where $\alpha_{F\xi} = \sigma_{\mathrm{AH}\xi}/\sigma_\xi$ ($\xi = \uparrow, \downarrow$) and $\sigma_{(\mathrm{AH})\xi}$ is the spin-polarized (anomalous Hall) conductivity. As shown in Fig. 1(a), a dc electric field $E_y\mathbf{e}_y$ is generated along the $y$ direction; similarly, an ac field $E_z(t)\mathbf{e}_z$ along the $z$ direction is shown in Fig. 1(b). Disregarding parasitic impedances and in the steady state, we obtain for the ac contribution along $z$

$$E_z(t) = \frac{4e/\hbar}{\sigma_N d_N + \sigma_F d_F}\,\mathrm{Re}\Bigg(\frac{\alpha_N j^y_{1s}(0)}{\kappa_2(\omega)}\tanh\frac{d_N\kappa_2(\omega)}{2} + \alpha_F j^y_{2s}(0)\lambda^F_{\mathrm{sd}}\tanh\frac{d_N}{2\lambda^N_{\mathrm{sd}}}\Bigg), \quad (13)$$

while the dc electric field along $y$ reads

$$E_y = \frac{2e/\hbar}{\sigma_N d_N + \sigma_F d_F}\Bigg[j^z_{1s}(0)\alpha_N\lambda^N_{\mathrm{sd}}\tanh\frac{d_N}{2\lambda^N_{\mathrm{sd}}} + j^z_{2s}(0)\alpha_F\lambda^F_{\mathrm{sd}}\tanh\frac{d_F}{2\lambda^F_{\mathrm{sd}}}\Bigg]. \quad (14)$$

AC contribution

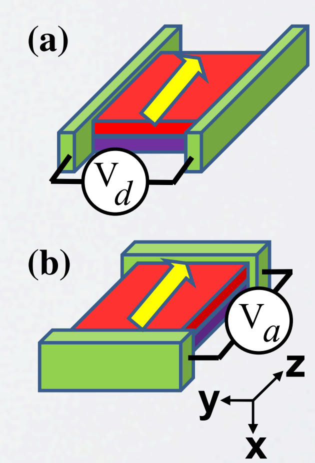
### Simplified expressions when ignoring backflow

$$E_y^{NB} = \frac{e\alpha_N f\sin^2\theta}{\sigma_N d_N + \sigma_F d_F}\frac{\mathrm{Re}\,g^{\uparrow\downarrow}}{A}\lambda^N_{\mathrm{sd}}\tanh\frac{d_N}{2\lambda^N_{\mathrm{sd}}}.$$

$$\frac{E_z^{NB}(t)}{\cos(\omega t + \delta)} = \frac{e\alpha_N f\sin\theta\cos\theta}{\sigma_N d_N + \sigma_F d_F}\frac{\mathrm{Re}\,g^{\uparrow\downarrow}}{A}\left|\frac{\tanh[\kappa_2(\omega)d_N/2]}{\kappa_2(\omega)}\right|$$

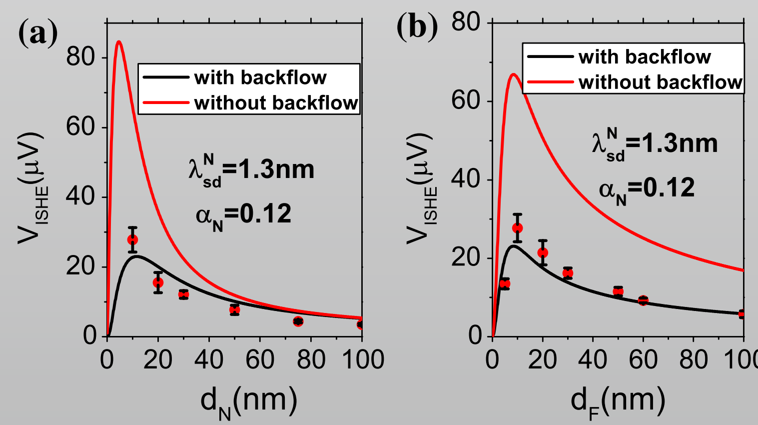
### Main differences

1. AC signal linear in $\theta$, DC quadratic
2. AC signal depends on spin dephasing length $\lambda_c^2 = D_N/\omega$, DC signal on spin-flip relaxation length $(\lambda_{sd}^N)^2 = D_N\tau_{sf}^N$



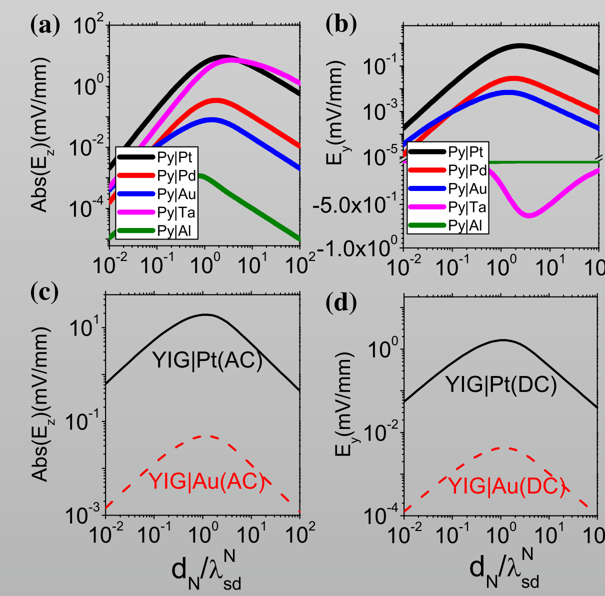
# CONCLUSIONS

## Backflow




1. Backflow strongly reduces signal for thin samples
2. Importance backflow reduces with increasing thickness
3. Total signal increases rapidly at small $d_N/\lambda_{sd}^N$, decreases at large $d_N/\lambda_{sd}^N$

## AC signal




1. AC signal order(s) of magnitude larger than DC
2. Dependance signal of thickness