

Two-dimensional **p-wave** superconducting states
with **magnetic moments**
on a conventional **s-wave** superconductor

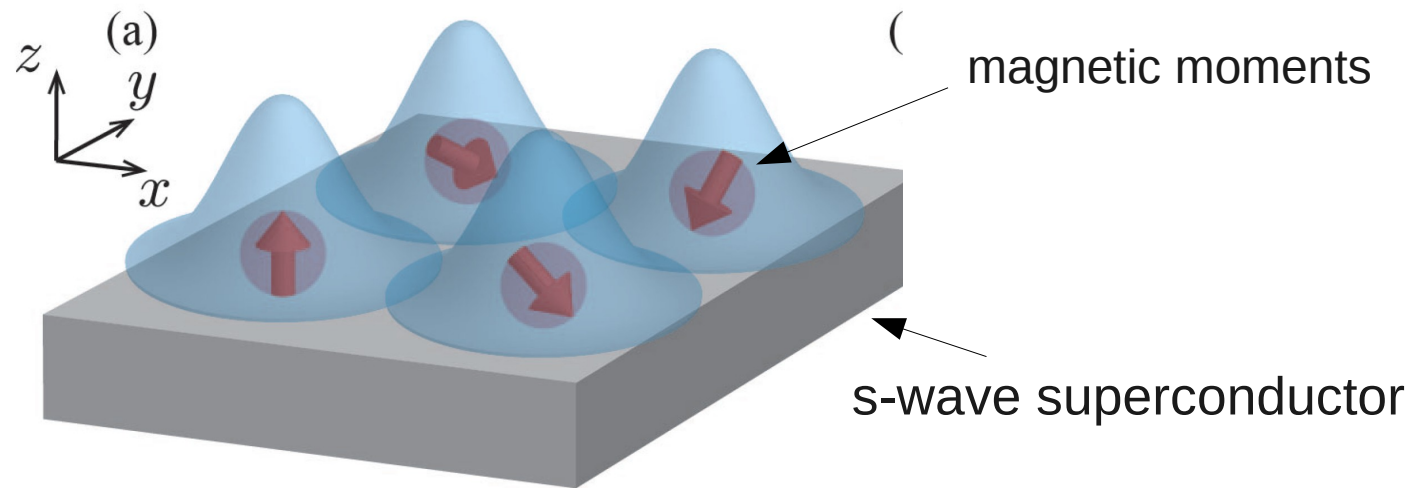
Sho Nakosai, Yukio Tanaka, and Naoto Nagaosa

arXiv:1306.3686

Journal club – Jelena Klinovaja
Basel – 25.06.13

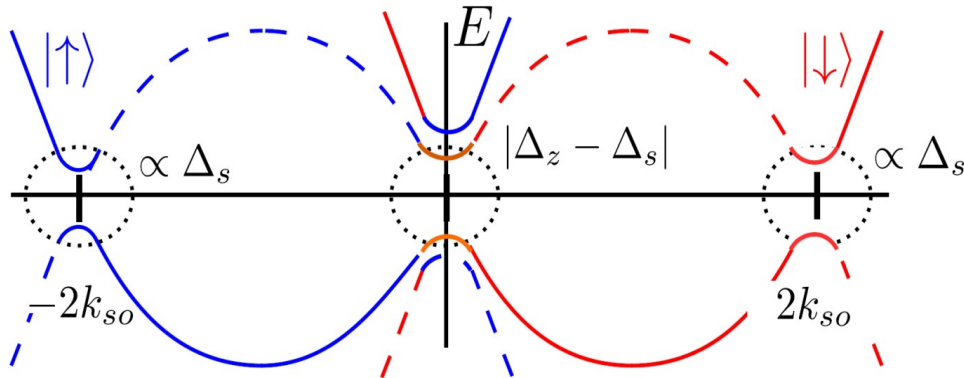
Outline

How to induce p-wave superconductivity?

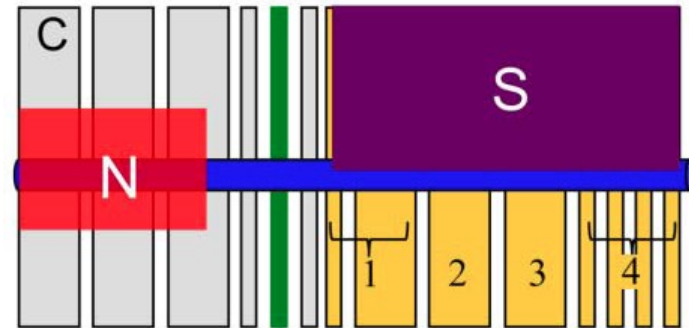
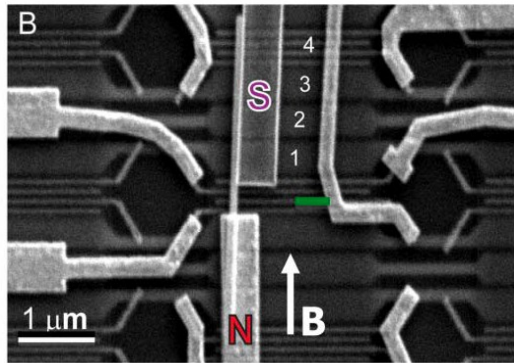


Possible motivation – 1D wires

spin-orbit interaction + s-wave \Rightarrow p-wave \Rightarrow Majorana



Klinovaja and Loss, PRB 86, 085408 (2012)



Mourik et al., Science 336, 1003 (2012)

Sato and Fujimoto, PRB 79, 094504 (2009)

Lutchyn et al., PRL 105, 077001 (2010)

Oreg et al., PRL 105, 177002 (2010)

Alicea, PRB 81, 125318 (2010)

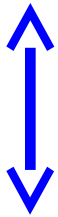
Possible motivation – 1D wires

The SOI interaction can be gauged 'away'!

Braunecker, Japaridze, Klinovaja, and DL, PRB 82, 045127 (2010)

Gauge trafo

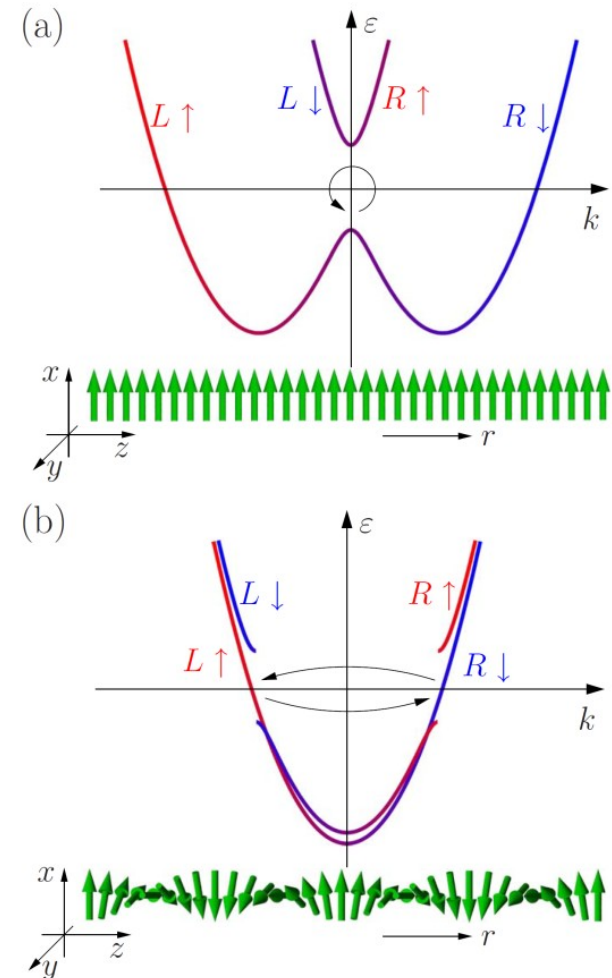
$$\Psi_\sigma = e^{-i\sigma k_{so}x} \tilde{\Psi}_\sigma$$



$B \neq 0$

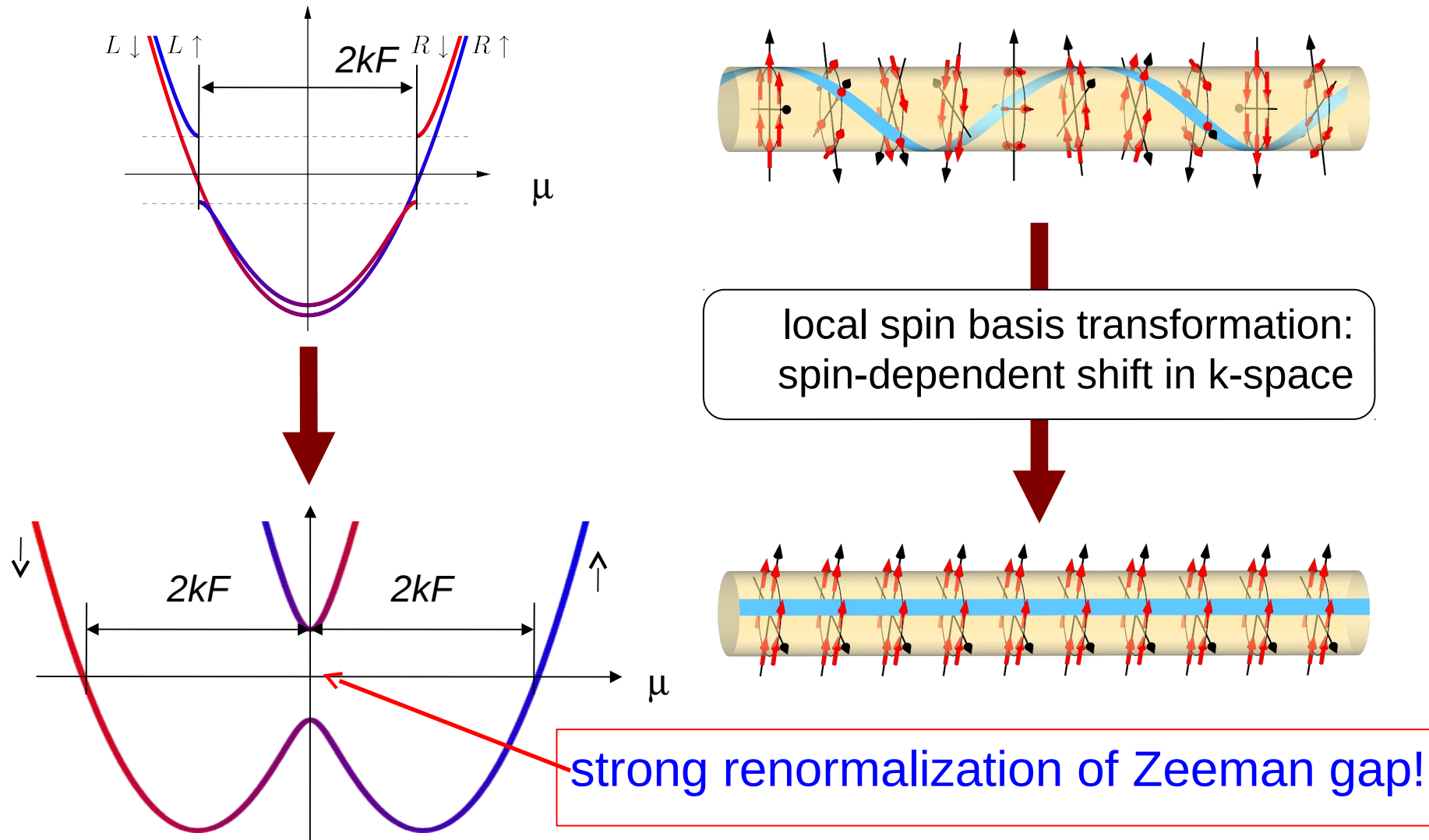
Only rotating field
and **no** SOI anymore!

$$\tilde{\mathbf{B}} = B(\cos(2k_{so}x)\mathbf{e}_x - \sin(2k_{so}x)\mathbf{e}_y)$$



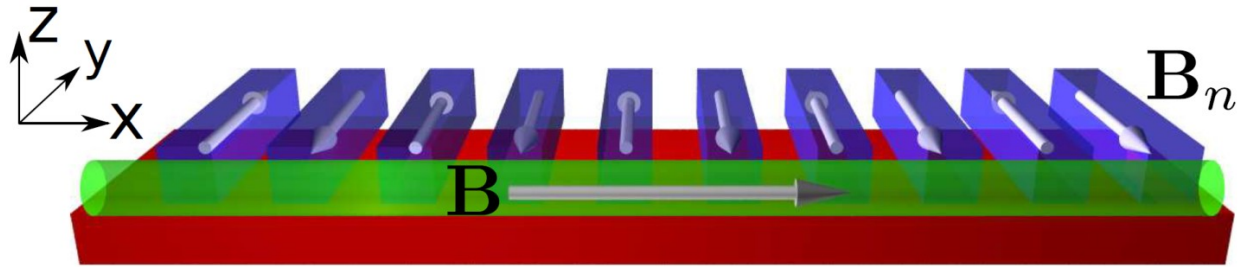
Rotating Field - Nuclear Helimagnet in 1D

Braunecker, Simon, and Loss, PRL 102, 116403 (2009) & PRB 80, 165119 (2009)



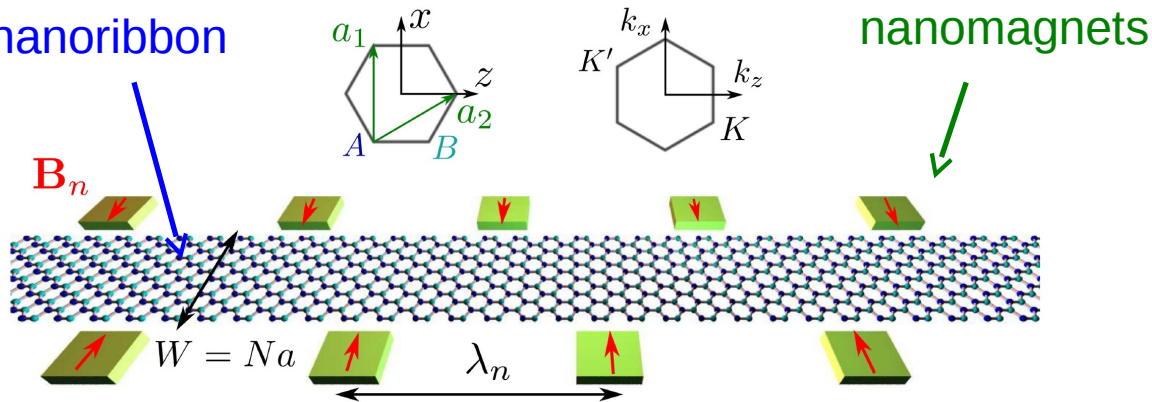
Confirmed by DMRG, see Stoudenmire, Alicea, Strykh, Fisher, PRB 84, 014503 (2011)

Rotating Field - Nanomagnets



Klinovaja, Stano, and Loss, PRL 109, 236801 (2012)

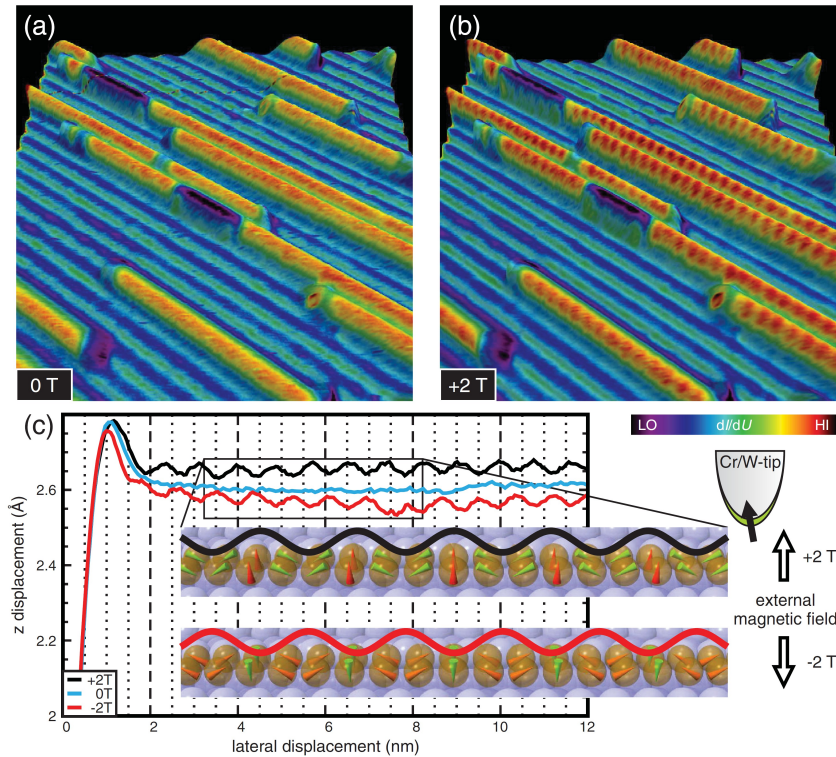
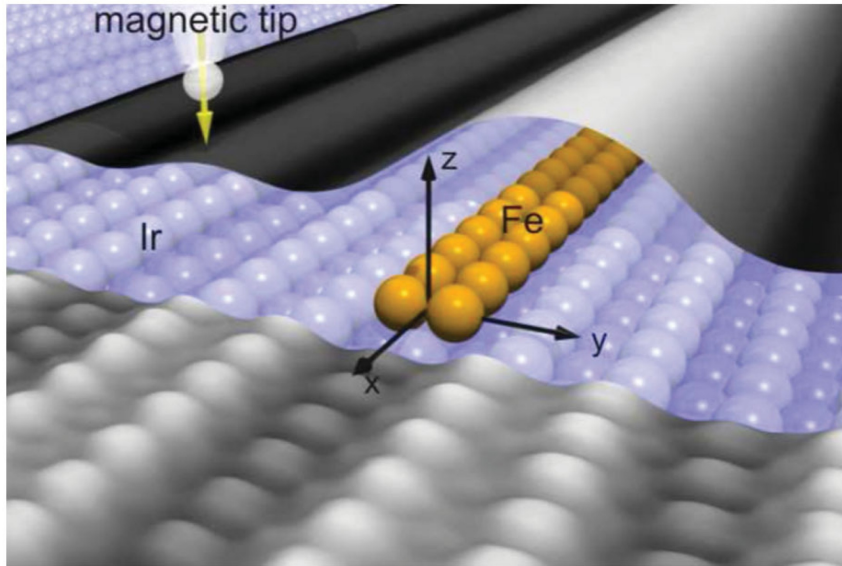
armchair
nanoribbon



Klinovaja and Loss, PRX 3, 011008 (2013)

$$\Delta_{\text{so}} = 10 \text{ meV for } \lambda = 200 \text{ nm}$$

Rotating Field – 2D - magnetic atoms



2D Lattice – Bound states (Shiba states) around magnetic atoms

$$H = \frac{1}{2} \sum_{\alpha\beta} \int d\mathbf{r} \left[\psi_{\alpha}^{\dagger}(\mathbf{r}) h_{\alpha\beta}(\mathbf{r}) \psi_{\beta}(\mathbf{r}) - \psi_{\alpha}(\mathbf{r}) h_{\alpha\beta}^T(\mathbf{r}) \psi_{\beta}^{\dagger}(\mathbf{r}) \right. \\ \left. + \psi_{\alpha}^{\dagger}(\mathbf{r}) \Delta_{\alpha\beta}(\mathbf{r}) \psi_{\beta}^{\dagger}(\mathbf{r}) + \psi_{\alpha}(\mathbf{r}) \Delta_{\alpha\beta}^{\dagger}(\mathbf{r}) \psi_{\beta}(\mathbf{r}) \right],$$

$$h_{\alpha\beta}(\mathbf{r}) = \xi(\mathbf{r}) \delta_{\alpha\beta} - J \mathbf{S}(\mathbf{r}) \cdot \boldsymbol{\sigma}_{\alpha\beta}$$

Kinetic energy: $\xi(\mathbf{r}) = \left[-\frac{1}{2m} \frac{\partial^2}{\partial \mathbf{r}^2} - \mu \right]$

Coupling to magnetic moments: $\mathbf{S}(\mathbf{r}) = \sum_i \mathbf{S}_i \delta(\mathbf{r} - \mathbf{r}_i)$

s-wave superconductivity: $\Delta_{\alpha\beta}(\mathbf{r}) = \Delta_0 i \sigma_{\alpha\beta}^y$

- H. Shiba, Prog. Theor. Phys. 40, 435 (1968)
A. I. Rusinov, Sov. Phys. JETP 29, 1101 (1969)
L. Yu, Acta Phys. Sin. 21, 75 (1985)

Shiba state

Single moment: $+s_z$ -direction

Bogoliubov-de Gennes (BdG)
equation

$$(\xi_k - E)u_{k\uparrow} - \frac{JS}{V} \sum_l u_{l\uparrow} + \Delta_0 v_{k\downarrow} = 0$$

$$(\xi_k + E)v_{k\downarrow} + \frac{JS}{V} \sum_l v_{l\downarrow} - \Delta_0 u_{k\uparrow} = 0$$

Solution:

$$E_0 = \Delta_0 \left[1 - (\pi JS N_0 / 2)^2 \right] / \left[1 + (\pi JS N_0 / 2)^2 \right]$$

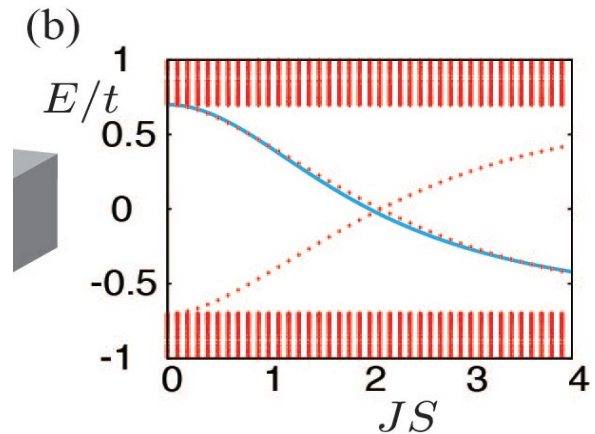
N_0 - the density of states in the normal phase

$$u_{\uparrow}(\mathbf{r}) \sim \frac{\sin(p_F r - \delta_+)}{p_F r} \exp \left[-\frac{r}{\xi_0} |\sin(\delta_+ - \delta_-)| \right]$$

$$v_{\downarrow}(\mathbf{r}) \sim \frac{\sin(p_F r - \delta_-)}{p_F r} \exp \left[-\frac{r}{\xi_0} |\sin(\delta_+ - \delta_-)| \right]$$

$$\tan \delta_{\pm} = \pm \pi JS N_0 / 2$$

$$\xi = v_F / (\pi \Delta)$$



H. Shiba, Prog. Theor. Phys. 40, 435 (1968)
A. I. Rusinov, Sov. Phys. JETP 29, 1101 (1969)
L. Yu, Acta Phys. Sin. 21, 75 (1985)

2D Lattice of Shiba states

Magnetic momentum: $S(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$

electron operators:

$$\psi_{\uparrow} = \sum_i \left[\cos \frac{\theta_i}{2} u_{\uparrow}(\mathbf{r} - \mathbf{r}_i) \alpha_i - e^{-i\phi_i} \sin \frac{\theta_i}{2} v_{\downarrow}^*(\mathbf{r} - \mathbf{r}_i) \alpha_i^{\dagger} \right]$$

$$\psi_{\downarrow} = \sum_i \left[e^{i\phi_i} \sin \frac{\theta_i}{2} u_{\uparrow}(\mathbf{r} - \mathbf{r}_i) \alpha_i + \cos \frac{\theta_i}{2} v_{\downarrow}^*(\mathbf{r} - \mathbf{r}_i) \alpha_i^{\dagger} \right],$$

$$H_{\text{eff}} = \sum_i E_0 \alpha_i^{\dagger} \alpha_i + \sum_{\langle ij \rangle} \left[\bar{t}_{ij} \alpha_i^{\dagger} \alpha_j + \left(\bar{\Delta}_{ij} \alpha_i^{\dagger} \alpha_j^{\dagger} + \text{H.c.} \right) \right]$$

$$\bar{t}_{ij} = \hat{z}_i^{\dagger} \hat{z}_j \bar{t}_0,$$

$$\bar{\Delta}_{ij} = \hat{z}_i^{\dagger} i \sigma_y \hat{z}_j^* \bar{\Delta}_0$$

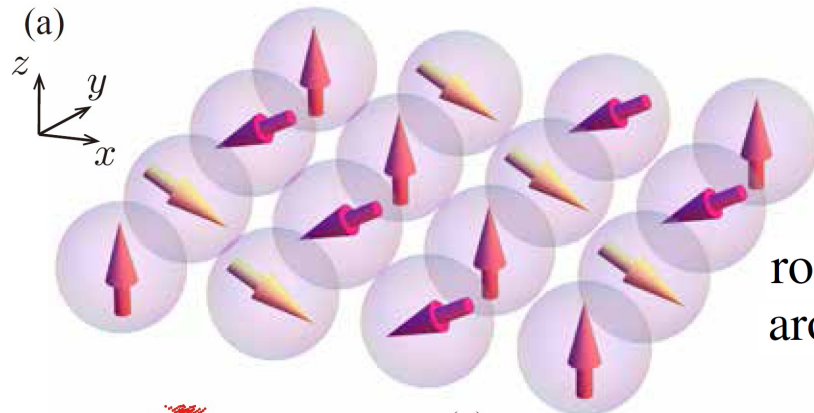
} depend only on the relative direction of the two moments

$$\hat{z}_i = \begin{pmatrix} \cos \frac{\theta_i}{2} \\ e^{i\phi_i} \sin \frac{\theta_i}{2} \end{pmatrix}$$

$$\bar{t}_0 = \int d\mathbf{r} \left[(u_i \xi(\mathbf{r}) u_j - v_i \xi(\mathbf{r}) v_j) + \Delta_0 (u_i u_j + v_i v_j) \right]$$

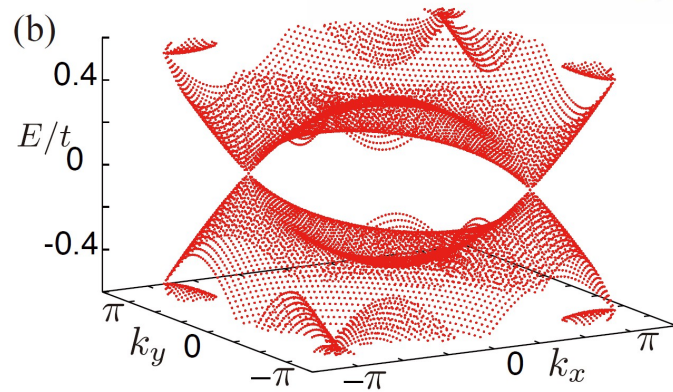
$$\bar{\Delta}_0 = \int d\mathbf{r} \left[(u_i \xi(\mathbf{r}) v_j + v_i \xi(\mathbf{r}) u_j) + \Delta_0 (u_i u_j - v_i v_j) \right]$$

Nodal superconductor: $p_x + p_y$ -wave pairing state.

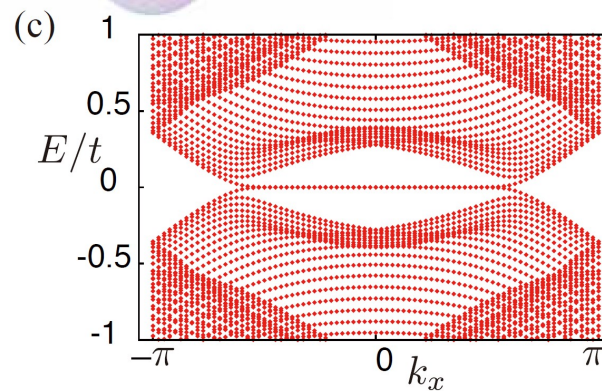


all the moments lie in the $s_x s_z$ -plane

rotate by $2\pi/3$
around s_y -axis along both x - and y -directions

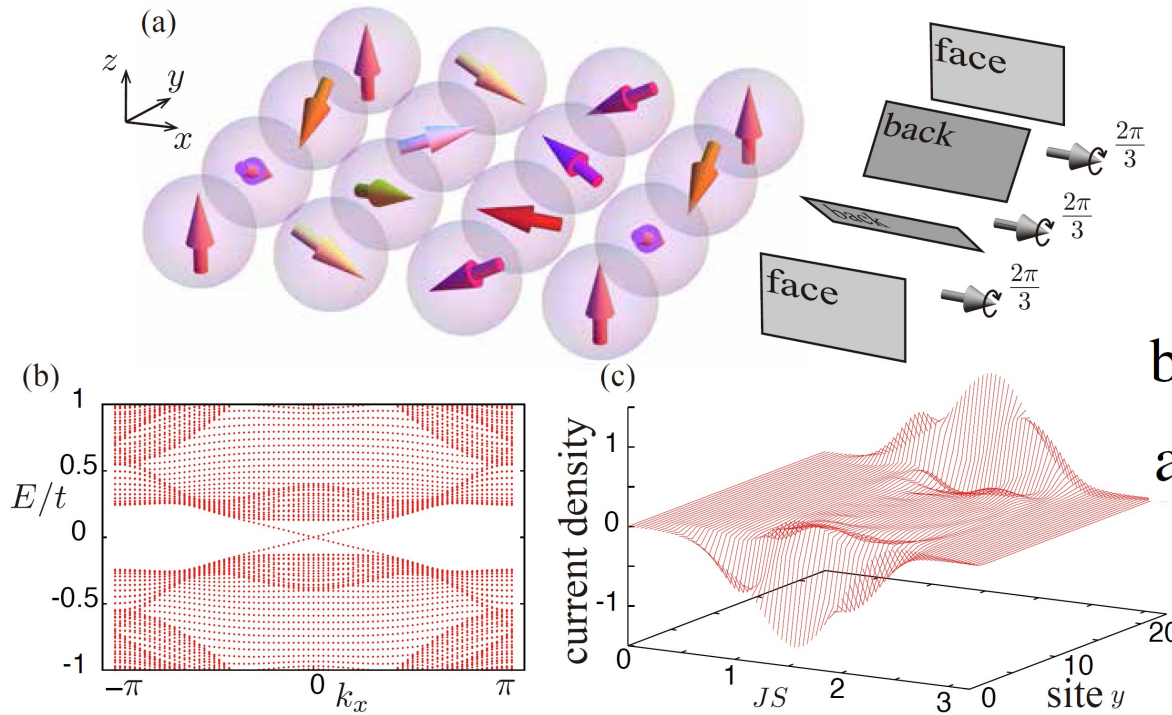


Nodes on the line $k_x = -k_y$



mostly flat Andreev bound states

Chiral p-wave superconductor: $p_x + ip_y$ -wave pairing



by $2\pi/3$ within

$s_x s_z$ -plane along x -direction

by $2\pi/3$

around s_x -axis along y -direction

two chiral Majorana edge channels localized at the open boundaries in the energy gap

current along x -direction:

$$j_i = \sum_{k_x \sigma} 2t \sin k_x \alpha_{i\sigma}^\dagger(k_x) \alpha_{i\sigma}(k_x)$$

topological criterion:

trivial phase: $JS > (JS)_c$

$$JS = (JS)_c \sim 2.7t$$

Results:

“We have proposed a new way of creating effective two-dimensional **unconventional superconductivity by local moments** on the conventional spin-singlet s-wave superconductor.”

