

## Stabilization of the Quantum Spin Hall Effect by Designed Removal of Time-Reversal Symmetry of Edge States

Huichao Li,<sup>1</sup> L. Sheng,<sup>1,\*</sup> R. Shen,<sup>1</sup> L. B. Shao,<sup>1</sup> Baigeng Wang,<sup>1</sup> D. N. Sheng,<sup>2</sup> and D. Y. Xing<sup>1,3,†</sup>

<sup>1</sup>*Department of Physics and National Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210093, China*

<sup>2</sup>*Department of Physics and Astronomy, California State University, Northridge, California 91330, USA*

<sup>3</sup>*National Center of Microstructures and Quantum Manipulation, Nanjing University, Nanjing 210093, China*

(Received 28 January 2013; published 24 June 2013)

The quantum spin Hall (QSH) effect is known to be unstable to perturbations violating time-reversal symmetry. We show that creating a narrow ferromagnetic region near the edge of a QSH sample can push one of the counterpropagating edge states to the inner boundary of the ferromagnetic region and leave the other at the outer boundary, without changing their spin polarizations and propagation directions. Since the two edge states are spatially separated into different “lanes,” the QSH effect becomes robust against symmetry-breaking perturbations.

DOI: [10.1103/PhysRevLett.110.266802](https://doi.org/10.1103/PhysRevLett.110.266802)

PACS numbers: 73.50.-h, 72.25.-b, 73.20.At, 73.43.-f

Based on:

Yang *et al.*, PRL **107**, 066602 (2011)

Liu *et al.*, PRL **101**, 146802 (2008)

Franziska Maier

July 09, 2013

# Quantum Spin Hall Effect (QSH)

Theory:

Graphene

Kane and Mele, PRL **95**, 226801 (2005)

GaAs with strain gradient

Bernevig and Zhang, PRL **96**, 106802 (2006)

Experiments:

HgTe quantum wells thicker than  $d_c$  (band inversion)

Bernevig *et al.*, Science **314**, 1757 (2006)

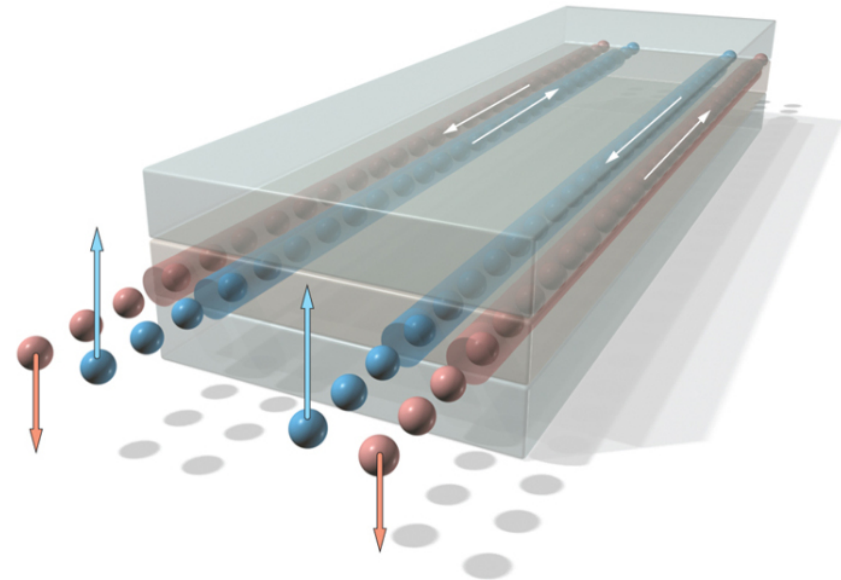
König *et al.*, Science **318**, 766 (2007)

Key ingredient:

Spin orbit interaction (SOI)

Time reversal symmetry (TRS) protected  
Immune to non-magnetic scattering, but  
sensitive to TRS breaking perturbations

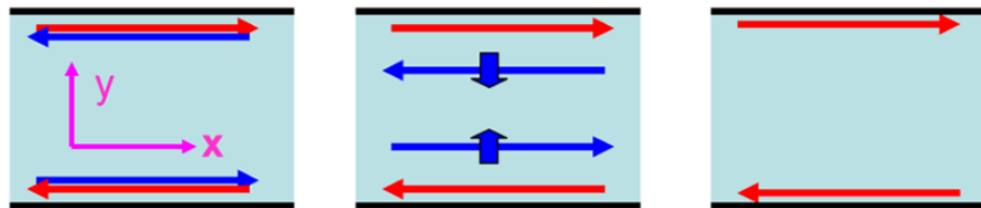
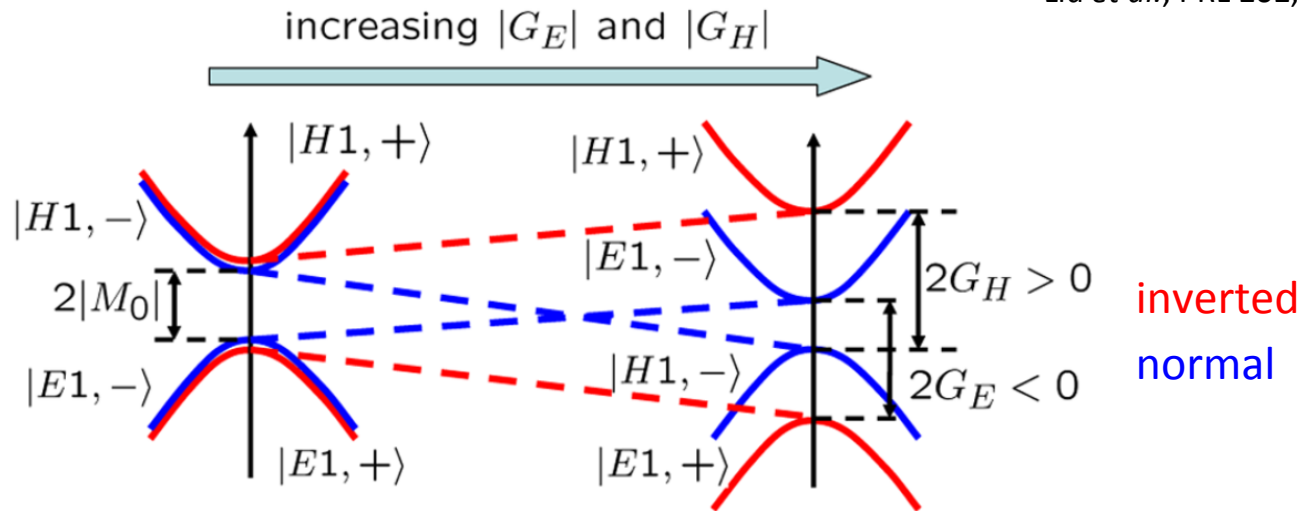
Characterized by spin Chern number  $C_{\pm}$



# Quantum Anomalous Hall Effect

Quantum Anomalous Hall effect created by TRS breaking due to exchange fields in  $\text{Hg}_{1-y}\text{Mn}_y\text{Te}$  quantum wells

Liu *et al.*, PRL **101**, 146802 (2008)



spin down states penetrate deeper and vanish

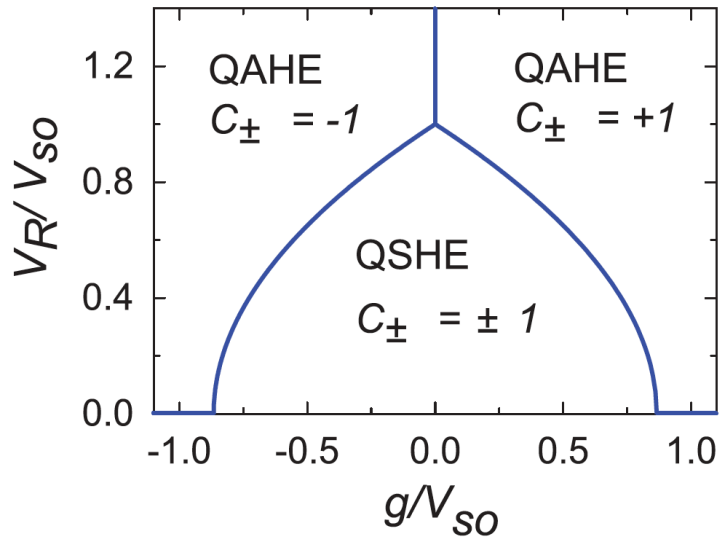
$G_i$  – spin splitting band  $i$

$$G_E G_H < 0$$

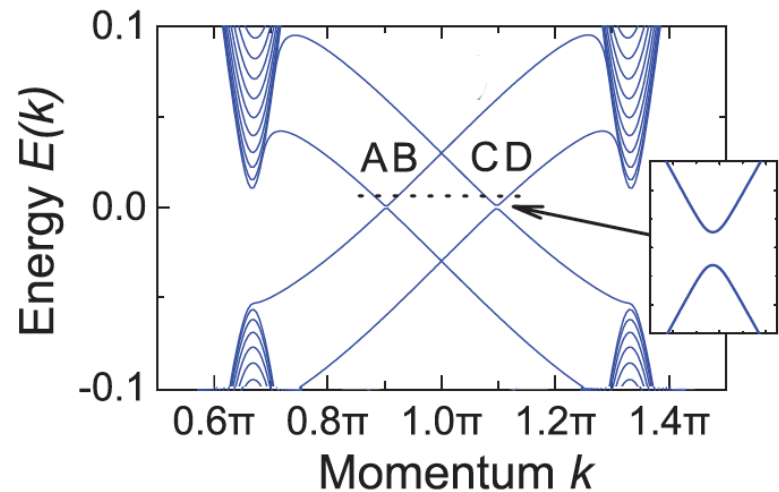
# QSH with broken TR symmetry

Honeycomb lattice with TRS breaking due to exchange field.  
Below a critical value band topology remains intact,  
allows for magnetic manipulation of QSH

Yang *et al.*, PRL **107**, 066602 (2011)



$g$  - Exchange field  
 $V_{SO}$  - intrinsic SOI  
 $V_R$  - Rashba SOI



# Hamiltonian including exchange field

$\hat{\tau}$  : spin

$\hat{\sigma}$  : e-h bands

$\hbar = 1$

$$H = H_0 + H_1$$

$$H_0 = v_F(\hat{\tau}_z k_x \hat{\sigma}_x + k_y \hat{\sigma}_y) + Dk^2 + (M_0 - Bk^2)\hat{\sigma}_z$$

$$H_1 = (g_0 \hat{\sigma}_z + g_1) \hat{\tau}_z$$

$$g_0 = \frac{1}{2}(G_H - G_E)$$

$$g_1 = \frac{1}{2}(G_H + G_E)$$

Non-zero band gap if

$$|g_1| < \max(|M_0|, |g_0|)$$

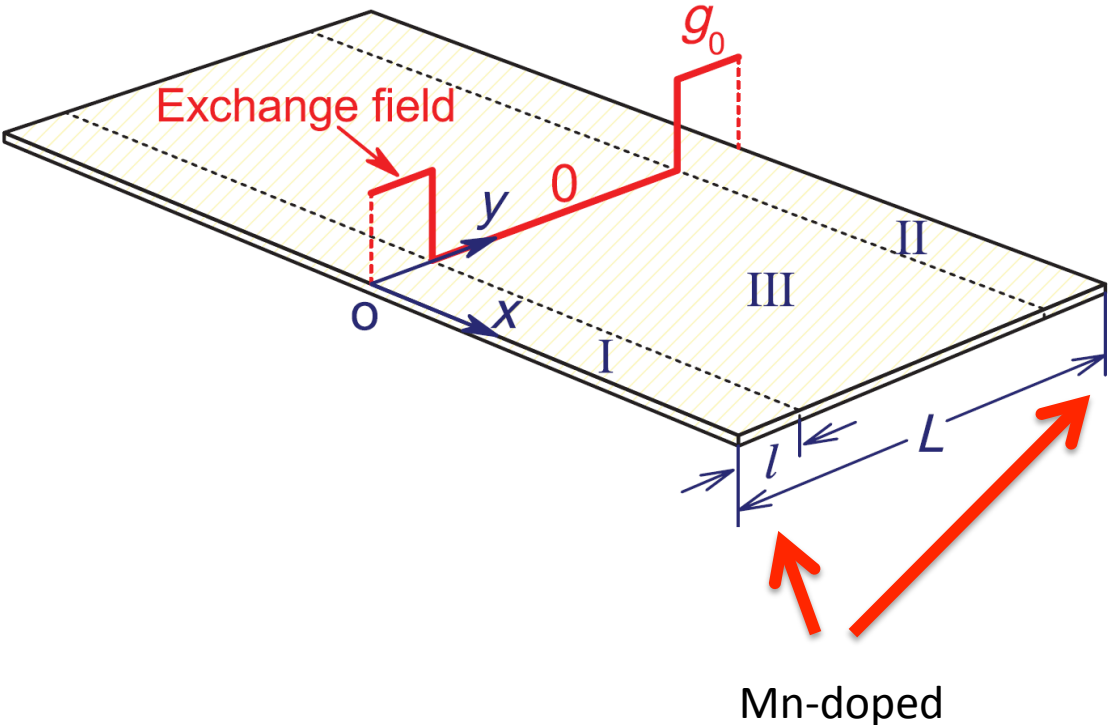
Spin Chern number:

$$C_{\pm} = \pm 1/2[\text{sgn}(B) + \text{sgn}(M_0 \pm g_0)]$$

$$g_0 = 0, \begin{cases} BM_0 > 0 : & C_{\pm} = \pm \text{sgn}(B) \\ BM_0 < 0 : & C_{\pm} = 0 \end{cases} \quad \begin{array}{l} \text{QSH phase} \\ \text{insulator} \end{array}$$

# QSH Sample

Mn breaks TRS  
(backscattering)



$$g_0 = \frac{1}{2}(G_H - G_E)$$

# Edge states I

$$E_{\pm}(k_x) = \pm v_F k_x$$

$$D = g_1 = 0$$

$$|\tau_z, \sigma_x\rangle$$

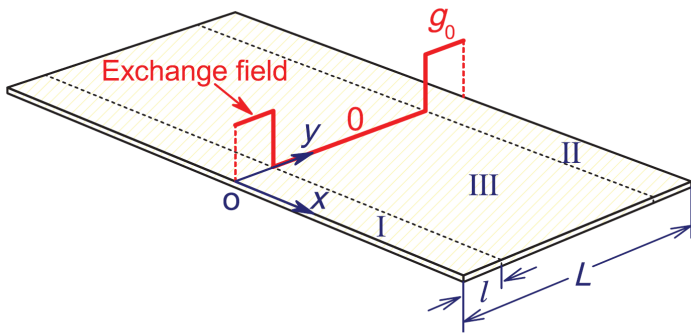
$$\varphi_+(k_x, y) = |1, 1\rangle \phi_+(k_x, y)$$

$$\varphi_-(k_x, y) = |-1, 1\rangle \phi_-(k_x, y)$$

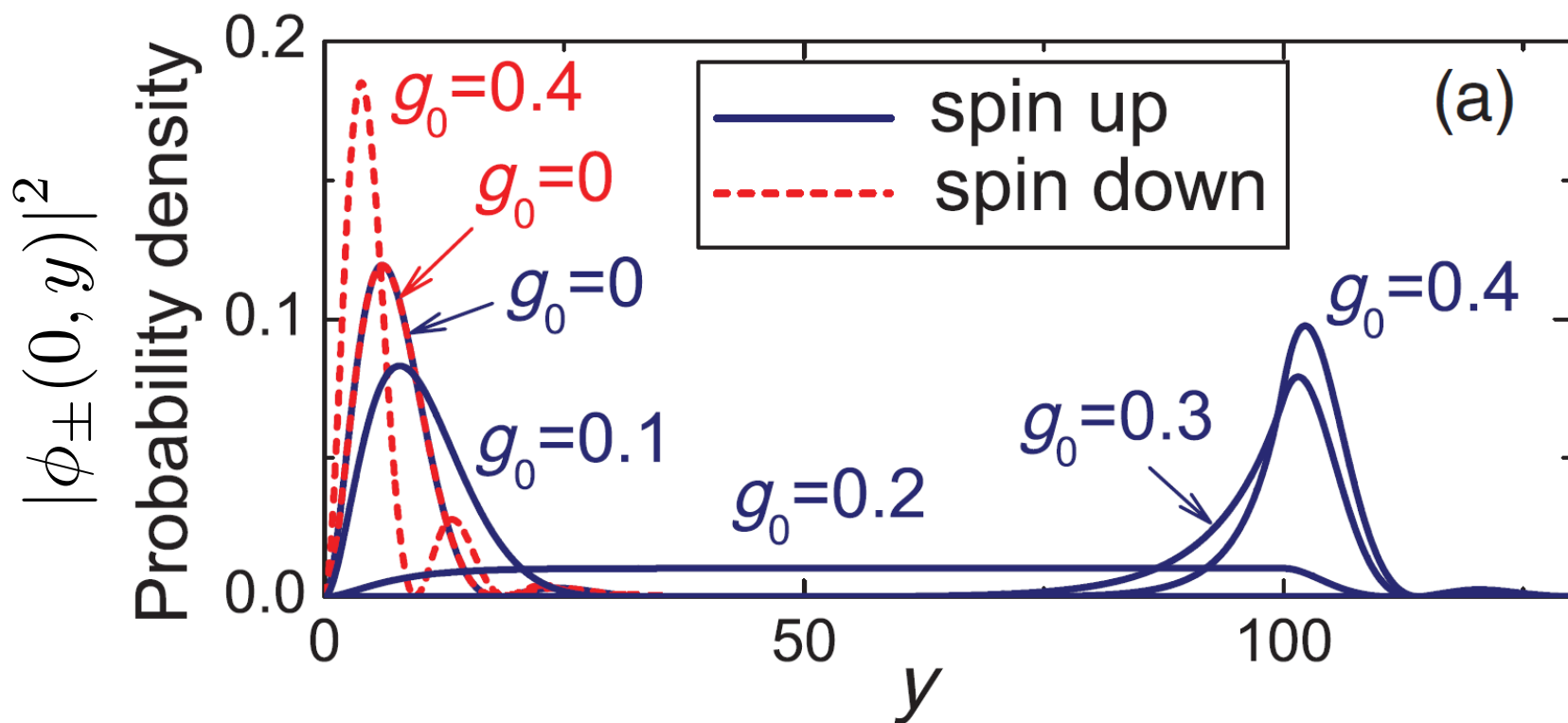
$$\phi_-(k_x, y) \propto \begin{cases} e^{-y/\xi_1(-g_0)} - e^{-y/\xi_2(-g_0)} \\ e^{-(y-l)/\xi_1(0)} - F_- e^{-(y-l)/\xi_2(0)} \end{cases}$$

$$\phi_+(k_x, y) \propto \begin{cases} e^{-y/\xi_1(g_0)} - e^{-y/\xi_2(g_0)} \\ e^{-(y-l)/\xi_1(0)} - F_+ e^{-(y-l)/\xi_2(0)} \end{cases}$$

$$\xi_{1,2}(\epsilon) = \frac{2|B|}{v_F \pm \sqrt{v_F^2 - 4B(M_0 - Bk_x^2 + \epsilon)}}$$



# Edge states II



increasing  $g_0$  separates  $|\phi_{\pm}(0, y)|^2$

$|\phi_{-}(0, y)|^2$  spreads out ( $g_0=0.2$ ) and localizes at  $y=l$

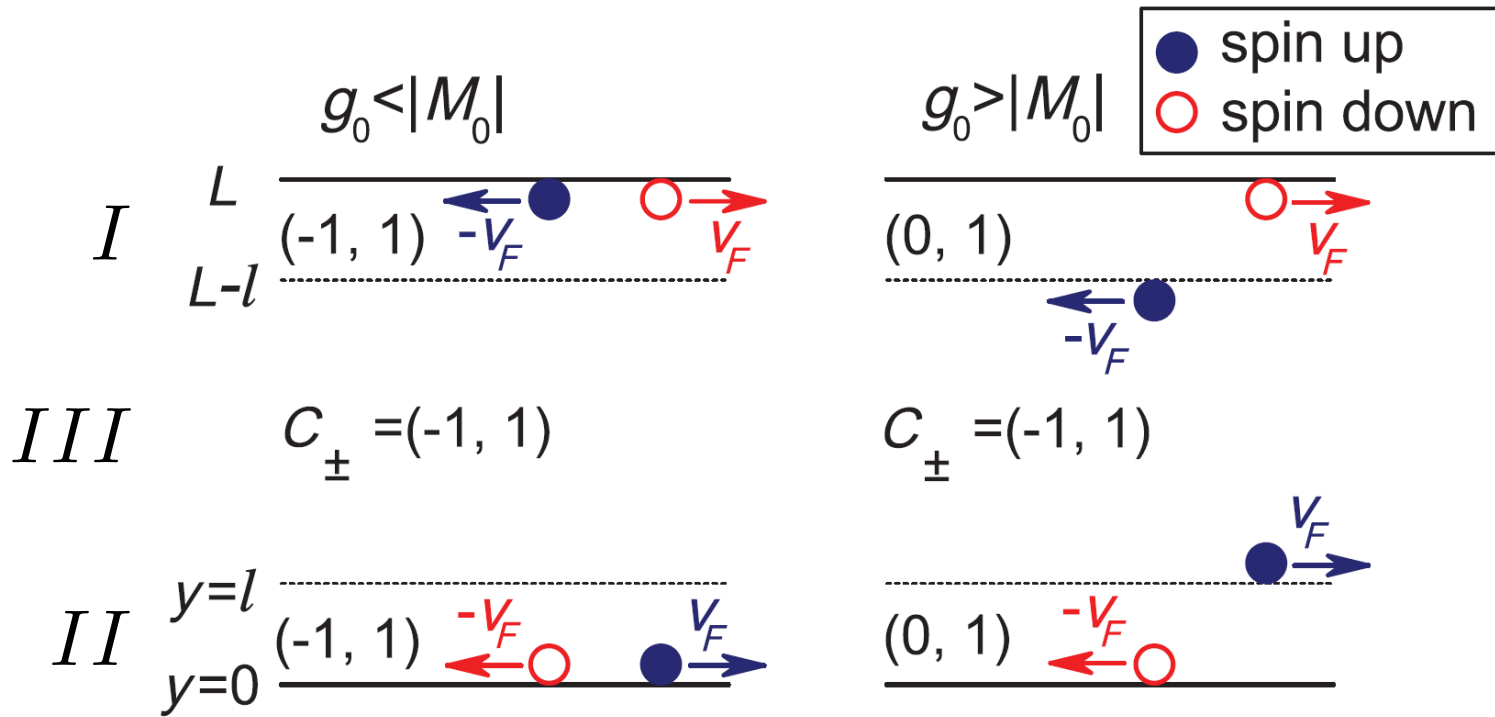
$$k_x = 0$$

$$l = 100$$



# Edge states III

$$C_{\pm} = (C_+, C_-)$$



No topological distinction between inner and outer regions

Outer region has  $C_+ = 0$ , i.e. is ordinary insulating for spin up

Separated lanes for + and -, protection against backscattering

# Numerical Model – HgTe QW

Randomly distributed Mn atoms, not fully aligned,  
provides exchange field and scattering potential

$$H_1 = -\frac{1}{\pi\lambda^2} \sum_{\alpha=0}^{N-1} (j_0\hat{\sigma}_z + j_1)\hat{\boldsymbol{\tau}} \cdot \mathbf{s}_\alpha e^{-|\mathbf{r}-\mathbf{R}_\alpha|^2/\lambda^2}$$

Mn spin orientation distribution

$$f(\mathbf{s}_\alpha) \propto e^{-\eta \cos \theta_\alpha}$$

$$M/M_s = -\langle \cos \theta_\alpha \rangle = \coth \eta - 1/\eta$$

Mean field recovers

$$H_1 = (g_0\hat{\sigma}_z + g_1)\hat{\boldsymbol{\tau}}_z$$

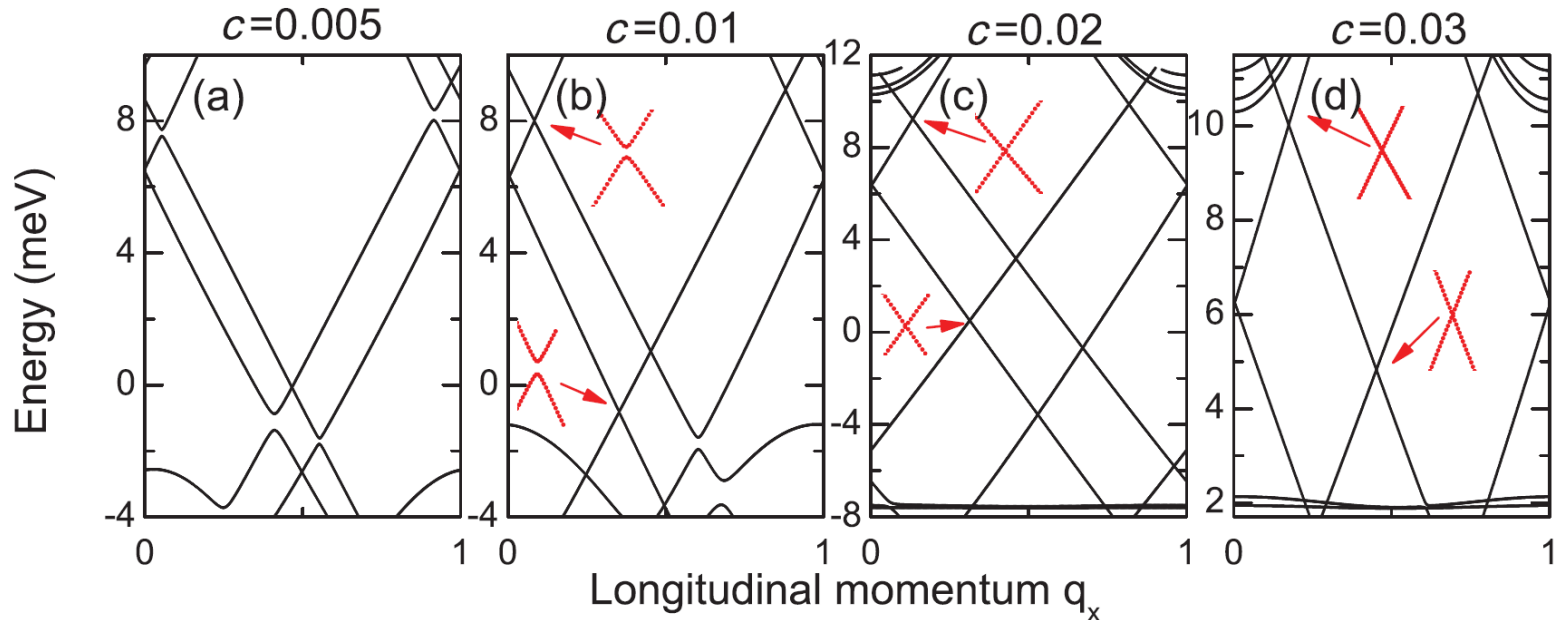
$$v_F = 364.5 \text{ meV nm}$$

$$B = -686 \text{ meV nm}^2$$

$$D = -512 \text{ meV nm}^2$$

$$M_0 = 10 \text{ meV}$$

# What about backscattering? - Spectrum



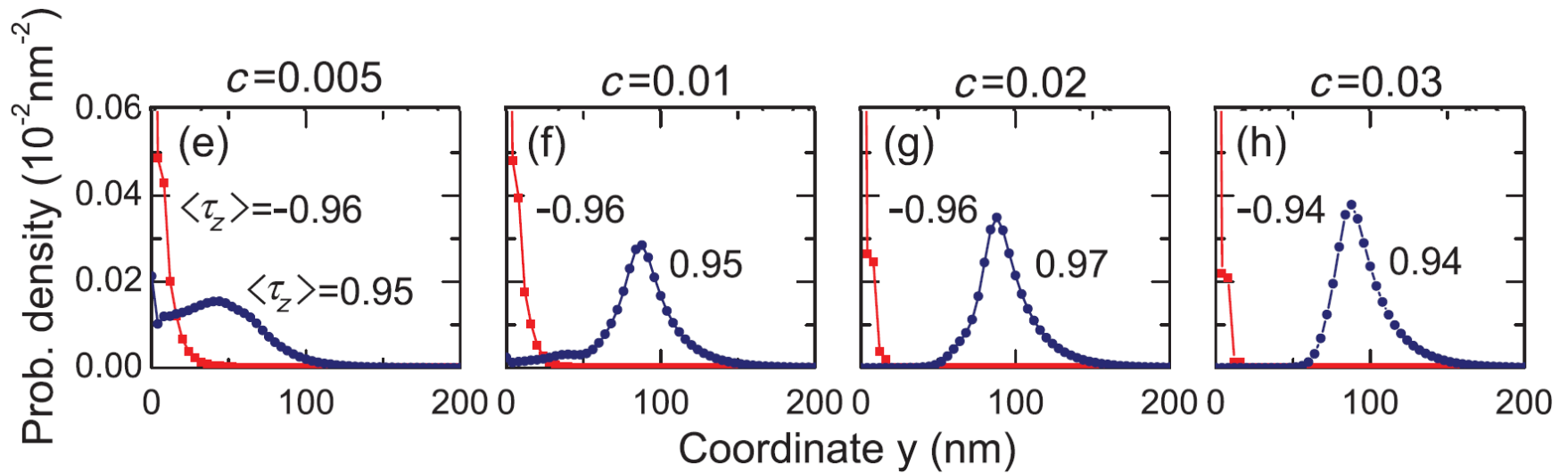
$c$  = doping concentration



Energy gaps vanish  
(backscattering is quenched)

$$[q_x] = 2\pi/L_x$$

# Numerical Model – Probability density



$c$  = doping concentration



Separation increases

$$E_F = 4 \text{ meV}$$

# Conclusions

Use QAH and QSH with broken TRS to separate QSH lanes

Numerical testing shows robustness

- Thank you -