Full counting statistics of Andreev tunneling

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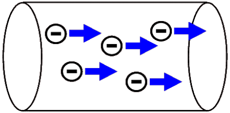
The full counting statistics (FCS) of charge transfer in nano-electronic circuits provides information about fundamental tunneling processes.[1–3] FCS is not limited to normal-state conductors, but may equally well describe charge fluctuations in superconducting structures. Nevertheless, despite considerable theoretical interest in the FCS of superconductors,[4–12] experiments have so far been restricted to normal-state electrons.[13–23] Here we measure the FCS of Andreev events in which Cooper pairs are either produced from electrons that are reflected as holes at a superconductor/normal-state interface or annihilated in the reverse process. Surprisingly, the FCS consists of quiet periods with no Andreev processes, interrupted by the tunneling of a single electron that triggers an avalanche of Andreev events giving rise to strongly super-Poissonian distributions. Our experiment is important for quantum metrological applications[24] and for entanglement generation using Cooper pair splitters.[25–27]

Full counting statistics (FCS)

How can we characterize a stream of particles (e.g. an electric current) ?



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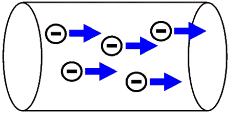

GOOD
Average current I

Full counting statistics (FCS)

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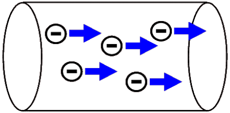

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Average current I

BETTER
I + shot noise S_{shot}

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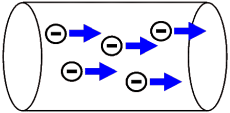

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FCS $p(n,t)$

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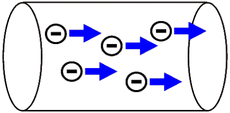

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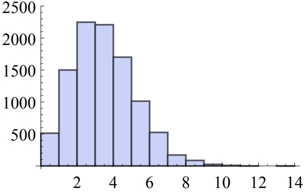
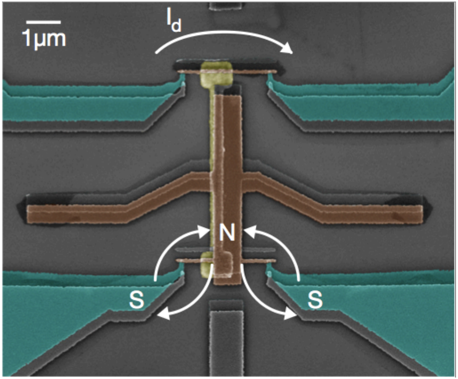
BEST
FCS $p(n,t)$

$$I = \frac{e}{T} \sum_n n p(n,T)$$

$$S_{\text{shot}} = \frac{e^2}{T} \sum_n n^2 p(n,T)$$

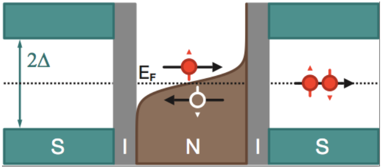


Experimental setup

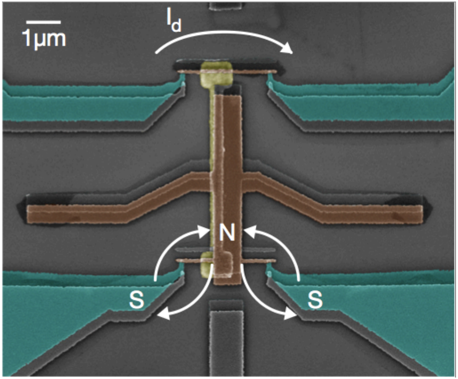


Single-electron transistor (I_d)

"SINIS" structure



Experimental setup



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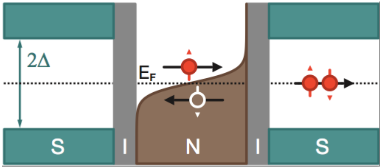


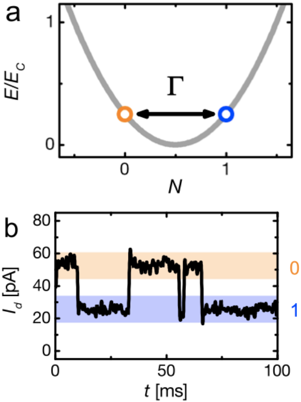
- Metallic island charging energy: $E_C = 40\ \mu\text{eV}$
- Superconducting gap (Al) $\Delta = 210\ \mu\text{eV}$
- Tunnel resistance $R_T = 490\ \text{k}\Omega$
- Dil. fridge: $T = 50\ \text{mK}$

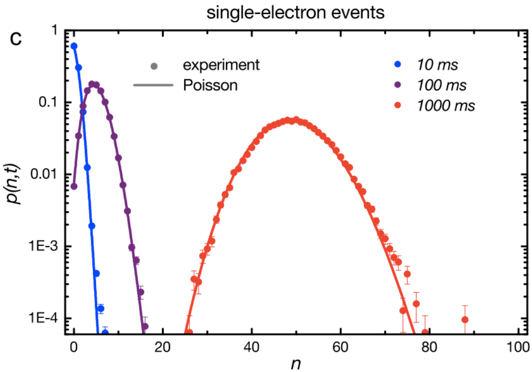
$E_C < \Delta$ necessary to observe Andreev tunneling processes

Single-electron events

Island charge states

$E = E_C(N - N_g)^2$





$$p(n,t) = \frac{(\Gamma t)^n}{n!} e^{-\Gamma t}$$

$$\Gamma = 49 \text{ Hz}$$

Single-electron events

Island charge states
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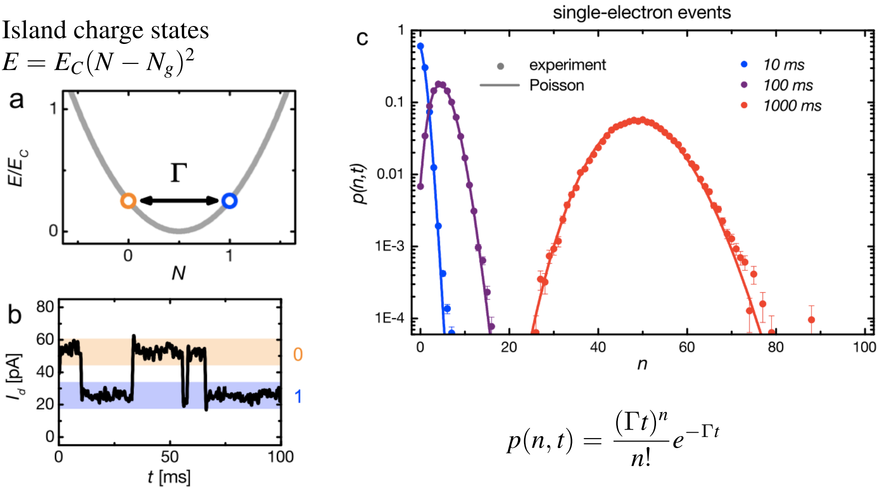

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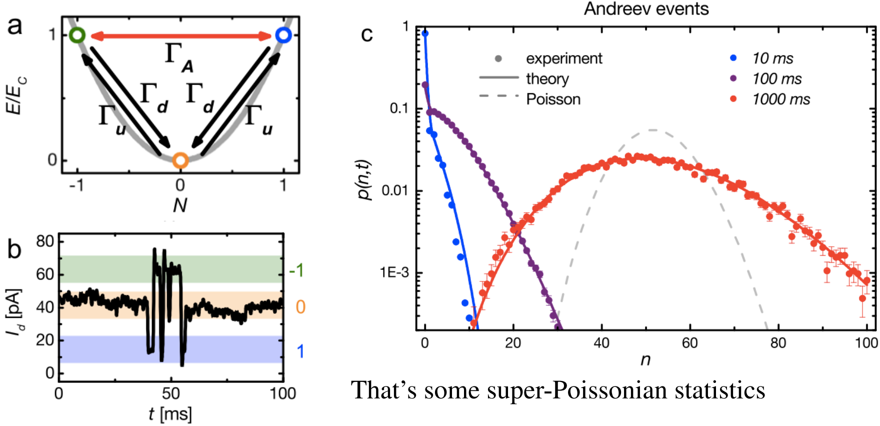
Poisson distributed events ⇒ independent tunneling events

Andreev processes

Island charge states



That's some super-Poissonian statistics

$\Gamma_u = 11.75$ Hz, $\Gamma_d = 252.0$ Hz, $\Gamma_A = 614.5$ Hz

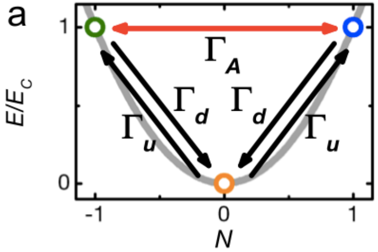
Calculation of the FCS, master equation

Classical description: only keep track of the probability $p_\alpha(n,t)$ of having observed n Andreev processes and being in the charge state α at time t

$$\dot{p}_d(n,t) = -2\Gamma_u\, p_d(n,t) + \Gamma_d\, p_u(n,t)$$
$$\dot{p}_u(n,t) = \underbrace{2\Gamma_u\, p_d(n,t) - \Gamma_d\, p_u(n,t)}_{\text{Single-electron processes}} + \underbrace{\Gamma_A\big(p_u(n-1,t) - p_u(n,t)\big)}_{\text{Andreev processes, change } n}$$

$$p_d(n,0) = \delta_{n0}\frac{\Gamma_d}{2\Gamma_u + \Gamma_d}$$
$$p_u(n,0) = \delta_{n0}\frac{2\Gamma_u}{2\Gamma_u + \Gamma_d}$$

Electron temperature $T \approx 140mK$



Calculation of the FCS

FCS: $p(n,t) = \sum_\alpha p_\alpha(n,t)$
Trick: compute the cumulant generating function

$$\mathcal{S}(\chi,t) = \log\left(\sum_n e^{i\chi n} p(n,t)\right)$$

Average $\langle n \rangle = -i\partial_\chi \mathcal{S}(\chi,t)|_{\chi=0}$, variance $(\Delta n)^2 = -\partial_\chi^2 \mathcal{S}(\chi,t)|_{\chi=0}, \ldots$

$$\begin{pmatrix} \dot{p}_u(\chi,t) \\ \dot{p}_d(\chi,t) \end{pmatrix} = \begin{pmatrix} \Gamma_A(e^{i\chi}-1)-\Gamma_d & 2\Gamma_u \\ \Gamma_d & -2\Gamma_u \end{pmatrix} \begin{pmatrix} p_u(\chi,t) \\ p_d(\chi,t) \end{pmatrix}$$

Calculation of the FCS, result

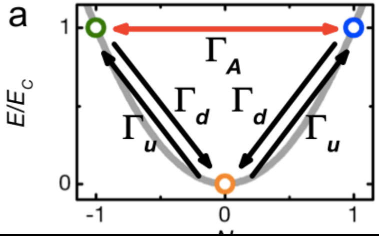
Result

$$\mathcal{S}(\chi,t) = 2\Gamma_u t \sum_{m=1}^{\infty} q(m)(e^{im\chi}-1) + \underbrace{\mathcal{O}(\Gamma_u^2)}_{\text{correlations between avalanches, small}}$$

$\Rightarrow$ sum of independent Poisson processes that with rate $2\Gamma_u$ generate *avalanches* of m Andreev events with probability $q(m)$

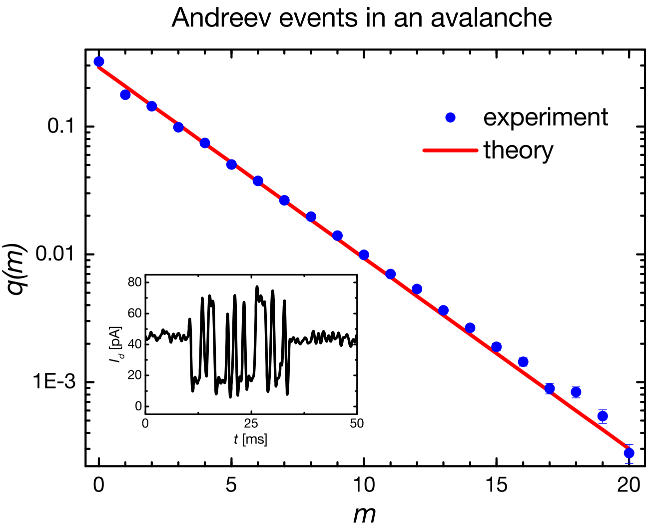
$$q(m) = \frac{\Gamma_d}{\Gamma_A+\Gamma_d}\left(\frac{\Gamma_A}{\Gamma_A+\Gamma_d}\right)^m .$$

a



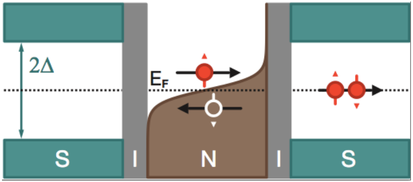
Single-electron tunneling events trigger avanlanches

Avalanche statistics

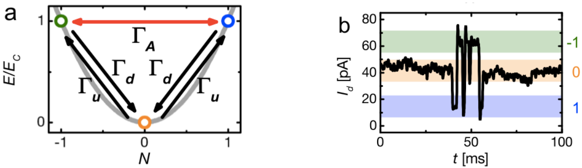


Conclusions

- (First?) FCS of Andreev processes in a SINIS structure



- Super-Poissonian statistics explained by avalanches



- Future: possible applications to Cooper pair splitters; metrology (SINIS turnstile as current standard)