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Transport via double constrictions in integer and fractional topological insulators

Chia-Wei Huang,[1] Sam T. Carr,[2,3] Dmitri Gutman,[1] Efrat Shimshoni,[1] and Alexander D. Mirlin[3,4,5]

[1] *Department of Physics, Bar-Ilan University, Ramat Gan, 52900, Israel*
[2] *School of Physical Sciences, University of Kent, Canterbury CT2 7NH, UK*
[3] *Institut fi¿œr Theorie der Kondensierten Materie and DFG Center for Functional Nanostructures, Karlsruher Institut fi¿œr Technologie, 76128 Karlsruhe, Germany*
[4] *Institut fi¿œr Nanotechnologie, Karlsruher Institut fi¿œr Technologie, 76021 Karlsruhe, Germany*
[5] *Petersburg Nuclear Physics Institute, 188300 St. Petersburg, Russia*

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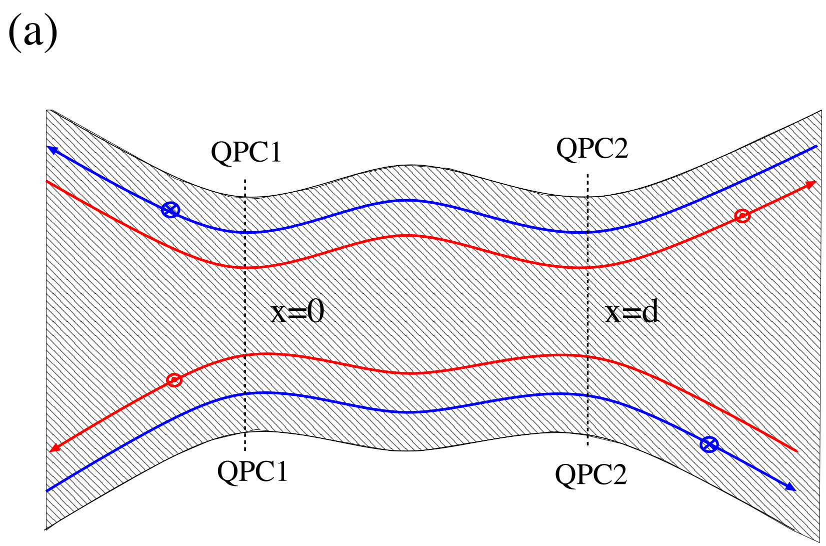
We study transport properties of the helical edge states of two-dimensional integer and fractional topological insulators via double constrictions. Such constrictions couple the upper and lower edges of the sample, and can be made and tuned by adding side gates to the system. Using renormalization group and duality mapping, we analyze phase diagrams and transport properties in each of these cases. Most interesting is the case of two constrictions tuned to resonance, where we obtain Kondo behavior, with a tunable Kondo temperature. Moving away from resonance gives the possibility of a metal-insulator transition at some finite detuning. For integer topological insulators, this physics is predicted to occur for realistic interaction strengths and gives a conductance G with two temperature T scales where the sign of dG/dT changes; one being related to the Kondo temperature while the other is related to the detuning.

Reminder: helical edge states in 2D Top. Insulators

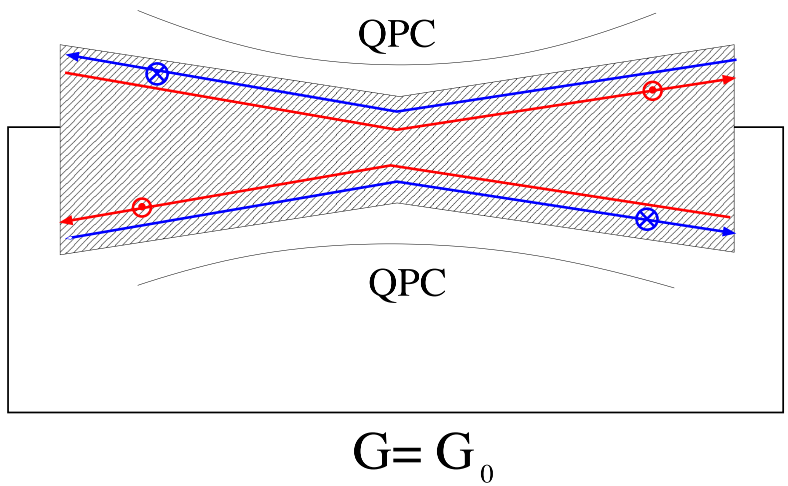


- Helical edge states: $\psi_{R\uparrow}, \psi_{L\downarrow}$; $\psi_{R\downarrow}, \psi_{L\uparrow}$
- Time-reversal invariance -> backscattering suppressed
- Backscattering possible by tunneling to the other edge!

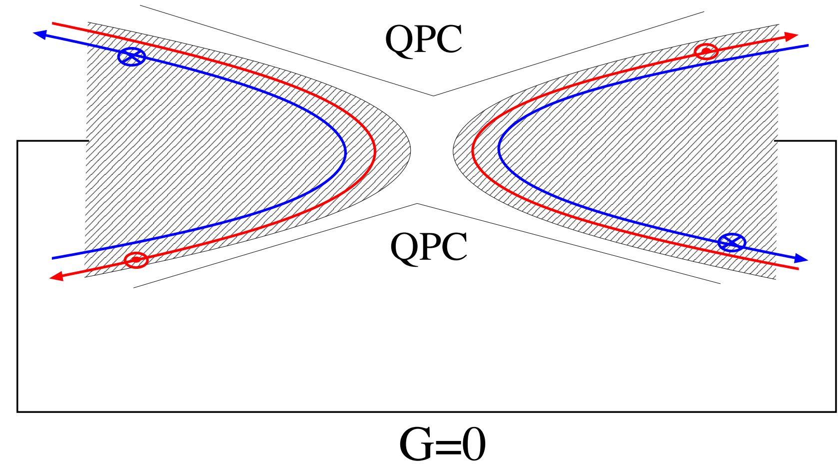
Outline of the paper

Review of a single contact:

(a)

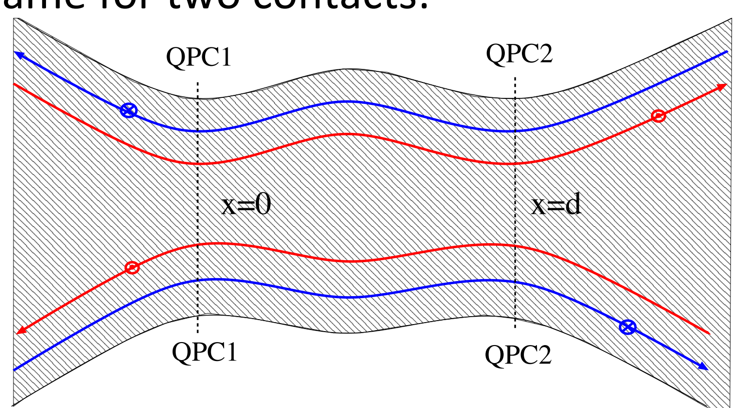


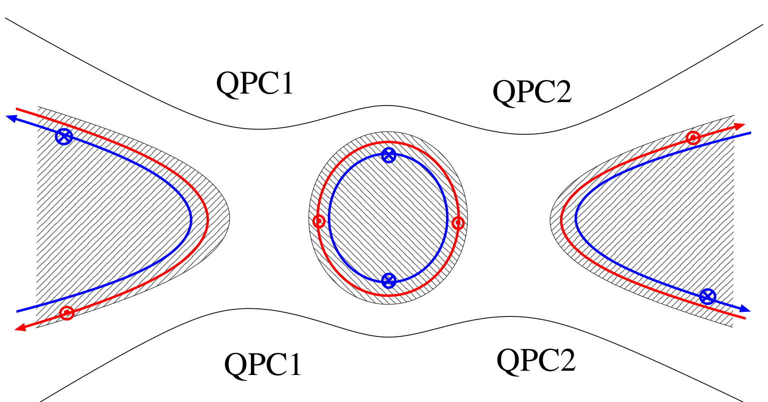
(b)



- Luttinger Liquid + bosonization to treat Coulomb interaction
- RG analysis for small and large tunnel coupling
- Identification of the possible conducting/insulating phases

The same for two contacts:





Bosonization for the single contact

- Tunneling couples the two edges
- Bosonize the Hamiltonian with help of charge and spin fields:

$\theta_c, \varphi_c \qquad \theta_s, \varphi_s$

$$H_0 = \frac{v}{4\pi} \sum_{a=c,s} \int dx \left[\frac{1}{g_a} \left(\partial_x \theta_a \right)^2 + g_a \left(\partial_x \varphi_a \right)^2 \right]$$

$$g_c = \frac{1}{g_s} = g$$

Tunneling happens by 3 mechanisms:

- Single tunneling events

$v_e \psi^\dagger_{R\uparrow} \psi_{L\uparrow} + \ldots$

- Pair backscattering

$v_c \psi^\dagger_{R\uparrow} \psi^\dagger_{R\downarrow} \psi_{L\uparrow} \psi_{L\downarrow} + \ldots$

- 2e charge transfer

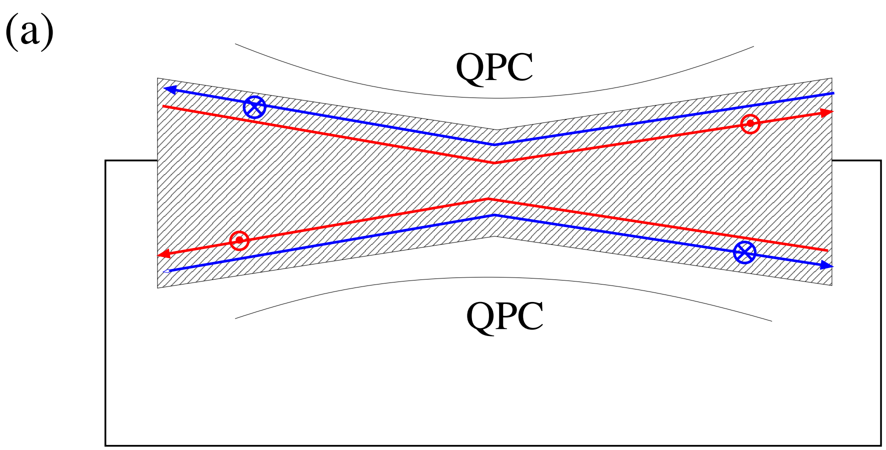
$v_s \psi^\dagger_{R\uparrow} \psi^\dagger_{L\downarrow} \psi_{L\uparrow} \psi_{R\downarrow} + \ldots$

Bosonized:

$$v_e \cos\theta_c \cos\theta_s + v_c \cos 2\theta_c + v_s \cos 2\theta_s$$

RG analysis of single contact

Almost open



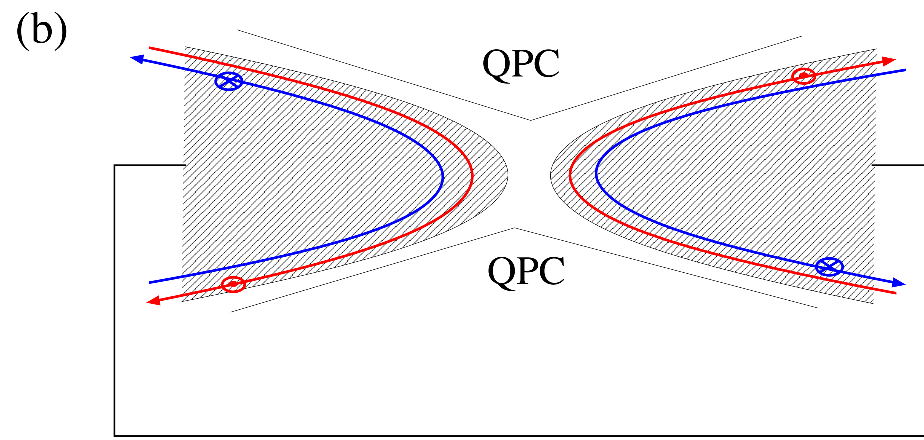
$G = G_0$

$$\frac{dv_e}{dl} = \left(1 - \frac{g + g^{-1}}{2}\right) v_e$$ Irrelevant

$$\frac{dv_c}{dl} = (1 - 2g) v_c$$ maybe relevant

$$\frac{dv_s}{dl} = (1 - 2g^{-1}) v_s$$ maybe relevant

Almost closed



$G = 0$

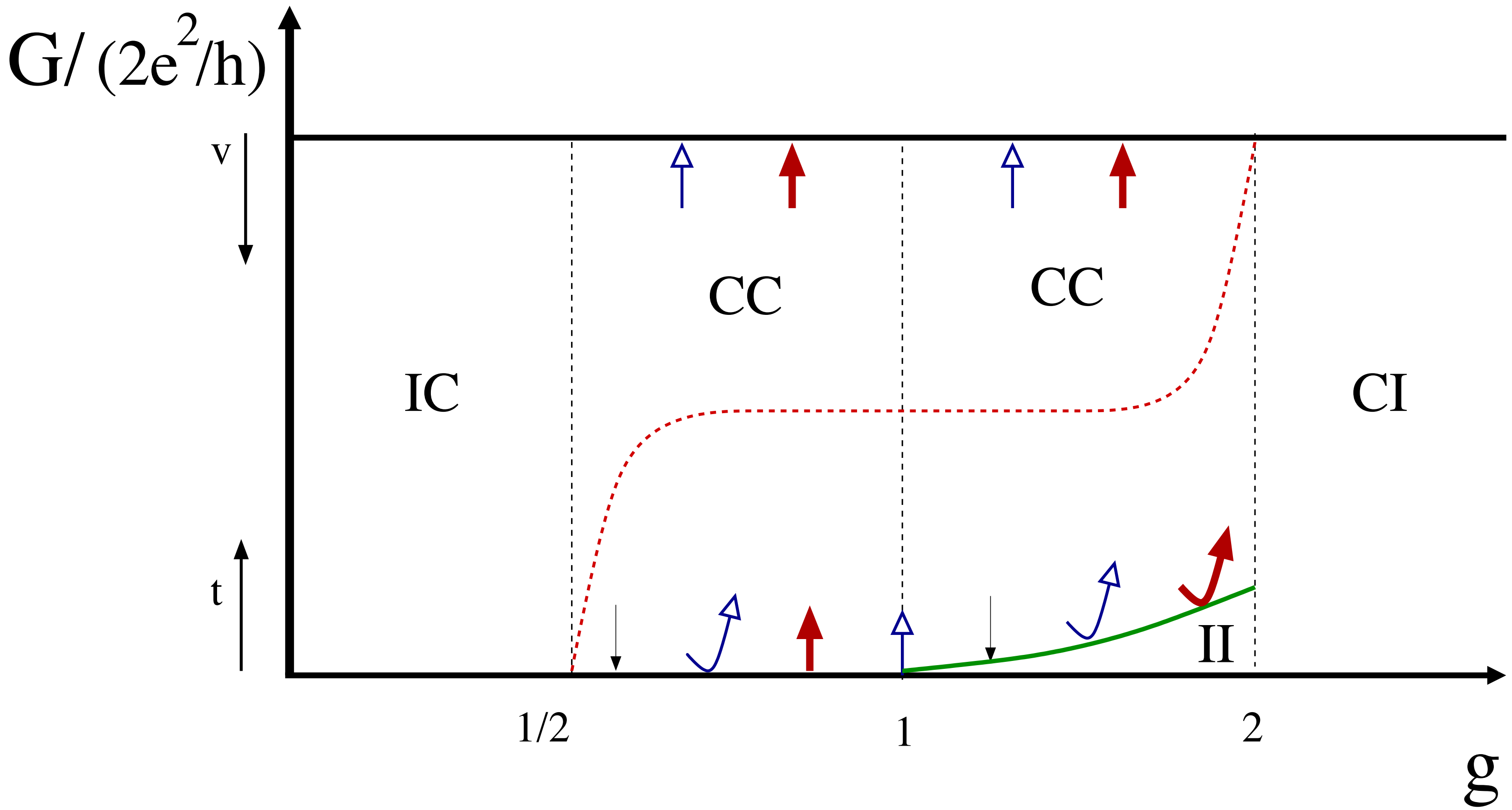
$$\frac{dv_e}{dl} = \left(1 - \frac{g + g^{-1}}{2}\right) v_e$$ Irrelevant

$$\frac{dv_c}{dl} = (1 - 2g) v_c$$ maybe relevant

$$\frac{dv_s}{dl} = (1 - 2g^{-1}) v_s$$ maybe relevant

For moderate interaction, all processes irrelevant. Both phases are stable

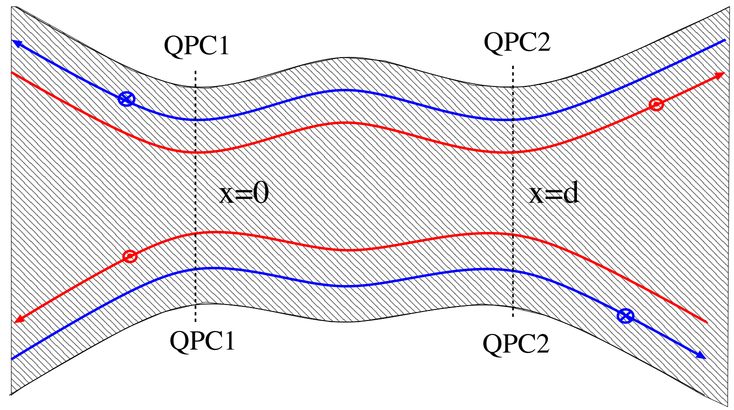
RG Flow, single contact



- Two stable fixed points, insulating and conducting
- In between one unstable fixed point, separating both phases

Two constrictions

(a)



- "Quantum dot" formed by double constriction
- Coulomb blockade
- Kondo physics

$$S = S_0 + \int d\tau V_{eff}$$

$$V_{eff} = V\left(\cos\frac{\theta_{+c}}{\sqrt{2}}\cos\frac{\theta_{-c}}{\sqrt{2}}\cos\frac{\theta_{+s}}{\sqrt{2}}\cos\frac{\theta_{-s}}{\sqrt{2}} + \sin\frac{\theta_{+c}}{\sqrt{2}}\sin\frac{\theta_{-c}}{\sqrt{2}}\sin\frac{\theta_{+s}}{\sqrt{2}}\sin\frac{\theta_{-s}}{\sqrt{2}}\right)$$
$$+\frac{U_c}{2}\left(\theta_{-c} - \theta_{-c_0}\right)^2 + \frac{U_s}{2}\left(\theta_{-s} - \theta_{-s_0}\right)^2$$

$\theta_{\pm c/s}$ Fields at x=0,x=d

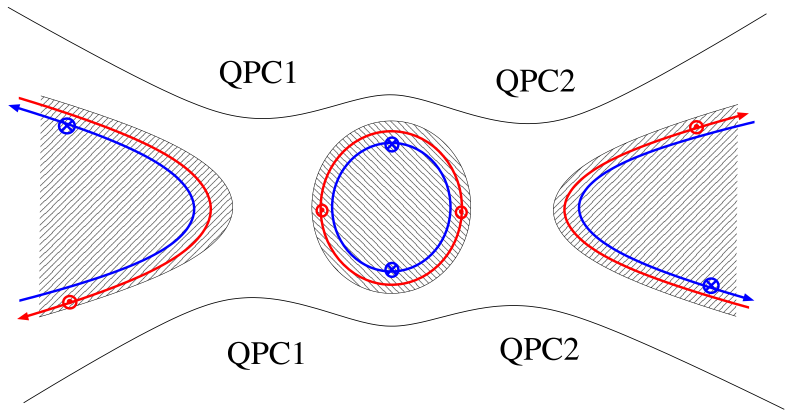
$\theta_{-s_0} = 0$ No ext. mag. field

$$\theta_{-c_0} = \frac{\pi}{\sqrt{2}}(2n+1)$$

Resonance condition

Strong constriction limit

(b)



- V largest energy scale
- Minimization of V: $\frac{\sqrt{2}}{\pi}\theta_{-c}, \frac{\sqrt{2}}{\pi}\theta_{-s}$ either both even or odd integers
- With the resonance condition: two degenerate spin states of the dot -> Kondo problem

The Kondo limit

- Tunneling Hamiltonian:

$$H_t^K = \frac{t_e}{2\pi\alpha}\cos\overline{\phi}_{+c}\cos\overline{\phi}_{+s} + \frac{\tilde{t}_c}{2\pi\alpha}\cos\overline{\phi}_{+c}\cos\overline{\phi}_{-s} + \frac{\tilde{t}_s}{2\pi\alpha}\cos\overline{\phi}_{+s}\cos\overline{\phi}_{-s}$$

- Can be obtained also from the last Hamiltonian via an instanton expansion

- t_e single electron through both constrictions without changing spin of the dot
- $\tilde{t}_c$ single charge transferred accompanied by spin-flip
- $\tilde{t}_s$ spin exchange between dot and leads

$$\frac{dt_e}{dl} = \left[1 - \frac{1}{2}(g + g^{-1})\right]t_e + AgK_s\tilde{t}_c\tilde{t}_s$$

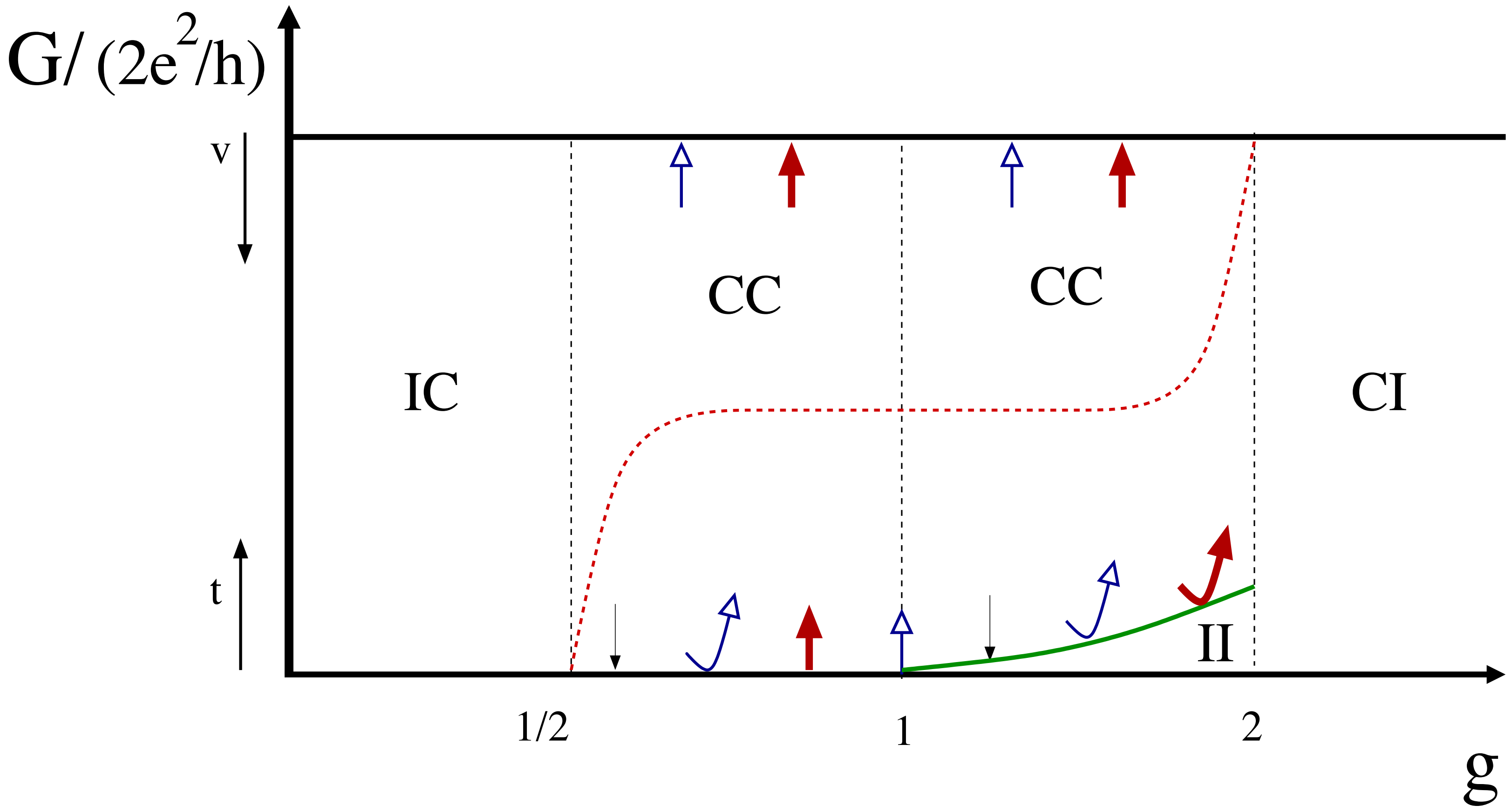
$$\frac{d\tilde{t}_c}{dl} = \left[1 - \frac{1}{2}(K_s g + g^{-1})\right]\tilde{t}_c + Agt_e\tilde{t}_s$$

$$\frac{d\tilde{t}_s}{dl} = \left[1 - \frac{g}{2}(1 + K_s)\right]\tilde{t}_s + \frac{At_e\tilde{t}_s}{g}$$

$$\frac{dK_s}{dl} = -\left(\frac{\tilde{t}_c^2}{g} + g\tilde{t}_s^2\right)K_s$$

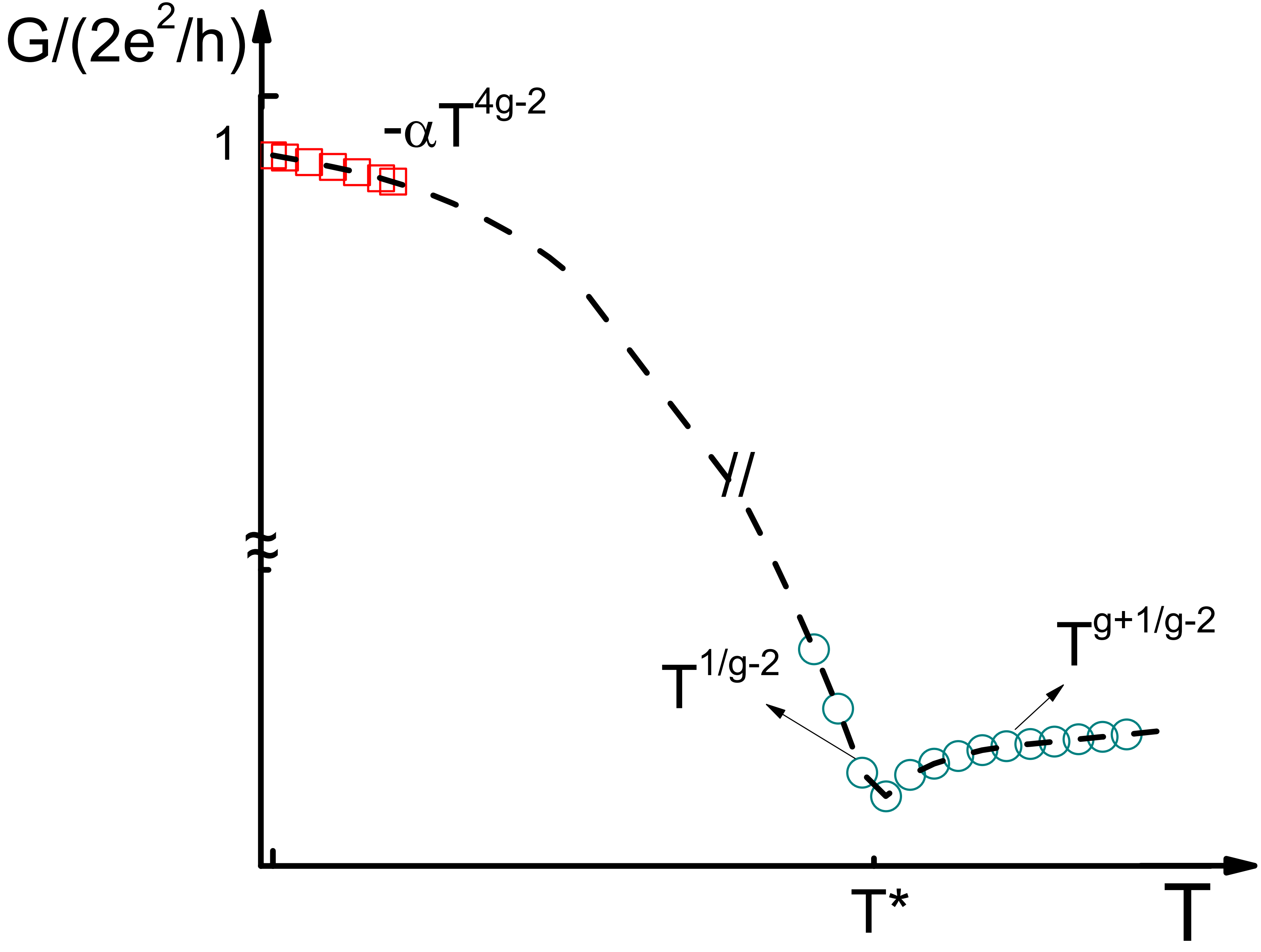
Phase diagram

- Without quadratic terms



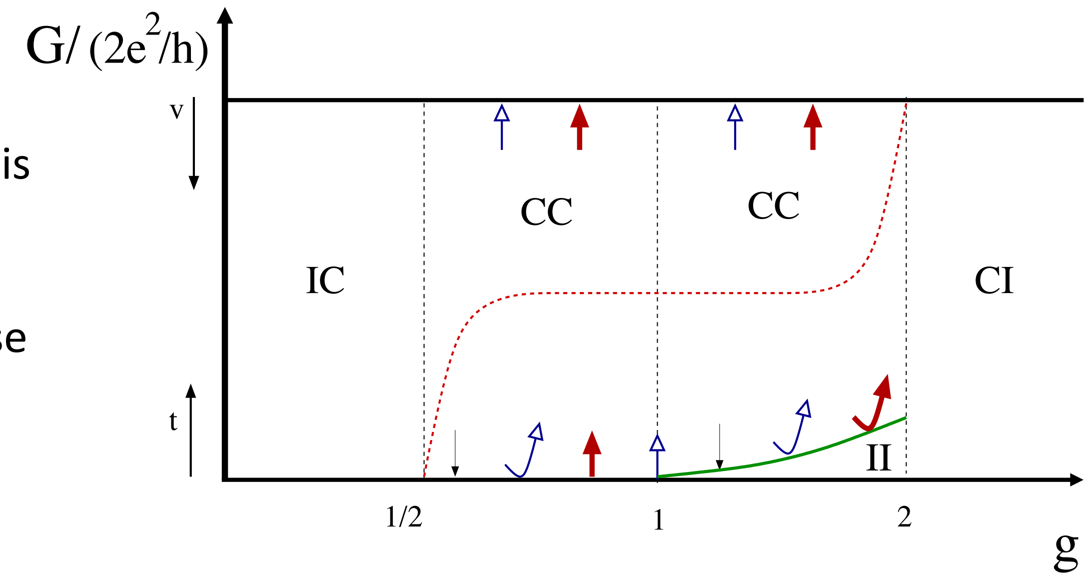
- CC phase corresponds to single channel Kondo fixed point

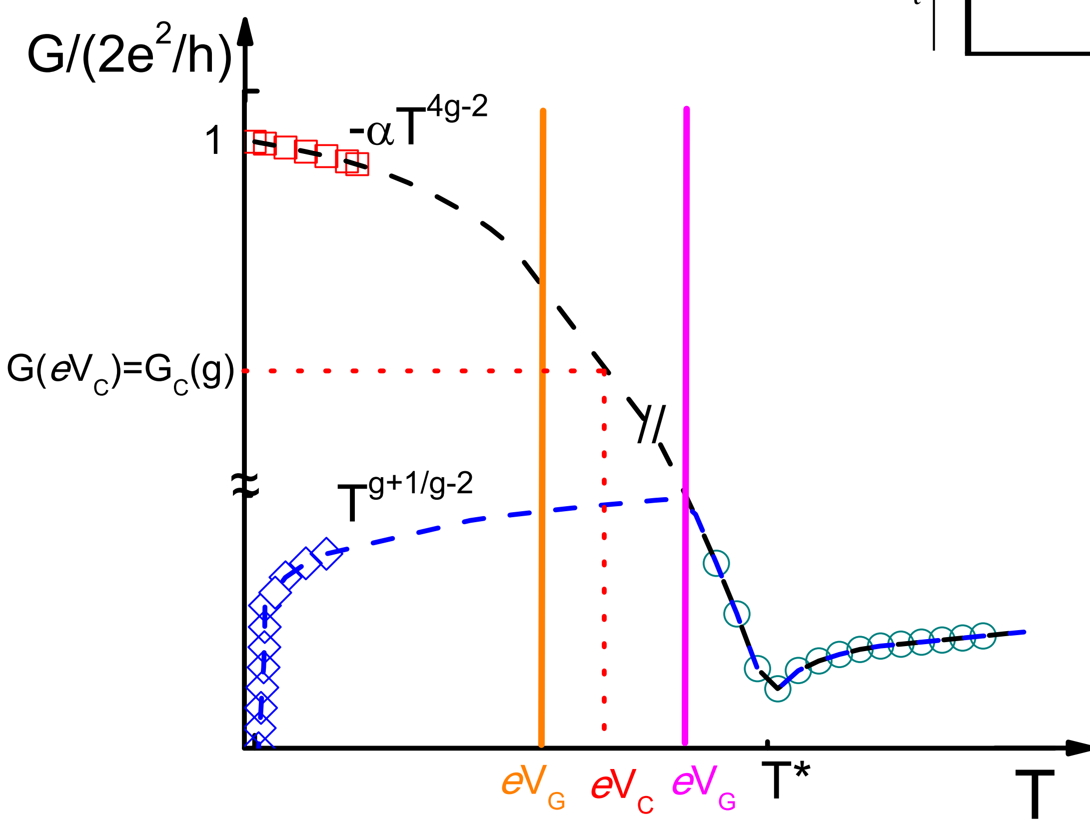
Temperature dependence



Off resonance

- New parameter V_G describes how far off resonance the dot is
- Large V_G equal to single constriction
- Small V_G equal to resonant case





- Critical V_C with metal-insulator transition

Conclusion

- Considered a QSHI with double constriction
- 3 important parameters: Luttinger parameter g, constriction strength v, of resonance condition V_G
- Non-monotonic dependence of conductance on T
- Metal insulator transition at $V_G = V_C$