Adiabatic Quantum Motors

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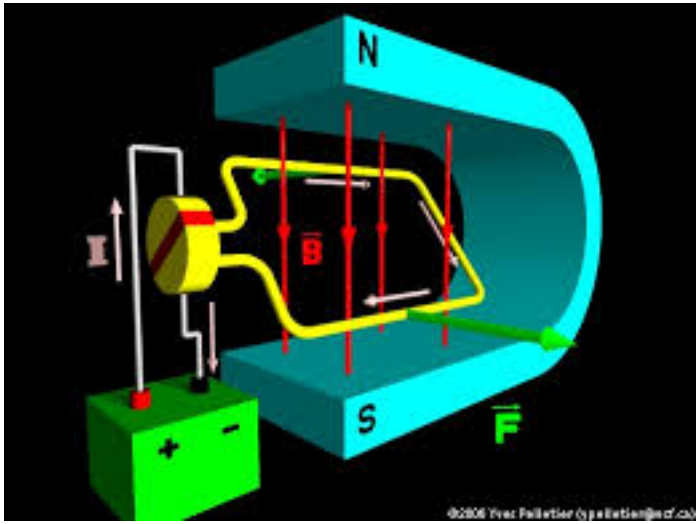
In 1960 Richard Feynamm announced a contest and promised a $1000 prize to the developer of an engine that fits a cube of side 1/64'' (~0.4 mm)

Introduction

Motor: device capable of converting energy into movement



kinetic energy -> rotational motion

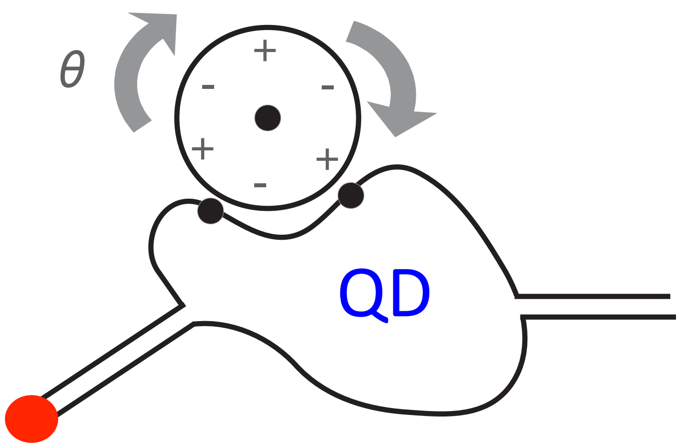
electric energy-> rotational motion

"As the dimensions of motors are reduced, it is natural to expect that quantum mechanics could be used to operate and to optimize nanomotors"

Quantum motors and quantum pumps

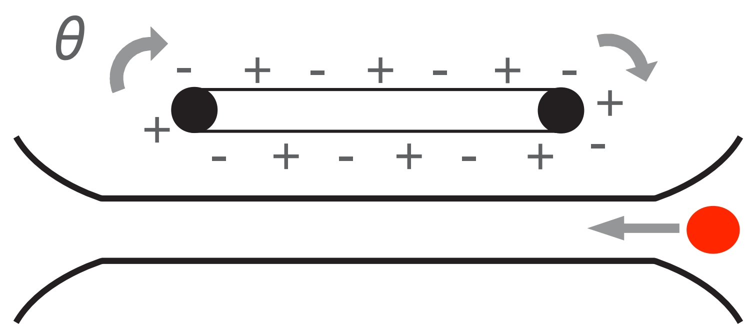
The main idea is to use the same operating principle as an electron pump, but in reverse

Chaotic QD pump



The motor (wheel) changes the shape of the QD

Thouless pump

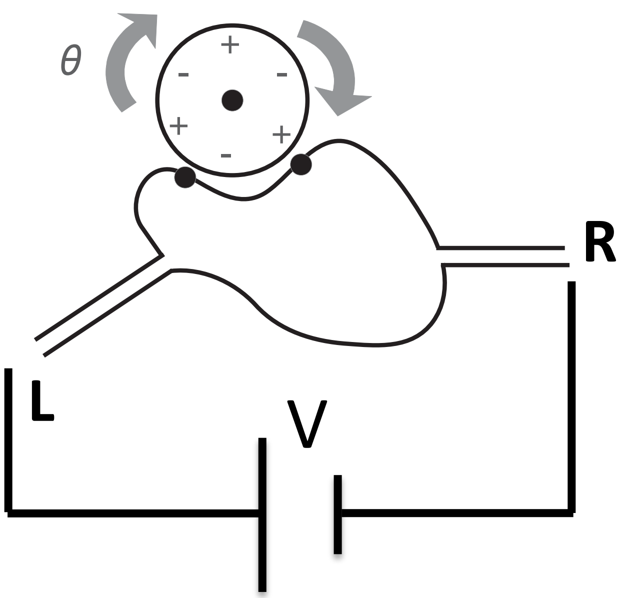


The conveyor belt generates a time-periodic potential felt by the electrons

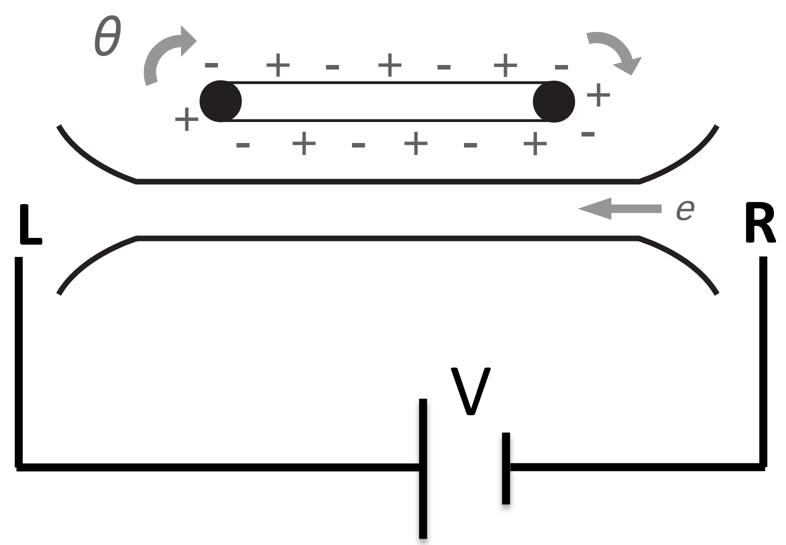
→ Electrons are pumped through the device

Quantum motors and quantum pumps

Chaotic QD pump

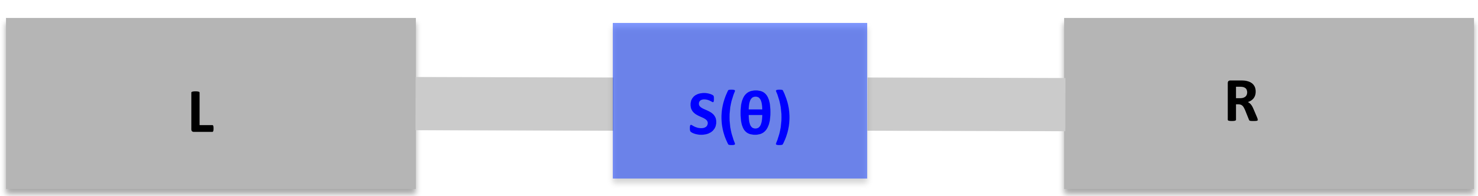


Thouless pump

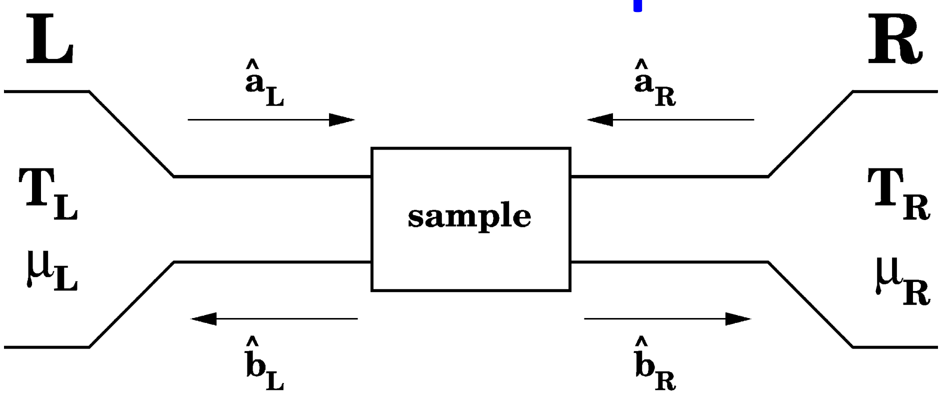


The voltage V induced a motion of the electrons that induce a force on the (classical) mechanical degree of freedom θ

The problem is treated within scattering theory formalism



Output Power



$$\begin{pmatrix} \hat{b}_{L1} \\ \cdots \\ \hat{b}_{LN_L} \\ \hat{b}_{R1} \\ \cdots \\ \hat{b}_{RN_R} \end{pmatrix} = s \begin{pmatrix} \hat{a}_{L1} \\ \cdots \\ \hat{a}_{LN_L} \\ \hat{a}_{R1} \\ \cdots \\ \hat{a}_{RN_R} \end{pmatrix}$$

Landauer-Büttiker $\quad s = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$

Here assume that the scattering matrix depends on several (classical) mechanical motor degrees of freedom X_ν

$$S = S(\mathbf{X})$$

Force exerted on mechanical degree of freedom [PRL **107**, 036804 (2011)]

$$F_\nu(\mathbf{X}) = \sum_\alpha \int \frac{d\epsilon}{2\pi i} f_\alpha \, \mathrm{Tr}\left(\Pi_\alpha S^\dagger \frac{\partial S}{\partial X_\nu} \right)$$

Π_α: Projects onto scattering channel in lead α

$\alpha = L, R$ $\quad \epsilon$: electron energy $\quad f_\alpha(\epsilon)$: Fermi distribution in lead α

Output Power

$$F_\nu(\mathbf{X}) = \sum_\alpha \int \frac{d\epsilon}{2\pi i} f_\alpha \,\mathrm{Tr}\left(\Pi_\alpha S^\dagger \frac{\partial S}{\partial X_\nu}\right)$$

For nonzero voltage V, the force F is NOT conservative

$$W_{\mathrm{out}} = \oint d\mathbf{X} \cdot \mathbf{F}(\mathbf{X}) \neq 0$$

In linear response $W_{\mathrm{out}} = \frac{ieV}{4\pi} \oint d\mathbf{X} \cdot \int d\epsilon f'(\epsilon) \,\mathrm{Tr}\left[(\Pi_L - \Pi_R) S^\dagger \frac{\partial S}{\partial \mathbf{X}}\right]$

This is proportional to the pumped charge into the system [PRB **58**, R10135 (1998)]

→ $W_{\mathrm{out}} = Q_p V$ and $P_{\mathrm{out}} = Q_p V / \tau$

Efficiency

The input power is given by the standard expression $P_{\text{in}} = \bar{I}V$

current induced by the voltage V

The efficiency of the motor is naturally defined as

$$\eta = P_{\text{out}}/P_{\text{in}} = Q_p/\bar{I}\tau$$

Define G(x) as the conductance of the device ⟶ $\bar{I} = \overline{G(\mathbf{X})}V + \frac{Q_p}{\tau}$

⟶ $$\eta = \frac{1}{1 + \bar{G}V\tau/Q_p}$$

Relation between the efficiency and the pumped charge!

QD pump is purely quantum mechanical effect

⟶ **Quantum** motor

Thouless pump

⟶ **Efficieny 1** for Fermi energy below the gap

Motor Dynamics

Assume the (classical) mechanical degree of freedom is subject to damping γ

$$\gamma\dot{\theta} = \frac{Q_p V}{2\pi} - F_{\text{load}}$$

Cycle period of the motor $\tau = 2\pi/|\dot{\theta}| = (2\pi)^2\gamma/(Q_p V - 2\pi F_{\text{load}})$

Use $\bar{I} = \overline{G(\mathbf{X})}V + \frac{Q_p}{\tau}$ → $$\frac{1}{\tau} = \frac{Q_p\bar{I} - 2\pi F_{\text{load}}\bar{G}}{Q_p^2 + (2\pi)^2\gamma\bar{G}}$$

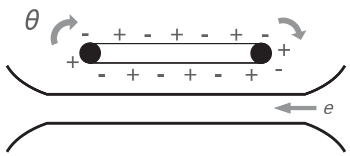
Thouless motor with conductance G=0 → $1/\tau = \bar{I}/Q_p$

current is solely due to pumped charge

→ maximal load $$F_{\text{load}}^{\max} = Q_p\bar{I}/2\pi\bar{G}$$

Thouless motor

Let us investigate the Thouless motor in more detail



$$\mathcal{H} = p^2/2m + 2\Delta\cos(2\pi x/a + \theta)\Theta(L/2 - |x|)$$

→ Periodic chemical potential

Choose Fermi wavevector close to $k_0 = \pi/a$

→ Hamiltonian can be linearlized

$$\mathcal{H} = v_F p \sigma^z + \Delta(\sigma^x \cos\theta + \sigma^y \sin\theta)\Theta(L/2 - |x|)$$

→ act in space of counterpropagating channels

A formula for the scattering matrix can then be determined

$$S = \frac{1}{M_{11}} \begin{pmatrix} -ie^{i\theta}\frac{\Delta}{\sqrt{E^2-\Delta^2}}\sin\lambda_L & 1 \\ 1 & -ie^{-i\theta}\frac{\Delta}{\sqrt{E^2-\Delta^2}}\sin\lambda_L \end{pmatrix}$$

$$\lambda_L = (L/\hbar v_F)[E^2 - \Delta^2]^{1/2}$$

$$M_{11} = \cos\lambda_L - i(E/\sqrt{E^2 - \Delta^2})\sin\lambda_L$$

Thouless motor

From the expression for the scattering matrix, we determine the conductance (Landauer)

$$G = \bar{G} = \frac{e^2}{h} \frac{|\Delta^2 - E_F^2|}{|\Delta^2 - E_F^2|\cos^2\lambda_L + E_F^2|\sin\lambda_L|^2}$$

From Brouwer's formula at zero temperature for an angular mechanical degree of freedom [PRB 58, R10135 (1998)] we obtained the pumped charge

$$Q_p = \frac{e\Delta^2|\sin\lambda_L|^2}{|\Delta^2 - E_F^2|\cos^2\lambda_L + E_F^2|\sin\lambda_L|^2}$$

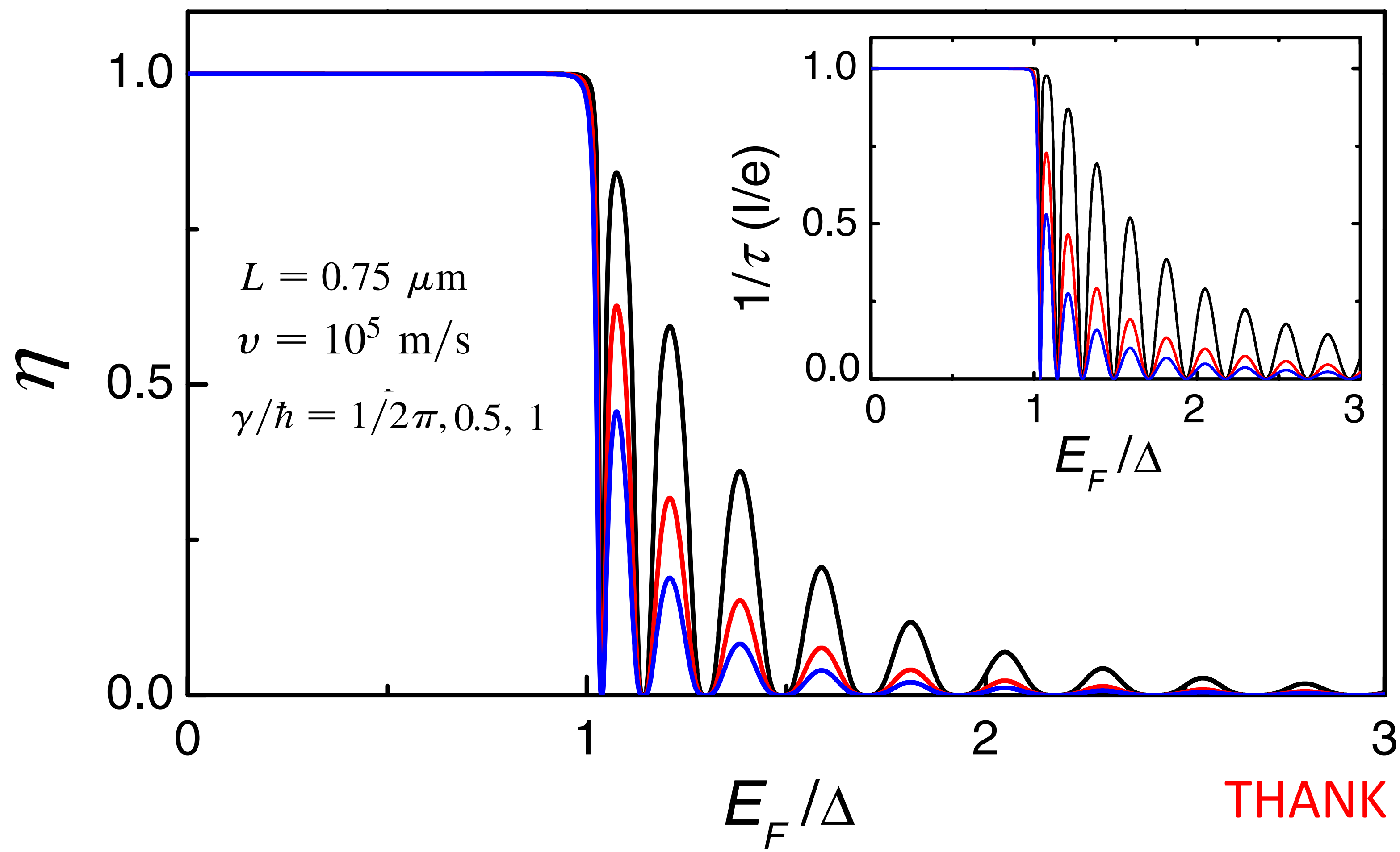
and thus the efficieny

$$\eta = \frac{1}{1 + \frac{2\pi\gamma}{\hbar} \frac{|E_F^2 - \Delta^2|}{\Delta^4|\sin\lambda_L|^4}[|E_F^2 - \Delta^2|\cos^2\lambda_L + E_F^2|\sin\lambda_L|^2]}$$

Thouless motor

$$\eta = \frac{1}{1 + \frac{2\pi\gamma}{\hbar}\frac{|E_F^2 - \Delta^2|}{\Delta^4|\sin\lambda_L|^4}\left[|E_F^2 - \Delta^2|\cos^2\lambda_L + E_F^2|\sin\lambda_L|^2\right]}$$

gap opened by the periodic potential



THANK YOU!

Conclusions