# **Adiabatic Quantum Motors** Raul Bùstos-Marun, Gil Refael, and Felix von Oppen PRL **111**, 060802 (2013)

*In* 1960 Richard Feynamm announced a contest and promised a \$1000 prize to the *developer of an engine that fits a cube of side 1/64'' (~0.4 mm)* 

# Introduction

## Motor: device capable of converting energy into movement





## $kinetic$  energy  $\rightarrow$  rotational motion

## electric energy-> rotational motion

"As the dimensions of motors are reduced, it is natural to expect that quantum mechanics could be used to operate and to optimize nanomotors" 

### Quantum motors and quantum pumps Adaman molors and douteum t  $\frac{1}{2}$ uantum motors and quantum pump

The main idea is to use the same operating principle as an electron pump, but in reverse The main idea is to use the same onerating principle as an base a control of the surface and the settle of the Thomas principie of a schematic of the Thomas principies o electron pump, but in reverse  $\epsilon$  main idea is to use the same operating principle as an  $\epsilon$ 

Chaotic QD pump **Thouless** pump  $\begin{array}{ccc}\n\text{Chanti ON numn} & \text{Thoulace numn}\n\end{array}$ 





The motor (wheel) The c Changes the shape of the contraction motor building on the contraction of  $\alpha$ the QD and the electrons on a chaotic quantum dot and  $\mathcal{L}$  and  $\mathcal{L}$  and  $\mathcal{L}$  and  $\mathcal{L}$ changes the shape of the QD

Inges the shape of The motors building time-periodic potential felt by quantum pump based on a chaotic quantum pump based on a change of  $\mathsf{Lip}$  and  $\mathsf{Lip}$  are changed on an and  $\mathsf{Lip}$  and  $\mathsf{Lip}$  are changed on an and  $\mathsf{Lip}$  and  $\mathsf{Lip}$  are changed on a change of  $\mathsf{Lip}$  and Thouless pump. When a voltage is applied to the pump, the The conveyor belt generates a



**Electrons are pumped through the device** Bectrons are pumped through the device

#### produce a rotation of the wheel. Alternatively, we could Quantum motors and quant such a motor is shown in Fig. 1(b). A single-channel control is shown in Fig. 1(b). A single-channel control i  $\overline{\phantom{a}}$ strength) originates from the adiabatic motion of, say, a  $s + 166$  motors and quentum pump Quantum motors and quantum pumps

### $q$ mechanical rotor degree of freedom. To operate this pump as Cha Chaotic QD pump **Thouless** pump



The voltage V induced a motion of the electrons that if adi $\frac{1}{6}$  $T$ cupe vinique a amouon or the creations that madee a force on The voltage V induced a motion of the electrons that induce a force on the (classical) mechanical degree of freedom  $\theta$ 

 $\tau$  become the second set of the set of the set of the society of the society  $\epsilon$  . The society of  $\epsilon$ The problem is treated within scattering theory formalism

**L S(θ R )** 



Here assume that the scattering matrix depends on several (classical) mechanical mechanical<br>motor degrees of freedom  $X_{\cdot\cdot}$ back to the left and right respectively. The original, rectangular blocks the original, rectangular blocks that  $\mathcal{N}_p$ motor degrees of freedom  $X_{\nu}$ the motor requires an international motor requires an international method in the set  $\mathbb{R}^n$ Here assume that the scattering matrix depends on several (classical) mechanical<br>motor degrees of freedom V degrees of freedom  $X_{..}$  $\mathcal{L}_{\mathcal{D}}$  and dependence on several model model

$$
S = S(\mathbf{X})
$$

 $T_{\text{C}}$  conservation in the matrix s is  $\frac{1}{2}$  in the matrix s is  $\frac{1$  $\mathbf{I}$ urs die presence of time-reversion matrix is also symmetric.  $\left( \frac{1}{1000} \right)$  in the presence of treedom [PRL  $\left( \frac{1}{107}, \frac{0}{0.05804} \right)$  (2011)] motor degrees of freedom can be expressed in terms of the Force exerted on mechanical degree of freedom [PRL 107<br> $\int d\epsilon = \int d\epsilon$ Retaining the dependence on several mode coordinates Force exerted on mechanical degree of freedom [PRL 107, 036804 (2011)]

$$
F_{\nu}(\mathbf{X}) = \sum_{\alpha} \int \frac{d\epsilon}{2\pi i} f_{\alpha} \operatorname{Tr} \left( \prod_{\alpha} S^{\dagger} \frac{\partial S}{\partial X_{\nu}} \right) \qquad \text{Projects onto scattering} \\ \text{channel in lead } \alpha
$$
\n
$$
\alpha = L, R \quad \epsilon \text{ : electron energy } f_{\alpha}(\epsilon) \text{ : Fermi distribution in lead } \alpha
$$

stribution in lead  $\alpha$ ! ıt le: ו<br>אי  $\alpha$  $\mathcal{H} = L, It \quad \mathbf{C}$  . Electron energy  $\int_{\partial \Omega} (\mathbf{c})$  . Fermi distribution in le denotes the conductance of the device for fixed X, the  $R$   $\epsilon$  : electron energy  $f_{\alpha}(\epsilon)$  : Fermi distribution in lead  $\alpha$  $\alpha$ ,  $\alpha$  $\alpha$  $\alpha=L,R\quad\epsilon$  : electron energy  $\ f_{\alpha}(\epsilon)$  : Fermi distribution in lead  $\alpha$ 

#### **Output Power** S matrix of the mesoscopic conductor  $\mathbf{C}$  $F_{\nu}(\mathbf{X}) = \sum$  $\alpha$  $\int d\epsilon$  $2\pi i$  $f_{\alpha}$  Tr $\left(\prod_{\alpha} S^{\dagger} \frac{\partial S}{\partial V}\right)$  $\partial X_{\nu}$  $\Gamma(\mathbf{v})$   $\mathbf{\nabla}$   $\int d\epsilon$   $\epsilon$   $\mathbf{\nabla}$   $\int \pi d\epsilon$  ${\color{red}\text{Output}}{\color{black} \text{Power}}$ **OUT POWER SHOWER SHOWER**  $\Gamma_{\nu}(\mathbf{A}) = \sum_{\alpha} \int \overline{2\pi i} \, \int \alpha^{-1} \left( \frac{\mathbf{I} \mathbf{I}_{\alpha} \mathbf{S} \cdot \overline{\partial X_{\nu}}}{\partial X_{\nu}} \right)$  $\int d\epsilon$  (i.e., force is conservative). In this case, force is case, for  $\partial S$   $\lambda$  $F_{\nu}(\mathbf{X}) = \sum \int \frac{d\boldsymbol{\epsilon}}{2\pi i} f_{\alpha} \operatorname{Tr} \left( \Pi_{\alpha} S^{\dagger} \frac{\partial S}{\partial X} \right)$

For nonzero voltage V, the force F is NOT conservative librium. Thus, the work performance performance  $\mathbf{r}$  performed by this force is force is the state is force is the state of  $\mathbf{r}$ or nonzero voltage v, th  $\alpha$ <br>orce F is NOT conservative onservative<br>. For nor ative<br>————————————————————

Wout <sup>i</sup>

 $\frac{1}{\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\left(\frac{1}{2}-\frac{1}{2}\right)}$ 

$$
W_{\text{out}} = \oint d\mathbf{X} \cdot \mathbf{F}(\mathbf{X}) \neq 0
$$

In linear response 
$$
W_{\text{out}} = \frac{ieV}{4\pi} \oint d\mathbf{X} \cdot \int d\epsilon f'(\epsilon) \text{Tr} \left[ (\Pi_L - \Pi_R) S^{\dagger} \frac{\partial S}{\partial \mathbf{X}} \right]
$$
  
This is proportional to the pumped charge into the  
system [PRB 58, R10135 (1998)]  
  
 $W_{\text{out}} = Q_p V$  and  $P_{\text{out}} = Q_p V / \tau$ 

The work performance performed by the adiabatic quantum motor performance  $\sigma$ 

 $\overline{\phantom{a}}$ 

### Efficiency and the slowly variable statement in the slowly state in the slowly sta  $\mathcal{L}(\mathbf{A}) = \mathbf{A} \mathbf{A}$ Efficiency of adiabatic quantum motors.—The applied as the ratio of output to input power For adiabatic motor degrees of freedom, the current I is made up of two contributions: the pumped charge and the pumped charge and the pumped charge and the pumped cha

The input power is given by the standard expression  $P_{\text{in}} = \bar{I}V$ on the motor contractions in the motor contractions in the motor contractions in the method in the method in t<br>Separate in the method in <br>  $P_{\text{in}} = IV$ line denotes an average over a single cycle.) The efficiency  $t = \frac{1}{\sqrt{2}}$  $\overline{\phantom{a}}$ adiabatic quantum motor. Thus,  $\frac{1}{\sqrt{2}}$ The input power is given by the standard expression  $\quad$   $\bm{l}$  $\mathcal{L} = \mathcal{L} \mathcal$  $\overline{\phantom{a}}$ 

the motor requires and  $\alpha$  in put  $\alpha$  in  $\alpha$  in  $\alpha$  in  $\alpha$  .

 $\mathcal{O}(\mathcal{O}(\log n))$  adiabatic quantum motor is the adiabatic quantum motor is then naturally defined in  $\mathcal{O}(\log n)$ as the ratio of output to the ratio of the ratio of output  $\alpha$ current induced by the voltage V  $\overline{\phantom{a}}$  of the adiabatic quantum motor is the adiabatic quantum motor is then naturally defined in  $\overline{\phantom{a}}$ urrent induced by the voltage V i kansleinung besteht bezeichnet.<br>I der gestigt besteht bezeichnet bezogstellt bezogstellt bezogstellt bezogstellt bezogstellt bezogstellt bezog<br>I der gestigt bezogstellt bezogstellt bezogstellt bezogstellt bezogstellt bez

 $The$  $\int P \left( \frac{P}{P} \right) dP = \int P \left( \frac{P}{P} \right) dP$ The efficiency of the motor is naturally defined as Retaining the dependence on several mode coordinates Here, filte, finder the fermion in the Fermi distribution in leaderships the Fermi of the Fermi of the Fermi o<br>The continuency of the filted in leadership denotes the conductance of the device for fixed X, the  $\frac{1}{2}$  also the public that the purpose of  $\frac{1}{2}$  also depend the voltage of voltage  $\frac{1}{2}$ 

$$
\eta = {P_{\rm out}}/{P_{\rm in}} = Q_p/\bar{I}\tau
$$

: De Define  $G(x)$  as the conductance of the device  $\mathcal{L}$  extens  $\mathcal{L}$  are the comparison motorities efficiency becomes

$$
G(\mathsf{x}) \text{ as the conductance of the device} \quad \longrightarrow \quad \bar{I} = \overline{G(\mathbf{X})}V + \frac{Q_p}{\tau}
$$

IV. (The over-

For adiabatic motor degrees of freedom, the current I is

$$
\eta = \frac{1}{1 + \bar{G}V\tau/Q_p}
$$
 Relation between the efficiency and the pumped charge!

 $\eta = \frac{1}{1 + \overline{G}V\tau/O_{\pi}}$  Relation between the efficiency and and the pumped charge!

QD pu ו<br>ו ا<br>I Gal effect<br>I Gffi  $QD$  p  $\mathsf{C}$ chanical effect  $\overline{1}$ QD pump is purely quantum mechanical effect

 $\longrightarrow$  $\longrightarrow$ **Note that in the absolute Separate Separa** the basis of quantum interference and become ineffective de to phase-breaking processes, in the term quantum processes,  $p$ 

conservative when the electronic conductor is out of equi-

ical effect and more reflect and thouless pump

I efficieny 1 for Fermi energy Note that the pumping current also depends on voltage on voltage on voltage on voltage on voltage on voltage o  $\bf 1$  below the gap

#### **Motor Dynamics**  $\mathbf{r}$ if the motor degree of  $\bf u$  $\blacksquare$ damping coefficient  $\mathbf{v}_1 \cup \mathbf{v}_2$  $\ln n$ analog yinamics, which y r Dynamics degree of the motor degree of  $\Gamma$ de priorities  $\sim$ (  $\alpha$  of the adiabatic quantum motor is the adiabatic  $\alpha$  of the adiabatic  $\alpha$  $\mathbf{I}$  the motor degree of  $\mathbf{I}$ **Motor Dynamics** motor follows from the (classical) condition  $\frac{1}{\sqrt{2}}$  $2 \frac{1}{2}$

Assume the (classical) mechanical degree of freedom is subject to damping γ Assume the (classical) mechanical degree of freedom is subject to damping  $\gamma$  $\frac{4}{3}$  $f$  freedom is subject to damping  $\gamma$  $T_{\rm eff}$  , we obtain for the motor  $T_{\rm eff}$  and  $T_{\rm eff}$  and  $T_{\rm eff}$   $\sim$   $\sim$ lassical) mechanical degree of freedom is subiect to damping v

$$
\gamma\dot{\theta}=\frac{\mathcal{Q}_pV}{2\pi}-F_{\text{load}}
$$

Cycle period of the motor  $\;\;\tau=\,2\pi/|\dot{\theta}|=(2\pi)^2\gamma/(Q_pV-2\pi F_{\rm load})$  $|\dot{\theta}| = (2\pi)^2 \gamma / (Q V - 2\pi F, )$  $|\theta| = (2\pi)^2 \gamma / (Q_p v - 2\pi F)_{0a}$  $d$  $T_{\text{max}}$  of the cycle period of the motor "  $\alpha$  $2\pi/|\dot{\theta}| = (2\pi)^2 \gamma/( \mathcal{Q}_p V - 2\pi F_{\rm load})$ denotes the conductance of the device for fixed X, the Cycle period of the motor  $\tau=z\pi$ or  $\tau = 2\pi/|\dot{\theta}| = (2\pi)^2 \gamma/(\Omega V - 2\pi F)$  $\begin{bmatrix} -2\pi r \ \text{load} \end{bmatrix}$  $\mathbf{d}$  $\dot{\theta} \rvert$  $\frac{1}{2}$  $\frac{1}{2} \gamma / (Q_p V - 2\pi)$  $\tau F_{\rm load}$ 

Use 
$$
\overline{I} = \overline{G(\mathbf{X})}V + \frac{Q_p}{\tau}
$$
  $\longrightarrow$   $\frac{1}{\tau} = \frac{Q_p \overline{I} - 2\pi F_{\text{load}} \overline{G}}{Q_p^2 + (2\pi)^2 \gamma \overline{G}}$ 

ductance G=0  $\longrightarrow$  1 ce G=0  $\longrightarrow$   $1/\tau = \overline{I}/Q_p$ Thouless i uless motor with conductand<br>U Thouless motor with conductance G=0  $\longrightarrow$   $1/\tau = \bar{I}/\mathcal{Q}_p$ in- and outgoing waves, respectively. This immediately. This immediately. This immediately  $\mathcal{L}$ d charge  $\sim$  $\mathcal{L}$ For an ideal Thouless motor with  $\mathcal{L}$ by the diese include with conductance of  $\sigma$  $\frac{1}{2}$ Thouless motor with conductance G=0  $\longrightarrow$   $1/\tau = \bar{I}/\mathcal{Q}_p$ factor that in this case, the entire current passing the entire current passing the device  $\mathcal{L}_\text{c}$  $m$  to pumping disperso.  $\blacktriangle$  $\frac{1}{2}$  ideal Thouless motor with G  $\frac{1}{2}$   $\frac{1}{2}$ factor that in this case, the entire current passing the entire current passing the device  $\mathcal{L}_\text{max}$ must be due to pumping. More generally, this remains a

Fortent is soldly due to pumped endige current is solely due to pumped charge current is solely due to pumped charge  $\sim$  $h = \ell$ good approximation as  $\mathcal{G}^2$  as  $\mathcal{G}^2$  as  $\mathcal{G}^2$  as  $\mathcal{G}^2$  as  $\mathcal{G}^2$  as  $\mathcal{G}^2$  as  $\mathcal{G}^2$ 

maximal load

$$
\longrightarrow \text{ maximal load} \qquad F_{\text{load}}^{\text{max}} = Q_p \bar{I}/2\pi \bar{G}
$$

is given by Fmax

p=ð2%Þ

 $\mathcal{I}=\mathcal{$ 

#### Thouless motor cal discussion. Consider a single-channel quantum wire single-channel quantum wire single-channel quantum wire  $\alpha$ mechanical rotor degree of freedom. To operate this pump as a motor, an applied bias voltage produces a charge quantum wire is located next to a convey of the conveyor belt with the conveyor belt with the conveyor belt with  $T_{\rm eff}$  and  $T_{\rm eff}$  acting for  $T_{\rm eff}$  and  $T_{\rm eff}$  acting for  $T_{\rm eff}$  acting for  $T_{\rm eff}$ analogy with the time-evolution operator in  $\mathbb{R}^n$  with the time-evolution operator in  $\mathbb{R}^n$ conductance vanishes while the pumped charge is  $\mathbf{r}$ Here, the '<sup>i</sup> denote the Pauli matrices in the space of the counterpropagation channels. We do not include the real propagation of  $\Gamma$ Motor dynamics.—The output power of a quantum motor depends on its dynamics through the cycle period  $\blacksquare$  $\det$ counterpropagating channels. We do not include the real

Let us investigate the Thouless motor in more detail us investigate the Thouless motor Let us investigate the Thouless motor in more d  $\begin{array}{c}\n\downarrow \\
\downarrow \\
\downarrow\n\end{array}$ tiga<sup>.</sup> o the Thouless motor in more detail Let us investigate the Thouless motor in more detail  $\overrightarrow{a}$ Motor dynamics.—The output power of a quantum  $\overline{\phantom{a}}$  , which is a single matrix  $\overline{\phantom{a}}$ the driving force and the load Fload acting on the angular Let us investigate the Thouless motor in moi



$$
\mathcal{H} = p^2/2m + 2\Delta \cos(2\pi x/a + \theta)\Theta(L/2 - |x|)
$$
  
Periodic chemical potential

Choose Fermi wavevector close to  $\quad k_0 = \pi/a$ first assume that there is only an outgoing wave on the  $\mathcal{A}$ CHOOSE FEITH WAVEVECTOR CHOSE TO  $k_0 = \pi/a$ of the motor. (This is realized for Thouless motors but Choose Fermi wavevector close to  $\;\; k_0 = \pi/a$  $\sigma$  as  $\overline{\sigma}$ mi wavevector close to  $k_0=\pi$ 

A Hamiltonian can be linearlized<br>
This results in an effective Hamiltonian can be linearlized  $a \int f(x) dx$  as the method  $a \int f(x) dx$  as the term of the  $\sigma = v_F \rho \sigma^2 + \Delta(\sigma^2 \cos \theta + \sigma^2 \sin \theta)$  $\mathcal{L}_{\text{max}}$  the chemical potential  $\mathcal{L}_{\text{max}}$  $\rightarrow$  act in space or counterpropagating chan  $\mathcal{H} = u \cdot n\sigma^z + \Lambda(\sigma^x \cos\theta + \sigma^z)$  $\mathcal{L}_{FP}$  berefore potential. Measuring momenta from  $\mathcal{L}_{FP}$ <sup>0</sup>=2m, one has  $\mathcal{H} = v_F \rho \sigma^z + \Delta(\sigma^x \cos \theta + \sigma^y \sin \theta) \Theta(L/2 - |x|)$  $\sum_{i=1}^n$  is  $\sum_{i=1}^n$  is the charge is the charge is  $\sum_{i=1}^n$ nterpropagating channels  $\mathbf P$  . Homitanian can be linearlized act in space of counterpropagating channels Mðor<br>1980 - Heimann Stoffen, fyrstur er fyrstu er fyrst<br>1980 - Fyrstu er  $\rightarrow$  and outgoing waves to the and  $\sim$ tion S11 is  $\frac{11}{\sqrt{2}}$  is  $\frac{1}{\sqrt{2}}$  is and the space of counter  $\blacksquare$ If the motor degree of freedom is subject to damping with  $\blacksquare$  $\rightarrow$  frampformal can be integritized 2 acc in space of cod  $\mathcal{O}(L/\mathcal{Z} - |\mathcal{X}|)$ lann  $\mathbf{r}$  $\longrightarrow$ -<br>Hamiltonian can be linearlized  $\epsilon$  $\sim$  1 normeoman can be integrated  $\mathcal{H} = v_F \rho \sigma^z + \Delta(\sigma^x \cos \theta + \sigma^y \sin \theta) \Theta$ first assume that there is only an outgoing wave on the  $\mathcal{L}_{\text{max}}$ right. Then, the left is on the left is on the lef act in space or counterprop  $\mathcal{F}_{FP}$  $\frac{1}{2}$  . This yields as the there is only an outgoing wave on the there is only an outgoing wave on the theorem  $V = \Omega Q(L/2)$  $1-2.$  Note that '2. Note  $\mathcal{F} \sin \theta$   $\Theta$  ( $L/2 = |x|$ ) counterpropagating channels T X Ö

the scattering matrix can then be determined  $\mathbf{0}$  $\alpha$  $\overline{a}$ motor depends on its discriming the control of the control in the control in the control in the control in the<br>Separate period in the control in t<br> for the scattering matrix can then be determined For the seationing matrix can their be determined A formula for the scattering matrix can then be determined  $\alpha$  formula for the scattering matrix can then be determined. A formula for the scattering matrix can then be dete  $T$  in equal behavior as  $T$  if  $\mathcal{L}$  as  $\mathcal{L}$  if  $\mathcal{L}$  sin(L, where  $\mathcal{L}$ A formula for the scattering matrix can then be determined for the seatiering matrix can then be determined for the scattering matrix can then be determined

$$
S = \frac{1}{M_{11}} \begin{pmatrix} -ie^{i\theta} \frac{\Delta}{\sqrt{E^2 - \Delta^2}} \sin \lambda_L & 1 \\ 1 & -ie^{-i\theta} \frac{\Delta}{\sqrt{E^2 - \Delta^2}} \sin \lambda_L \end{pmatrix} \frac{\lambda_L}{M_{11}} = \frac{-(L/hv_F)[E^2 - \Delta^2]^{1/2}}{\cos \lambda_L - i(E/\sqrt{E^2 - \Delta^2}) \sin \lambda_L}
$$

## Thouless motor  $T_{\text{beam}}$ Fermi energy EF takes the formulation of the formulation  $\mathbf{F}$

From the expression for the scattering matrix, we determine the conductance<br>(Landauer) (Landauer) From t for the efficiency of the Thouless motor.  $Fr$  $\frac{1}{2}$ the condu

$$
G = \bar{G} = \frac{e^2}{h} \frac{|\Delta^2 - E_F^2|}{|\Delta^2 - E_F^2|\cos^2 \lambda_L + E_F^2|\sin \lambda_L|^2}
$$

 $\frac{1}{2}$  acting for  $\frac{1}{2}$  $From$  $\overline{\mathsf{I}}$  of tre In article that the factor and the factor with the periodic potential the potential opensy of the periodic pot<br>In a the periodic potential opensy of the periodic potential opensy of the potential opensy of the potential o of freedom [PRB 58, R10135 (1998)] we obtained the pumped charge jef and becomes oscillatory and finite in L for  $\mathcal{L}_{\mathcal{F}}$  for  $\mathcal{L}_{\mathcal{F}}$ From Brouwer's formula at zero temperature for an angular mechanical degree  $\overline{\phantom{a}}$  Fig. 1  $\overline{\phantom{a}}$ Prouver's fermule at zero temperature fer an engular meebenies Brouwer's formula at zero temperature for an angular mechanical degree<br>Nom IPPP 58, P10125 (1998)] We obtained the numped charge

$$
Q_p = \frac{e\Delta^2 |\sin\lambda_L|^2}{|\Delta^2 - E_F^2|\cos^2\lambda_L + E_F^2|\sin\lambda_L|^2}
$$

 $and t$ 'k0. This results in an effective Hamiltonian H with counterpropagating linear channels and backscattering  $\mathbf{F}_{\mathbf{r}}$  for  $\mathbf{r}$  for  $\mathbf{r}$  the gap, the charge pumped is the gap, the charge pumped is  $\mathbf{r}$ and thus the efficieny and thus the efficieny  $\mathbf{p}$ 

nus the efficiency  
\n
$$
\eta = \frac{1}{1 + \frac{2\pi\gamma}{\hbar} \frac{|E_F^2 - \Delta^2|}{\Delta^4 |\sin \lambda_L|^4} \left[ |E_F^2 - \Delta^2| \cos^2 \lambda_L + E_F^2| \sin \lambda_L|^2 \right]}
$$



# **Conclusions**