# Superconducting Proximity Effect in Silicene: Spin-Valley Polarized Andreev Reflection, Non-Local Transport, and Supercurrent

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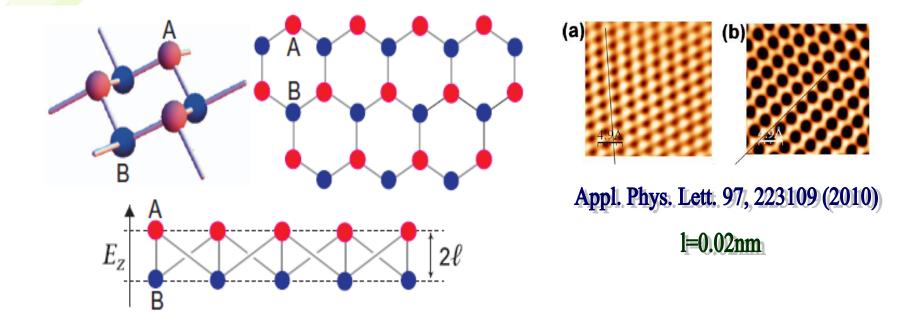
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# Outline of the talk

- Introduction to Silicene
- **◆** Proximity effect in Silicene
- **♦** Conclusion

## Introduction to Silicene

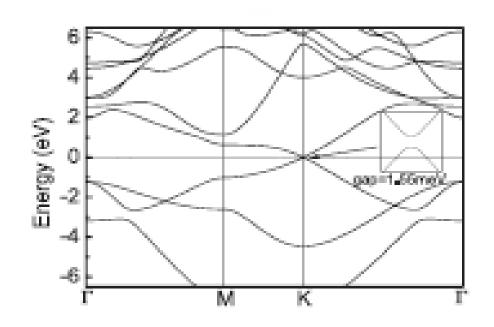


**Figure 1.** Illustration of the buckled honycomb lattice of silicene. A honeycomb lattice is distorted due to a large ionic radius of a silicon atom and forms a buckled structure. The A and B sites form two sublattices separated by a perpendicular distance  $2\ell$ . The structure generates a staggered sublattice potential in the electric field  $E_z$ , which leads to various intriguing pheneomena.

## **Model Hamiltonian**

$$\begin{split} H = -t \sum_{\langle i,j \rangle \alpha} c^{\dagger}_{i\alpha} c_{j\alpha} + \mathrm{i} \frac{\lambda_{\mathrm{SO}}}{3\sqrt{3}} \sum_{\langle \langle i,j \rangle \rangle \alpha\beta} \nu_{ij} c^{\dagger}_{i\alpha} \sigma^{z}_{\alpha\beta} c_{j\beta} - \mathrm{i} \frac{2}{3} \lambda_{\mathrm{R}} \sum_{\langle \langle i,j \rangle \rangle \alpha\beta} \mu_{ij} c^{\dagger}_{i\alpha} (\vec{\sigma} \times \vec{d}^{0}_{ij})^{z}_{\alpha\beta} c_{j\beta} \\ + \ell \sum_{i\alpha} \zeta_{i} E^{i}_{z} c^{\dagger}_{i\alpha} c_{i\alpha}. \end{split}$$

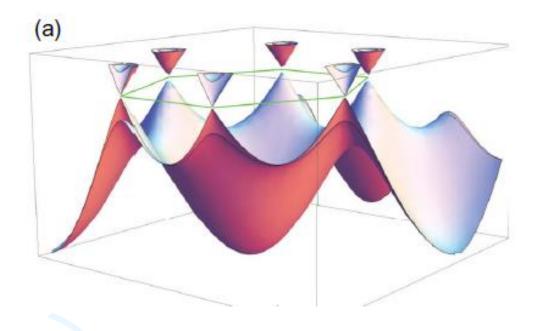
$$t = 1.6eV$$
  
 $\lambda_{SO} = 3.9meV$   
 $\lambda_{R} = 0.7meV$   
 $l = 0.23\mathring{A}$ 



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## Low energy Dirac theory

$$\mathcal{E}_{\eta} = \pm \sqrt{\hbar^2 v_{\rm F}^2 k^2 + \left(\ell E_z - s \sqrt{\lambda_{\rm SO}^2 + a^2 \lambda_{\rm R}^2 k^2}\right)^2} \qquad s = \eta s_z$$



**Figure 3.** Band structure of silicene at the critical electric field  $E_c$ . (a) A bird's-eye view. Dirac cones are found at six corners of the hexagonal Brillouin zone.

#### For k=0

$$\Delta (E_z) = 2 |\ell E_z - \eta s_z \lambda_{SO}|$$

$$E_{\rm c} = \lambda_{\rm SO}/\ell = 17 \,{\rm meV \, \AA}^{-1}$$
.

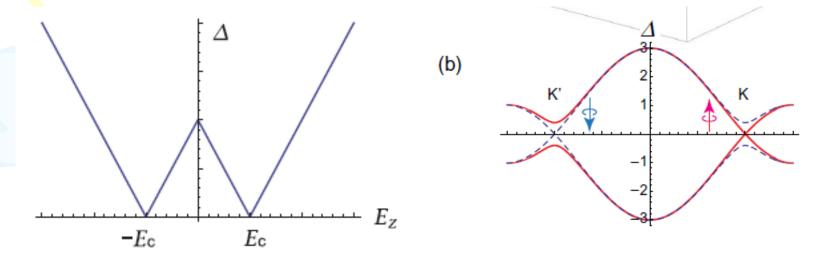
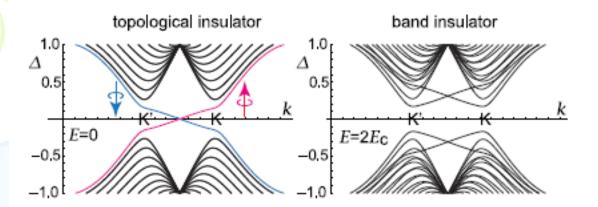


Figure 2. The band gap  $\Delta$  as a function of the electric field  $E_z$ . The gap is open for  $E_z \neq \pm E_c$ , where silicene is an insulator. It can be shown that it is a topological insulator for  $|E_z| < E_c$  and a band insulator for  $|E_z| > E_c$ .

## Silicene nanoribbon

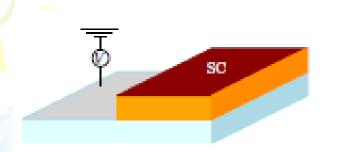


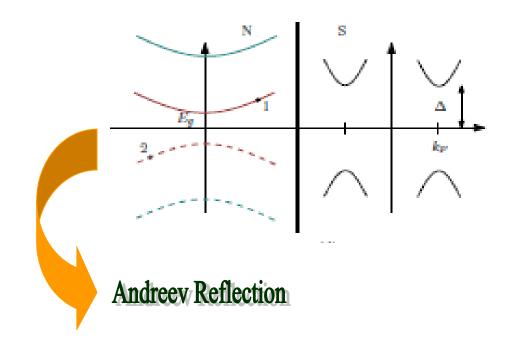
**Figure 4.** 1D energy bands for a silicene nanoribbon. (a) The bands crossing the gap are edge states, demonstrating that it is a topological insulator. There are two edge states since a nanoribbon has two edges (red and blue lines for the left and right edges). (b) All states are gapped, demonstrating that it is a band insulator.

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# Proximity effect in Silicene

## **NS Junction**





## Hamiltonian

$$\begin{split} H &= -t \sum_{\langle i,j \rangle,\alpha} c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{\mathrm{i}\lambda}{3\sqrt{3}} \sum_{\langle \langle i,j \rangle \rangle,\alpha,\beta} \mathsf{v}_{ij} c^{\dagger}_{i\alpha} \sigma^z_{\alpha\beta} c_{j\beta} \\ &+ l \sum_{i\alpha} \zeta_i E^i_z c^{\dagger}_{i\alpha} c_{i\alpha} - \mu \sum_{i\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} + \sum_{i\sigma} (\sigma \Delta_0 c^{\dagger}_{i\sigma} c^{\dagger}_{i,-\sigma} + \mathrm{h.c.}) \end{split}$$

## Low energy Spectrum

$$E_{\eta,\sigma}(\mathbf{k}) = \pm \sqrt{\left(\sqrt{k^2 + (lE_z - \eta \sigma \lambda_{SO})^2} \pm \mu_S\right)^2 + |\Delta_0|^2}.$$

#### At Interface x=0

$$\begin{split} \psi_{N} &= \frac{1}{\sqrt{2E\tau_{+}}} [\eta k_{F} \mathrm{e}^{\mathrm{i}\eta\theta}, \tau_{+}, 0, 0] + \frac{r_{e}}{\sqrt{2E\tau_{+}}} [-\eta k_{F} \mathrm{e}^{-\mathrm{i}\eta\theta}, \tau_{+}, 0, 0] \\ &+ \frac{r_{h}}{\sqrt{2E\tau_{-}}} [0, 0, \eta k_{F} \mathrm{e}^{-\mathrm{i}\eta\theta}, \tau_{-}], \\ \psi_{S} &= \frac{t_{e}}{\sqrt{2}} [\eta \mathrm{e}^{\mathrm{i}\eta\theta_{S}} u_{+}, u_{+}, \eta \mathrm{e}^{\mathrm{i}\eta\theta_{S}} u_{-} \mathrm{e}^{-\mathrm{i}\phi}, u_{-} \mathrm{e}^{-\mathrm{i}\phi}] \\ &+ \frac{t_{h}}{\sqrt{2}} [-\eta \mathrm{e}^{-\mathrm{i}\eta\theta_{S}} u_{-} \mathrm{e}^{\mathrm{i}\phi}, u_{-} \mathrm{e}^{\mathrm{i}\phi}, -\eta \mathrm{e}^{-\mathrm{i}\eta\theta_{S}} u_{+}, u_{+}]. \end{split}$$
(3)

#### Conductance

$$G/G_N = \frac{1}{4} \sum_{\eta,\sigma} \int_{-\pi/2}^{\pi/2} d\theta \cos\theta (1 + |r_{\eta,\sigma}^h|^2 - |r_{\eta,\sigma}^e|^2)$$

### Normal incidence $\theta = 0$

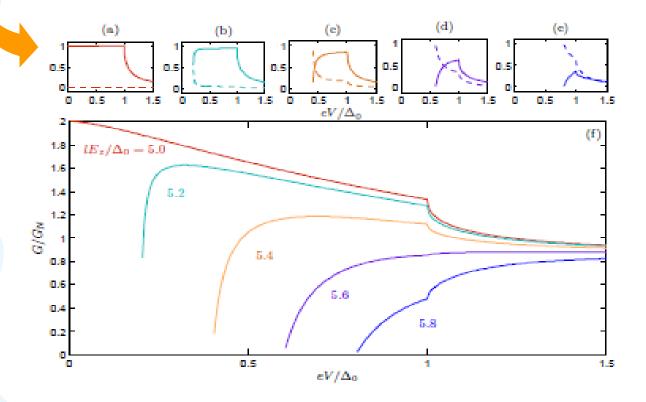
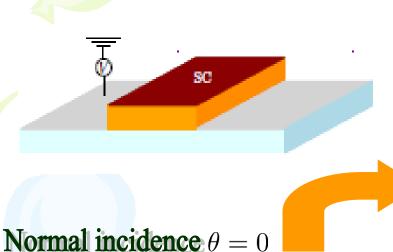


FIG. 2: (Color online) (a)-(e): Normal (dashed lines) and Andreev reflection (full lines) probabilities for an N|S junction with  $\eta = \sigma = +1$ ,  $\lambda_{SO}/\Delta_0 = 5.0$ , and  $lE_z/\Delta_0$  ranging from 5.0 to 5.8 from (a) to (e). In (f), the conductance  $G/G_N$  (averaged over spin and valleys) is plotted vs. bias voltage for the same choices of  $lE_z/\Delta_0$ .

## **NSN Junction**



$$(a)m_L = 0; m_R \neq 0$$

$$(b)m_L \neq 0; m_R \neq 0$$

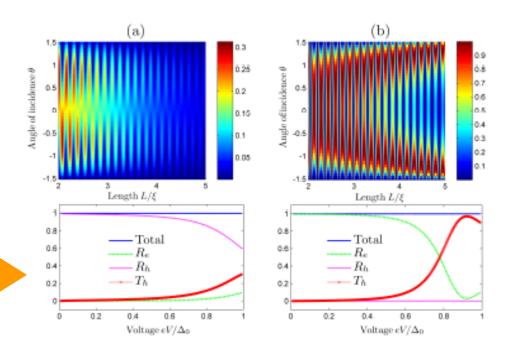


FIG. 3: (Color online) *Top row:* Contour-plot of the CAR probability for a bias voltage  $eV/\Delta_0=0.9$  vs. angle of incidence  $\theta$  and junction length L. Bottom row: Probabilities for the different scattering events for a fixed junction length  $L/\xi=2.1$  for normal incidence. In column (a), we consider scenario (i) as described in the main text (non-gapped/superconductor/gapped) whereas in (b) we consider scenario (ii) as described in the main text (gapped/superconductor/gapped). We have considered in all cases a strongly doped superconducting region with  $\mu_S/\Delta_0=20$  and set  $m_L/\Delta_0=0$  and  $m_R/\Delta_0=5$  in (a) whereas  $m_L/\Delta_0=m_R/\Delta_0=5$  in (b). The coefficients  $(R_e,R_h,T_h)$  are the probabilities for normal, Andreev, and crossed Andreev reflection, respectively.

#### onductance

$$\frac{G_{\rm nl}}{G_0} = \frac{1}{4} \sum_{\eta,\sigma} \int_{-\pi/2}^{\pi/2} {\rm d}\theta \mathcal{P}_h |t_h|^2 \sqrt{q_F^h - k_y^2},$$

$$\mathcal{P}_h = 1/(E - \mu_R), \quad q_F^h = \sqrt{(\mu_R - E)^2 - m_R^2}$$

$$k_y = k_L \sin \theta$$

0.6

0.8

### Non local current is spin valley polarized

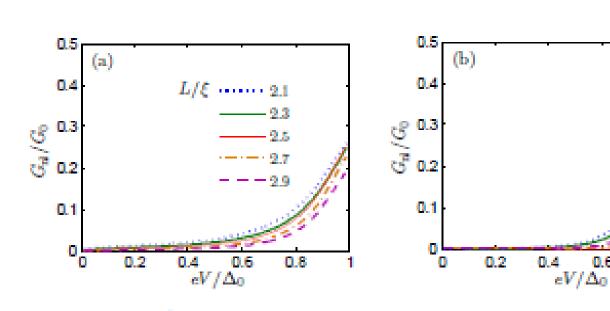


FIG. 4: (Color online) Non-local conductance for (a) gapless and (b) gapped silicene on the left side [corresponding to scenarios (i) and

## **SNS Junction**

$$\frac{\varepsilon(\Delta\phi)}{\Delta_0} = \pm \sqrt{\frac{4\mathcal{M}^2\cos^2(\Delta\phi/2) + L^2(\mathcal{M}^2 - k^2)^2}{4\mathcal{M}^2 + L^2(\mathcal{M}^2 - k^2)^2}} \qquad L \ll \xi$$

$$\mathcal{M} = \mu_N + (\eta \sigma \lambda_{SO} - lE_z)$$

## Josephson Current T~0

$$\frac{I(\Delta\phi)}{I_0} = \sum_{\eta\sigma} \frac{\mathcal{M}^2 \sin \Delta\phi}{[4\mathcal{M}^2 + L^2(\mathcal{M}^2 - k^2)^2] \epsilon(\Delta\phi)}.$$



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# Conclusions



- Tunable nonlocal current and supercurrent by electric field unlike graphene
- Interesting to study transport through other junctions with silicene like NIS, NMS etc

